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1 Introduction

Finite automata and regular languages are one of the oldest topics in formal languages theory. The basic properties of this class of languages were investigated in 1950s and 1960s. Although regular languages are the simplest languages in Chomsky hierarchy, some challenging problems are still open. The most famous is the question of how many states are sufficient and necessary for two-way deterministic automata to simulate two-way nondeterministic automata, which is connected to the well-known NLOGSPACE vs. DLOGSPACE problem [1].

In last three decades, we can observe a new interest in regular languages which have applications in software engineering, programming languages, and other areas of computer science. However, they are also interesting from the theoretical point of view [22]. Various properties of this class are now intensively studied. One of them is descriptional complexity which studies the cost of description of languages by formal systems such as deterministic and nondeterministic automata, or grammars.

Rabin and Scott in 1959 [31] defined nondeterministic finite automata (NFAs), described an algorithm known as the "subset construction" which shows that every n-state nondeterministic automaton can be simulated by at most 2^n-state deterministic finite automaton (DFA). In 1962 Yershov [33] then showed that this construction is optimal. Maslov [29] investigated the state complexity of union, concatenation, and star, and also some other operations. Birget in [2, 3] examined intersection and union. He also considered the question of the size of nondeterministic automaton for the complement of a language. The complement of a formal language L over an alphabet Σ is the language $L^c = \Sigma^* \setminus L$, where Σ^* is the set of all strings over an alphabet Σ. The complementation is an easy operation on regular languages represented by deterministic finite automata (DFAs) since to get a DFA for the complement of a regular language, it is enough to interchange the final and non-final states in a DFA for this language.

On the other hand, complementation on regular languages represented by NFAs is an expensive task. We first must apply the subset construction to a given NFA, and only after that, we may interchange the final and non-final states. This gives an upper bound 2^n.

Sakoda and Sipser [32] presented an example of languages over a growing alphabet size meeting this upper bound. Birget claimed the result for a three-letter alphabet in [3], and later corrected this to a four-letter alphabet. Holzer and Kutrib [20] obtained the lower bound 2^{n-2} for a binary n-state NFA language. Finally, binary n-state NFA languages meeting the upper bound 2^n were described by Jirásková in [24]. In the case of a unary alphabet, the complexity of complementation is in $e^{\Theta(\sqrt{n \ln n})}$ [20, 24].

Birget [2] described a lower-bound technique for proving minimality of NFAs. The technique is known as a fooling-set method. Although in some cases there is a large gap between the size of a fooling set and the size of minimal nondeterministic automaton [23], in a many other cases, the fooling sets can be used to prove the minimality of nondeterministic machines, and we successfully use this method throughout our thesis.

The systematic study of the state complexity of operations on regular languages began in the paper by Yu et al. [34]. The nondeterministic state complexity of operations was investigated by Holzer and Kutrib [20], and some improvements of their results can be found in [24]. Some special operations were examined as well: proportional removals in [10], shuffle in [8], and cyclic shift in [26].

Recently, researchers investigated subclasses of regular languages such as, for example, prefix- and suffix-free languages [9, 15, 18], ideal languages [5], closed languages [6], bifix-, factor-, and subword-free languages [4], union-free languages [25], or star-free languages [7]. In some of these classes, the operations have smaller complexity, while in the others, the complexity of operations is the same as in the general case of regular languages.

Prefix-free languages are used in codes like variable-length Huffman codes or country calling codes. In a prefix-code, there is no codeword which is a proper prefix of any other codeword. Therefore, a receiver can identify each codeword without any special marker between words. This was a motivation for investigating this class of languages in last few years [11, 12, 17, 19, 28].

The non-deterministic state complexity of operations on prefix-free and suffix-free languages was studied by Han *et al.* in [15–17, 19]. For the nondeterministic state complexity of complementation, they obtained an upper bound $2^{n-1} + 1$ in both classes, and lower bounds 2^{n-1} and $2^{n-1} - 1$ for prefix-free and suffix-free languages, respectively. The questions of tightness remained open. In the first part of this thesis, we solve both of these open questions, and we prove that in both classes, the tight bound is 2^{n-1}. To prove tightness, we use a ternary alphabet. Hence the nondeterministic state complexity of complementation on prefix- or suffix-free languages defined over an alphabet that contains at least three symbols is given by the function 2^{n-1}. We also show that this upper bound cannot be met by any binary prefix- or suffix-free language. We get a similar result in the class of factor-free languages, and moreover we obtain the tight upper bounds on the nondeterministic complexity of each considered operation in each of the four free classes. We also study the unary free languages, and, besides some other results, we prove that the nondeterministic state complexity of complementation is in $\Theta(\sqrt{n})$ in the each of the four classes of free languages.

Then we deal with the operations of intersection, union, concatenation, star, reversal, and complementation on prefix-, suffix-, factor-, and subword-closed languages, and on right (left, two-sided, and all-sided) ideal languages. In all cases, we get tight upper bounds on the nondeterministic complexity for all operations. Except for three cases, our witnesses are defined over small fixed alphabets.

Finally, we use our results to show that the nondeterministic complexities of basic regular operations, except for complementation, in the classes of prefix-, suffix-, factor-, and subword-convex languages are the same as in the general case of regular languages. As for complementation, the complexity in the class of suffix-convex languages is 2^n which is one of the most interesting results of this thesis.

2 Aims

1. Summarize known results concerning deterministic and nondeterministic complexity of basic operations in the class of regular languages and its subclasses.

2. Investigate properties of nondeterministic finite automata accepting languages in some special subregular classes (prefix-, suffix-, factor-, and subword-free languages, closed languages, convex languages and ideal languages).

3. Use the properties of nondeterministic automata to get nondeterministic complexity of operations union, intersection, concatenation, star, reversal and complementation in above mentioned subregular classes.

3 Main results

We present our main results on nondeterministic state complexity of basic regular operations as union, intersection, concatenation, (Kleene) star, reversal, complementation in subclasses of regular languages. We consider these subclasses: prefix (suffix, factor, subword) - free (closed, convex) and right (left, two-sided, all-sided) ideals.

For finding upper bounds we use properties of languages belonging to mentioned subclasses. For lower bounds we use very useful tools for estimation the number of states in NNFAs based on fooling set techniques [2, 3, 14, 21]. Recall that set of pairs of strings $\{(u_1, v_1), (u_2, v_2), \ldots, (u_n, v_n)\}$ is called a *fooling set* for a language L if for all i, j in $\{1, 2, \ldots, n\}$,

(F1) $u_i v_i \in L$,

(F2) if $i \neq j$, then $u_i v_j \notin L$ or $u_j v_i \notin L$.

Lemma 3.1 ([2, Lemma 1], **Lower bound method for NNFAs**). *Let $\mathcal{F}$ be a fooling set for a language L. Then every NNFA for the language L has at least $|\mathcal{F}|$ states.*

We use more variants of this method.

3.1 Free languages

A language is prefix-free if it does not contain two distinct strings such that one of them is a prefix of the other. Suffix-, factor-, and subword-free languages are defined analogously. We use the notion of a free language for a language belonging to one of these four classes.

Table 1 provides an overview of complexities of operations on unary-free languages and compares them to the known results on regular unary languages from [20]. Notice that the exact complexity of concatenation in the case of regular languages is still not known. Table 2 summarizes our results on the nondeterministic complexity of operations on prefix-, suffix-, factor-, and subword-free languages and compares them to the results on regular languages which are from [20,24]. Notice that the complexity of each operation in each class is always smaller than in the general case of regular languages, except for the reversal operation on suffix-free languages. All our wittnes languages are defined over small fixed alphabet which are always optimal, except for intersection and complementation on subword-free languages where it remains open whether the upper bounds can be met by subword-free languages defined over smaller alphabets. We conjecture that the bound mn is assymptotically tight for intersection of binary subword-free languages.

We assume nondeterministic complexity of languages K, L are m, n, respectively in the next tables.

	$K \cap L$	$K \cup L$	KL	L^*	L^c
Unary free	$m = n$	$\max\{m, n\}$	$m + n - 1$	$n - 1$	$\Theta(\sqrt{n})$
Unary regular [20]	mn; $\gcd(m, n) = 1$	$m + n + 1$; $\gcd(m, n) = 1$	$\geq m + n - 1$ $\leq m + n$	$n + 1$	$2^{\Theta(\sqrt{n \log n})}$

Table 1: Nondeterministic complexity of operations on unary free languages.

Class	Regular [20, 24]		Prefix-free		Suffix-free		Factor-free		Subword-free	
$K \cap L$	mn	2	$mn-(m+n-2)$	2	.	2	$mn-2(m+n-3)$	2	.	$m+n-5$
$K \cup L$	$m+n+1$	2	$m+n$	2	$m+n-1$	2	$m+n-2$	2	.	2
KL	$m+n$	2	$m+n-1$	1	.	1	.	1	.	1
L^*	$n+1$	1	n	2	.	2	$n-1$	1	.	1
L^R	$n+1$	2	n	1	$n+1$	2	n	1	.	1
L^c	2^n	2	2^{n-1}	3	.	3	$2^{n-2}+1$	3	.	2^{n-2}

Table 2: Nondeterministic complexity of operations on free classes. The dot means that the complexity is the same as in the previous column.

3.2 Closed languages

A language L is prefix-closed if $w \in L$ implies that every prefix of w is in L. Suffix-, factor-, and subword-closed languages are defined analogously.

We investigated the nondeterministic state complexity of basic regular operations on the classes of closed languages. For each class and for each operation, we obtained the tight upper bounds. To prove tightness we usually used a binary alphabet. In all the cases where we used a larger alphabet for describing witness languages, it remains open whether the obtained upper bounds can be met also by languages defined over smaller alphabets. We also considered the unary case. Our results are summarized in the following tables. The tables also display the size of alphabet used to describe witness languages.

Class	$K \cap L$	$\|\Sigma\|$	$K \cup L$	$\|\Sigma\|$	$K \cdot L$	$\|\Sigma\|$
Prefix-closed	mn	2	$m+n+1$	2	$m+n$	3
Suffix-closed	mn	2	$m+n+1$	2	$m+n$	3
Factor-closed	mn	2	$m+n+1$	2	$m+n$	3
Subword-closed	mn	2	$m+n+1$	2	$m+n$	3
Unary closed	$\min(m,n)$		$\max(m,n)$		$m+n-1$	
Regular	mn	2	$m+n+1$	2	$m+n$	2
Unary regular	mn; $\gcd(m,n)=1$		$m+n+1$; $\gcd(m,n)=1$		$\geq m+n-1$ $\leq m+n$	

Table 3: The nondeterministic complexity of union, intersection, and concatenation on closed languages. The results for regular languages are from [20].

Class	L^*	$\|\Sigma\|$	L^R	$\|\Sigma\|$	L^c	$\|\Sigma\|$
Prefix-closed	n	2	$n+1$	2	2^n	2
Suffix-closed	n	2	$n+1$	3	$1+2^{n-1}$	2
Factor-closed	1	1	$n+1$	3	$1+2^{n-1}$	2
Subword-closed	1	1	$n+1$	$2n-2$	$1+2^{n-1}$	2^n
Unary closed	1		n		$n-1$	
Regular	$n+1$	1	$n+1$	2	2^n	2
Unary regular	$n+1$		n		$2^{\Theta(\sqrt{n \log n})}$	

Table 4: The nondeterministic complexity of star, reversal, and complementation on closed languages. The results for regular languages are from [20, 24].

3.3 Ideal languages

A language L over an alphabet Σ is a right (left, two-sided, all-sided) ideal if $L = L\Sigma^*$ ($L = \Sigma^* L$, $L = \Sigma^* L \Sigma^*$, $L = L \sqcup\!\sqcup \Sigma^*$, respectively).

We investigated the nondeterministic state complexity of basic regular operations on the classes of ideal languages. For each class and for each operation, we obtained the tight upper bounds. These bounds are the same as in the general case of regular languages for intersection and star on all four classes, and reversal on left ideals, while in the remaining cases the complexity is always smaller than for regular languages.

To prove tightness we usually used a binary alphabet which is always optimal. In all the cases where we used a larger alphabet for describing witness languages, It remains open whether the obtained upper bounds can be met also by languages defined over smaller alphabets. We also considered the unary case. Our results are summarized in the following tables.

Class	$K \cap L$	$\|\Sigma\|$	$K \cup L$	$\|\Sigma\|$	$K \cdot L$	$\|\Sigma\|$
Right ideal	mn	2	$m+n$	2	$m+n-1$	1
Left ideal	mn	2	$m+n-1$	2	$m+n-1$	1
Two-sided ideal	mn	2	$m+n-2$	2	$m+n-1$	1
All-sided ideal	mn	2	$m+n-2$	2	$m+n-1$	1
Unary ideal	$\max(m,n)$		$\min(m,n)$		$m+n-1$	
Regular	mn	2	$m+n+1$	2	$m+n$	2
Unary regular	mn; $\gcd(m,n)=1$		$m+n+1$; $\gcd(m,n)=1$		$\geq m+n-1$ $\leq m+n$	

Table 5: The nondeterministic complexity of intersection, union, and concatenation on ideal languages. The results for regular languages are from [20].

Class	L^*	$\|\Sigma\|$	L^R	$\|\Sigma\|$	L^c	$\|\Sigma\|$
Right ideal	$n+1$	2	n	1	2^{n-1}	2
Left ideal	$n+1$	2	$n+1$	3	2^{n-1}	2
Two-sided ideal	$n+1$	2	n	1	2^{n-2}	2
All-sided ideal	$n+1$	2	n	1	2^{n-2}	2^{n-2}
Unary ideal	$n-1$		n		$n-1$	
Regular	$n+1$	1	$n+1$	2	2^n	2
Unary regular	$n+1$		n		$2^{\Theta(\sqrt{n \log n})}$	

Table 6: The nondeterministic complexity of star, reversal, and complementation on ideal languages. The results for regular languages are from [20].

3.4 Convex languages

A language L is prefix-convex if $u, w \in L$ and u is a prefix of w imply that each string v such that u is a prefix of v and v is a prefix of w is in L. Suffix-, factor-, and subword-convex languages are defined analogously. Except for complementation on factor- and subword-convex languages, we always obtain tight upper bounds.

Tables 7 and 8 summarizes our results on convex languages. In the second table, the $\cdot$ means that the complexity is the same as in the previous column. This table also displays the sizes of alphabet used for describing wittnes languages. Whenever the alphabet is binary or unary, it is always optimal, otherwise we do not know whether the upper bounds are tight also for smaller alphabets. The exact complexity of complementation in the classes of factor-convex and subword-convex languages remains open.

	$K \cap L$	$K \cup L$	KL	L^*	L^c
Unary convex	$\max\{m,n\}$	$\max\{m,n\}$	$m+n-1$	$n-1$	$n+1$
Unary regular [20]	mn; $\gcd(m,n)=1$	$m+n+1$; $\gcd(m,n)=1$	$\geq m+n-1$ $\leq m+n$	$n+1$	$2^{\Theta(\sqrt{n \log n})}$

Table 7: Nondeterministic complexity of operations on unary convex classes.

	Regular [20, 24]		Prefix-		Suffix-		Factor-		Subword-convex	
$K \cap L$	mn	2	$\cdot$	2	$\cdot$	2	$\cdot$	2	$\cdot$	2
$K \cup L$	$m+n+1$	2	$\cdot$	2	$\cdot$	2	$\cdot$	2	$\cdot$	2
KL	$m+n$	2	$\cdot$	3	$\cdot$	3	$\cdot$	3	$\cdot$	3
L^*	$n+1$	1	$\cdot$	2	$\cdot$	2	$\cdot$	2	$\cdot$	2
L^R	$n+1$	2	$\cdot$	2	$\cdot$	2	$\cdot$	2	$\cdot$	$2n-2$
L^c	2^n	2	$\cdot$	2	$\cdot$	5	$\geq 2^{n-1}+1$ $\leq 2^n$	2	$\geq 2^{n-1}+1$ $\leq 2^n$	2^n

Table 8: Nondeterministic complexity of operations on convex classes. The $\cdot$ means that the complexity is the same as in the previous column.

4 Conclusions

In this thesis, we studied the nondeterministic state complexity of basic unary and binary operations on the subregular classes of free, closed, ideal, and convex languages. After providing basic definitions and notations, we summarized the known results concerning the complexity of basic operations on the above mentioned classes in the deterministic case, and on the class of regular languages in the nondeterministic case. We described upper and lower bound methods used throughout this thesis.

We examined the operations on the classes of prefix-, suffix-, factor-, and subword-free languages, and we obtained tight upper bounds in each case. The most interesting result of this part of the thesis is obtaining the complexity of complementation for prefix-, suffix-, and factor-free languages. In each of these three classes, we described witness languages over a ternary alphabet, and we were able to show that the upper bounds cannot be met by any binary languages.

In the next parts we studied closed and ideal languages. For each of these eight subclasses, we again found the exact nondeterministic complexity of each considered operation. Except for three cases, all our witness languages are desribed over a fixed alphabet of size at most three, and moreover binary alphabets are always optimal.

For convex languages we used our previous results to show that the complexity of each operation, except for complementation, in the class of convex languages is the same as in the general case of regular languages. A careful reader might notice that the classes of prefix-free, prefix-closed, and right ideal languages are subclasses of the class of prefix-convex languages; and we have similar inclusions in the other three convex classes. In the case of complementation on suffix-convex languages, we obtained another very interesting result of this thesis. We described a proper suffix-convex language, that is, a suffix-convex language which is neither suffix-free, nor suffix-closed, nor left ideal, meeting the upper bound 2^n for its complementation. We had to find such a special language because the complexity of complementation on the classes of suffix-free, suffix-closed, and left ideal languages is less than 2^n.

Some problems remained open. For complementation on subword-free languages, we defined witnesses over a growing alphabet. It is open whether the upper bound is tight for some fixed alphabet. In the classes of closed and ideal languages, some of our witnesses were described over a ternary alphabet. We do not know whether or not a binary alphabet can be used to describe the corresponding witnesses. The exact nondeterministic state complexity of complementation in the classes of factor-convex and subword-convex languages remains open as well.

5 The list of my published papers

1. **Conference papers [the number of citations]**

   (a) Mlynárčik,P.: *On average complexity of InsertSort.* ITAT 2005, Information Technologies - Applications and Theory, Proceedings, Slovakia, 117-122
   [cited: 0]

   (b) Čevorová, K., Jirásková, G., Mlynárčik, P., Palmovský, M., Šebej, J.: *Operations on Automata with All States Final.* Z. Ésik and Z. Fülöp (Eds.): Automata and Formal Languages 2014 (AFL 2014) EPTCS 151, 2014, pp. 201Ű215, doi:10.4204/EPTCS.151.14
   [cited: 2 (self)]

   (c) Jirásek, J., Jirásková, G., Krausová, M., Mlynárčik, P., Šebej, J.: *Prefix-Free Languages: Right Quotient and Reversal* In: H. Jürgensen et al. (Eds.): DCFS 2014, LNCS 8614, pp. 210-221. Springer International Publishing Switzerland (2014)
   [cited: 1 (self)+1 (non-self)]

   (d) Jirásková, G., Mlynárčik, P.: *Complement on Prefix-Free, Suffix-Free, and Non-Returning NFA Languages.* In: H. Jürgensen et al. (Eds.): DCFS 2014, LNCS 8614, pp. 222-233. Springer International Publishing Switzerland (2014)
   [cited: 3 (self)+1 (nonself)]

   (e) Mlynárčik,P.: *Complement on Free and Ideal Languages.* In: Shallit, Okhotin (Eds.): DCFS 2015, LNCS 9118, pp. 185-196. Springer International Publishing Switzerland (2015)
   [cited: 2 (self)]

   (f) Hospodár, M., Jirásková, G., and Mlynárčik, P.: *Nondeterministic Complexity of Operations on Closed and Ideal Languages.* In: Han YS., Salomaa K. (Eds.): Implementation and Application of Automata. CIAA 2016. LNCS 9705, pp. 125-137. Springer (2016)
   [cited: 1 (self)]

   (g) Hospodár, M., Jirásková, G., and Mlynárčik, P.: Nondeterministic Complexity of Operations on Free and Convex Languages. In: Carayol A., Nicaud C. (Eds.): Implementation and Application of Automata. CIAA 2017. LNCS 10329, pp. 138-150. Springer (2017)
   [cited: 0]

2. **Journal papers**

   (a) Jirásek, J., Jirásková, G., Krausová, M., Mlynárčik, P., Šebej, J.: *Prefix-free languages: Left and right quotient and reversal.* (2014) Theoretical Computer Science

cited: 0

Non-self citations in:

1. Yuan Gao and Nelma Moreira and Rogério Reis and Sheng Yu: *A Survey on Operational State Complexity.* CoRR, abs/1509.03254, (2015), http://arxiv.org/abs/1509.03254

2. Michal Hospodár: *Complexity of unary union-free and unary star-free languages.* In: Henning Bordihn, Rudolf Freund, Benedek Nagy, and Gyrgy Vaszil (eds.): Eighth Workshop on Non-Classical Models of Automata and Applications, NCMA 2016, Debrecen, Hungary, August 29-30, 2016. Short Papers. Computersprachen TU Wien, pp. 15-23 (2016) ISBN 978-3-200-04725-9

6 The list of given talks

This part contains the list of my talks in significant conferences concerning to the topic of my thesis.

1. *16th International Workshop on Descriptional Complexity of Formal Systems.*
   August 5-8, 2014, Turku, Finland

2. *Workshop on Černý's Conjecture and Optimization Problems.*
   November 2014, Opava, Czech Republic

3. *DCFS 2015 Descriptional Complexity of Formal Systems*
   June 25-27, 2015, Waterloo, Ontario, Canada

4. *CIAA 2016 21st International Conference on Implementation and Application of Automata*
   July 19-22, 2016, Seoul, South Korea

5. 18. Konferencia košických matematikov
   April 20-22, 2017, Herľany, Slovak Republic

6. *Workshop on Černý's Conjecture and Optimization Problems of Finite Automata.*
   May 16-17, 2017, Opava, Czech Republic

7. *CIAA 2017 22st International Conference on Implementation and Application of Automata*
   June 27-30, 2017, Paris (Marne-la-Vallée), France

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