Notes on a family of preimage resistant functions

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Security requirements

- Preimage resistance
- Second preimage resistance
- Collision resistance

The old construction

Let $P(X) \in \mathbb{Z}[X]$ be a fixed monic polynomial of degree $n \geq 3$ having no multiple roots. Denote by $\alpha_1, \ldots, \alpha_n$ the roots of P and put

$$L_i(\underline{X}) := \sum_{j=1}^{m} \alpha_i^{j-1} X_j \quad for \quad i = 1, \ldots, n \quad and \quad m \leq n.$$

Define the norm form corresponding to the polynomial P by

$$\mathcal{N}_P(\underline{X}) := \prod_{i=1}^{n} L_i(\underline{X}).$$

Aumassons attack

- Non-standard notion of collision resistance
- Iterated scheme
- Circulant matrices
- Implementation flaws

New construction

Theorem 1 *Let* $f(\underline{X}) \in \mathbb{F}_q[X_1, \dots, X_m]$ *be a polynomial such that*

$$f(\underline{X}) := b(X_1, \dots, X_m) + a(X_1, \dots, X_m)$$

with homogeneous polynomials $a(\underline{X}), b(\underline{X})$ *satisfying* $k = \deg a(\underline{X}) < \deg b(\underline{X}) = n$, $\deg_{X_i} b(\underline{X}) = n$ *for* $1 \leq i \leq m$. *Further, s uppose that there exist indices* $1 \leq j_1 < j_2 \leq n$ *such that the binary form*

$$b_0(X_{j_1}, X_{j_2}) := b(0, \dots, 0, X_{j_1}, 0, \dots, 0, X_{j_2}, 0, \dots, 0) \tag{1}$$

has no multiple zero.

Let $N(f, \gamma, q)$ *denote the number of solutions of the equation* $f(x_1, \dots, x_m) = \gamma$ *in* $x_1, \dots, x_m \in \mathbb{F}_q$. *Then*

$$|N(f, \gamma, q) - q^{m-1}| \leq (n-1)(n-2)q^{m-3/2} + 5n^{13/3}q^{m-2}. \tag{2}$$

Moreover, if $q > 15n^{13/3}$, *then*

$$|N(f, \gamma, q) - q^{m-1}| \leq (n-1)(n-2)q^{m-3/2} + (5n^2 + n + 1)q^{m-2}. \tag{3}$$

Practical considerations

Lemma 1 *Let* $f(\underline{X}) := b(\underline{X}) + a(\underline{X})$ *such that* $b(\underline{X}) = \beta_1 X_1^r + \cdots + \beta_m X_m^r$*,* $a(\underline{X}) = \alpha_1 X_1^s + \cdots + \alpha_m X_m^s$ *and* $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_m \neq 0$*. If* $0 < s < r < q$ *and* r *is odd if* $q = 2^f$*, then* $f(\underline{X})$ *satisfies all assumptions of Theorem 1.*

Practical considerations

- Odd characteristic arithmetic
- Even characteristic arithmetic

Strict avalanche criterion

If a function is to satisfy the strict avalanche criterion, then each of its output bits should change with a probability of one half whenever a single input bit is complemented.

Asymptotic behavior

Theorem 2 *Let us define* $f \in \mathbb{F}_{2^k}[x_1, \ldots, x_m]$ *as* $f(x_1, \ldots, x_m) = \sum_{i=1}^{m} \alpha_i x_i^n + \sum_{i=1}^{m} \beta_i x_i$ *where* $n = 2^l + 1$ *such that* $(l, k) = 1$*. Then*

$$(1 - q\varepsilon)^{m-1}\left(\frac{1}{q} - \varepsilon\right) \leq$$

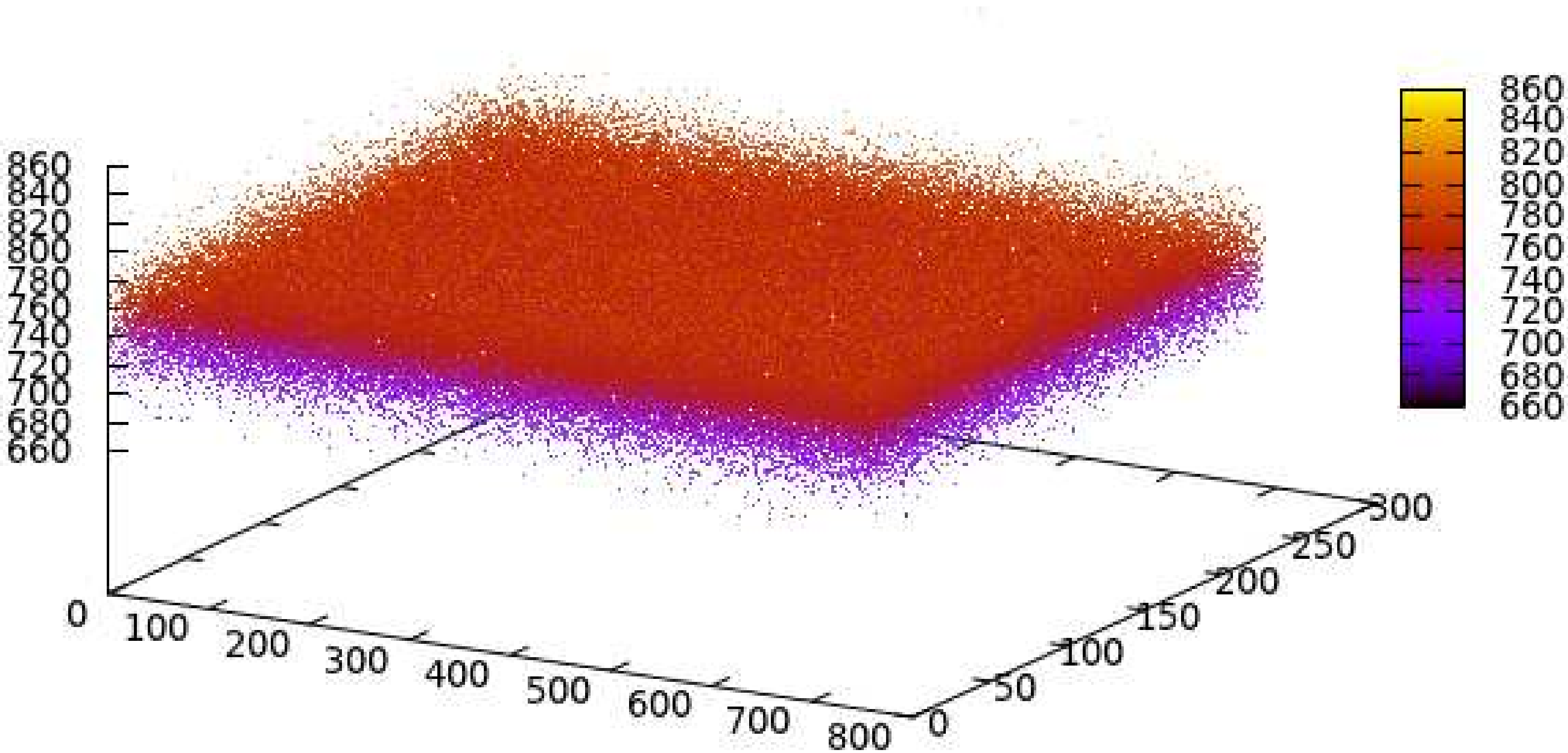
$$P(f(x_1, \ldots, x_m) - f(x_1 + \delta_1, \ldots, x_m + \delta_m) = \gamma)$$

$$\leq (1 + q\varepsilon)^{m-1}\left(\frac{1}{q} + \varepsilon\right)$$

where $0 \leq \varepsilon \leq (q - n)q^{-\frac{3}{2}}$*.*

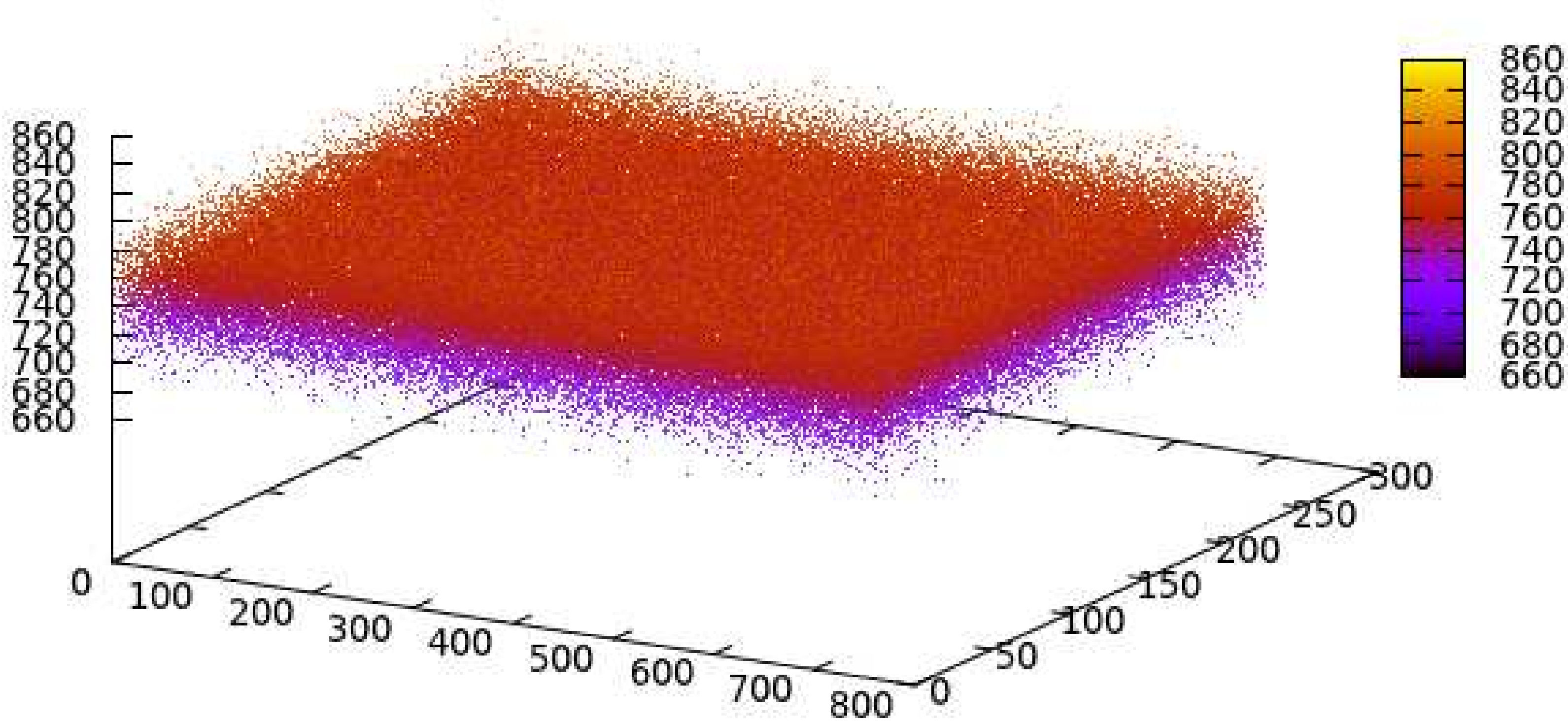
Test results 1.

Base



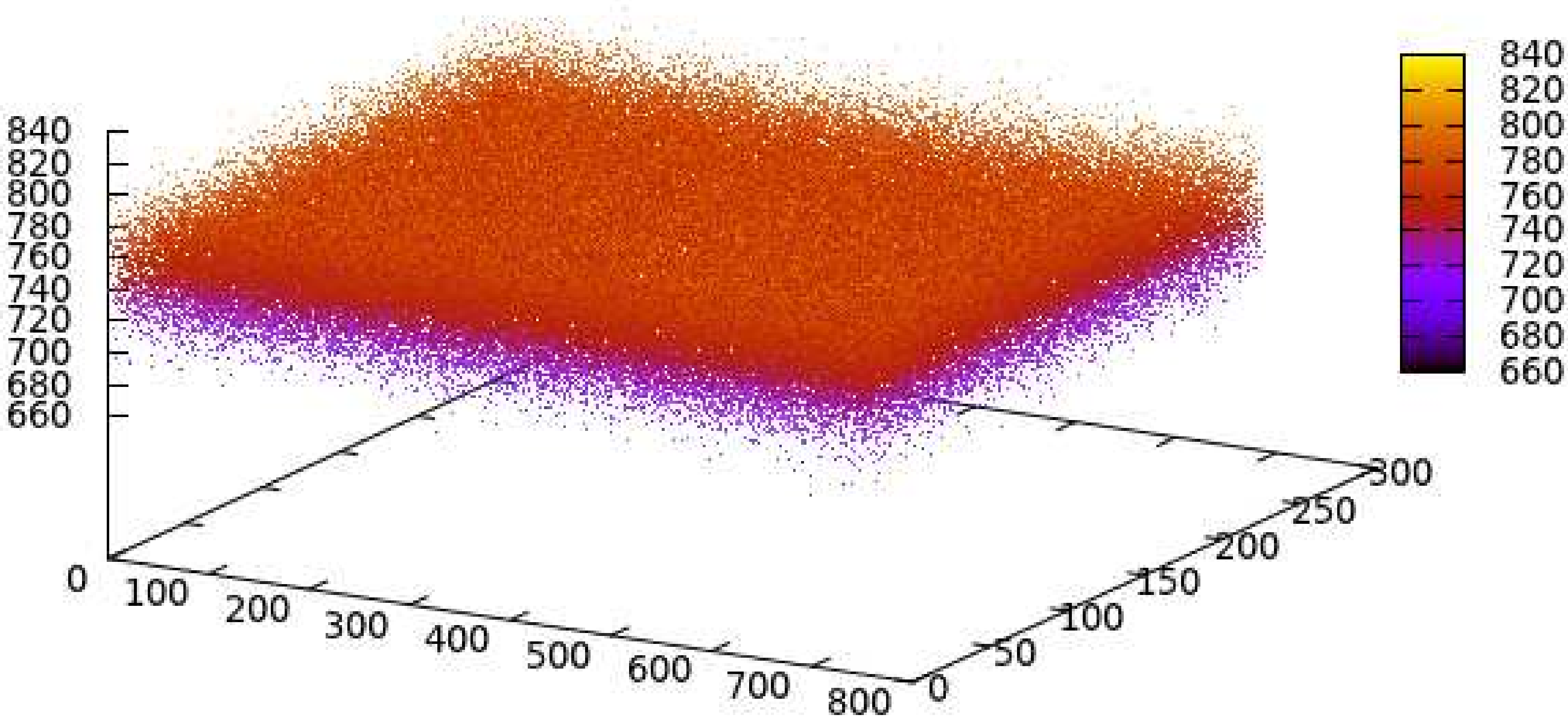
Test results 2.

Coefficients changed



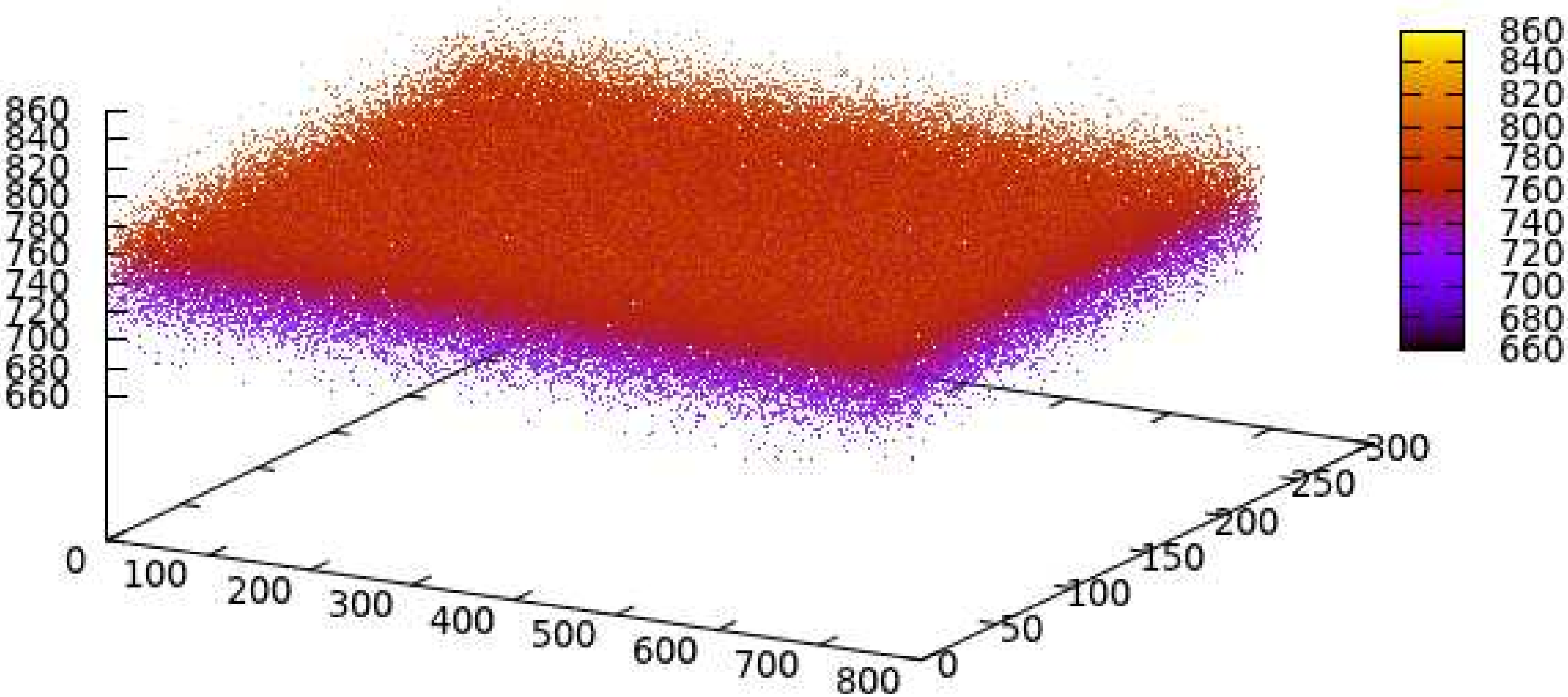
Test results 3.

Small exponent



Test results 4.

Low weight exponent



Thank you for your attention!

References

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