Questionnaire
Summary of the main activities of a research institute of the Slovak Academy of Sciences

Period: January 1, 2016 - December 31, 2021

1. Basic information on the institute:

1.1. Legal name and address

Mathematical Institute, SAS
Štefánikova 49,
SK-81473 Bratislava
Slovakia

1.2. URL of the institute web site

https://www.mat.savba.sk/

1.3. Executive body of the institute and its composition

Directoriat	Name	Age	Years in the position, from - to
Director	Karol Nemoga	69	2015 -
Deputy director	Anatolij Dvurečenskij	72	2015 -
Scientific secretary	Marek Hyčko	43	2016 -

Add more rows for any changes during the evaluation period

1.4. Head of the Scientific Board

doc. RNDr. Ľubica Holá, DrSc. - up to Oct. 17, 2016
Mgr. Anna Jenčová, DrSc. - from Nov. 8, 2016

1.4.1 Composition of the International Advisory Board

- Prof. Antonio Di Nola, University of Salerno, Salerno, Italy,
- Prof. Lajos Molnár, DSc., Dep. of Analysis, Bolyai Institute, University of Szeged, Szeged, Hungary,
- RNDr. Jiří Rákosník, CSc., former director of the Institute of Mathematics, Academy of Sciences, Czech Republic, Prague, Czech Republic

1.5. Basic information on the research personnel

1.5.1. Fulltime equivalent work capacity of all employees (FTE all), FTE of employees with university degrees engaged in research projects (FTE researchers)

2016		2017		2018		2019		2020		2021		2016-2021	
FTE all	FTE researchers	FTE all	FTE researchers	FTE all	FTE researchers	FTE all	FTE researchers	FTE all	FTE researchers	FTE all	FTE researchers	average FTE all per year	average FTE researchers per year
42,89	29,06	43,51	28,68	42,54	28,03	40,29	27,73	43,41	29,59	42,81	31,36	**42,58**	**29,08**

1.5.2. If applicable, add also a short information on the merger of the institute in the evaluation period. You can also add rows in the above table corresponding to the founding institutes

1.6. Basic information on the funding of the institute

1.6.1. Institutional salary budget, other salary budget[1], non-salary budget[2]

Salary budget	2016	2017	2018	2019	2020	2021	average
Institutional salary budget *[millions of EUR]*	0,758	0,782	0,844	0,988	1,146	1,128	**0,941**
Other salary budget *[millions of EUR]*	0,022	0,024	0,020	0,025	0,023	0,287	**0,067**
Total salary budget *[millions of EUR]*	0,780	0,806	0,864	1,013	1,169	1,415	**1,008**
Non-salary budget *[millions of EUR]*	0,191	0,202	0,188	0,192	0,186	0,201	**0,193**

1.7. Mission Statement of the Institute as presented in the Foundation Charter indicating the years when it was adopted and revised

The Mathematical Institute of the Slovak Academy of Sciences, is a scientific organization with a budgetary economy. It was founded on March 1, 1959. The Institute is concentrating on the basic research in mathematics (mainly logic and set theory, number theory, algebraic and topological structures, quantum structures, discrete mathematics, real and functional analysis, dynamical systems, differential equations, probability theory, and mathematical statistics). In computer science, the research is focused on the theory of algorithms and computational complexity and on the theory of formal languages, automata, and numerical systems.

The Institute provides advisory and expert analysis concerning the main scientific activity in the area of applications of mathematical methods. It could according to, Act 133/2002 On Slovak Academy of Sciences, perform enterprise activity with respect to its main scientific activity like performing program tools, their sale and updating in the area of applied

[1] Salary budget originating outside the regular budgetary resources of the organization, e.g. from the project funding.
[2] Includes Goods and Services and PhD fellowships

mathematics, and organize specialized seminars and conferences, and publishing publications in the area of applied mathematics.

The Institute participates in the pedagogical process of PhD studies according to valid legal regulations.

The Institute ensures publications of results concerned with research activity through the mathematical journals.

The last revision of the Foundation Charter was adopted on October 28, 2021, with effect on January 1, 2022.

1.8. Summary of R&D activity pursued by the institute during the evaluation period in both national and international contexts. Describe the scientific importance and societal impact of each important result/discovery. Explain on general level – the information should be understandable for a non-specialist (recommended 5 pages, max. 10 pages for larger institutes with more than 50 average FTE researchers per year as per Table 1.5.1.)

Number Theory and Cryptography

The classical part of mathematics is number theory. Here we studied the distribution of sequences and also problems of uniform distribution theory, among them, for example, the distribution of leading digits of imaginary parts of Riemann zeta zeros. Results are contained in the book "O. Strauch, Distribution of Sequences: A Theory." The book includes a method for exceeding the Benford law to real sequences x_n, which respect to the distribution functions of x_n. Cooperating with Ohkubo we shall extend this method to sequence y_n of zeros of Riemann's zeta functions.

In cryptology, we studied the design of the McElliece cryptosystem with special matrices. The problem of generating invertible circulant binary matrices with a prescribed number of ones was solved.

We devoted attention also to implementing new post-quantum methods and testing such implementation in cooperation with colleagues from Spain and Malta.

During the whole period were solved problems for the Ministry of Defense SR concerning security levels of various methods, and designing new systems. All these solutions are secret.

Uncertainty Modeling by Statistical Methods, Quantum Structures, and Fuzzy Sets

Uncertainty in our everyday life is a phenomenon which accomplished mankind through the whole history. Therefore, generations were looking for it, how to determine it in order to use it, and improve it if it wasn't favorable. Therefore, we know till today names of the famous antic oracle centers as Pythia in Delphi, Greece, or Sibyl in Cumae, Italy. Games, mainly hazard games, are needed to evaluate the chance of winning or losing. A hazard game is even mentioned in the New Testament when the soldiers who crucified Jesus played dice for his dress. Therefore, there appeared the first combinatorial notions and elementary probability. Till the beginning of the 20^{th} century, the probability theory was assumed more-less as a mathematical puzzle. In the thirties, A.N. Kolomogorov posed axiomatical foundations of the probability theory, which is a stable base

for mathematical statistics till nowadays, and it was the first model for describing a kind of uncertainty.

Appearing quantum mechanics, it was immediately recognized that due to the Heisenberg uncertainty principle, the Kolmogorov probability model does not fit all quantum mechanical events which appear during a simultaneous measuring process of two quantum mechanical values. The problem is that when we measure quantum mechanical observables, we are going into a micro-world where some, not neglectable phenomena appear, and they influence the measuring process. These phenomena are not observed in classical physics. Therefore, there appeared a need how to use mathematical methods to describe measurements at the quantum mechanical level. Today, we know that similar problems appear also in different areas of human activity, e.g. in psychiatry.

The mathematical foundations of quantum mechanics were a fruitful source of problems interesting physicists, mathematicians, logicians, philosophers, and others. Mathematical Institute has over 30 years of tradition in the research of uncertainty problems of quantum mechanics. Our famous protagonists are Profs. S. Pulmannová, A. Dvurečenskij, A. Jenčová, M. Plávala, as well late Profs. B. Riečan and R. Frič. The methods used in the theory of quantum structures combine statistical and probabilistic methods, algebraic methods, methods of the theory of Hilbert spaces, theory of fuzzy sets, theory of categories, theory of measurements, etc. Our Institute is very famous not only in the Slovak milieu but also because our results are famous in the international measure. Two of our colleagues were presidents of the International Quantum Structure Association, which is the main platform for the foundations of quantum mechanics.

In the evaluated period, we have solved two APVV grants about the uncertainty, and two Ph.D. students finished their studies; one of them has presently time a fellowship of A. von Humboldt Foundations. Important notions of the theory of quantum structures are observable and state that is quantum-mechanical analogs of measurable functions and probability measures, respectively. An observable is assumed to be a kind of σ-homomorphism from the set of all Borel sets into a quantum structure which can be a Boolean σ-algebra in the classical measurements or a kind of effect algebras in the general case. If we restrict an observable to the set of infinite intervals of the form $(-\infty,t)$ for each real number t, we obtain a spectral resolution. A resolution can be characterized as a non-decreasing mapping from reals into a quantum structure with simple properties. There is a natural question when does a spectral resolution can be extended to an observable? We have published a series of articles where we have solved this important question for different kinds of effect algebras. This problem was also solved for higher dimensions. The results were applied to a question of defining different types of joint n-dimensional observables of n one-dimensional observables. Moreover, the results also allowed us to define a special kind of a sum of two observables on MV-algebras.

MV-algebras is the realm of effect algebras, which are today the most important algebraic structure of quantum structures, play an analogous role as do Boolean algebras in the realm of quantum logics. Therefore, MV-algebras met the interests also of specialists in quantum structures. MV-algebras were generalized in many ways, including non-commutative pseudo MV-algebras. A. Dvurečenskij established in the past a representation of every pseudo MV-algebras as an interval in a unital l-groups. This result was a starting point for introducing EMV-algebras and pseudo EMV-algebras. A basic result about pseudo EMV-algebras says that every EMV-algebra/pseudo EMV-algebra M without top element can be embedded into a bounded pseudo EMV-algebra N, that is in fact a pseudo MV-algebra, as a maximal and normal ideal of N, and every element of N is either from the image of M or a complement of the image of some from M. We have also developed a theory of states on these algebras and introduced a weak pseudo EMV-algebras.

Quantum Information Theory and Quantum Foundations.

Relative entropies are fundamental quantities in information theory. They are used for the description of the performance of procedures in various tasks like communication through noisy channels or hypothesis testing and are an indispensable technical tool. We studied a class of Rényi relative entropies in the most general setting of quantum information theory and proved a number of their important properties. These results also found applications in quantum field theory.

Incompatibility of quantum measurements is one of the essential phenomena appearing in quantum mechanics and is nowadays the basis for many quantum information protocols. Our works contributed to a mathematical description, characterization, and quantification of incompatibility, as well as to clarification of its relations to other nonclassical effects present in quantum mechanical systems. We have shown connections between purely mathematical, or geometric quantities, e.g., "the largest hypercube that can be mapped into a ball" with operational characteristics such as "the amount of noise that can destroy incompatibility of a set of measurements" The insight obtained from such connections can be fruitful both ways.

The task of quantum foundations is to study mathematical structures suitable for a description of quantum systems. We cooperated in the development of one such structure, the synaptic algebra. We also studied and compared generalized notions of spectrality, an important property connected with the possibility of decomposition of quantum effects into simpler elements.

Aggregation Theory

Uninorms are aggregation functions which are due to their nice properties used in many domains, such as neural networks, Expert systems, Decision making, Image processing, Information fusion, Fuzzy system modeling, and many others. Therefore, many authors focused on the study of uninorms, especially those with continuous underlying functions. However, the complete characterization of this class of uninorms remained an open problem for a long time. In the collection of works we were able to offer the complete characterization of uninorms with continuous underlying functions and show their decomposition via ordinal sum construction, description of their set of points of discontinuity related to their characterizing set-valued function, and show similar results also for other related functions. We also showed several results related to convex combinations of uninorms.

Discrete Structures, Graph Theory and Algebra

We investigated many problems, among them:

- Construction of a polynomial-time algorithm such, that, for a graph X with 4p vertices (p is prime), finds (if any) a Cayley representation of X over the group $C_2 \times C_2 \times C_p$. This result shows that recognizing and testing isomorphism of Cayley graphs over an abelian group of order 4p can be done in polynomial time. So far, polynomiality of recognition and isomorphism testing was known only for cyclic groups.
- Skew morphisms of cyclic groups. Skew morphisms of groups are certain generalisations of automorphism groups arising naturally in investigation of Grothendieck dessins with maximal symmetry groups. In particular, so called reciprocal pairs skew morphisms of cyclic groups are of interest. We proved that if ϕ is a skew morphism which is an automorphism, then any skew morphism ϕ' such that ϕ, ϕ' are reciprocal must be `smooth'. We also investigated the classification of unique regular dessins with a prescribed nilpotent automorphism group.
- We have a long-term project aimed to investigate relationship between category of graphs with branched coverings and Riemann surfaces. Branched coverings between graphs can be considered as a one-dimensional counterpart of branched coverings between surfaces. Groups of automorphisms of graphs acting freely on the set of arcs of a graph X correspond to regelar branched coverings defined on X. Discrete versions of Robert Accola's results on Riemann surfaces with automorphism groups admitting partitions were derived. We found a condition for γ-

hyperelliptic involution on a graph to be unique. We constructed an infinite family of graphs with more than one γ-hyperelliptic involution.

- In a classical result of 1972 Singerman classifies the inclusions between triangle groups. We have generalized Singerman's classification of triangle group inclusions to the broader class of generalized quadrangle groups, that is, plane Fuchsian groups generated by three or four elliptic generators.
- We were able to enumerate orientable hypermaps of any fixed genus. The numbers of maps and hypermaps appear as coefficients in 2D gravity models in theoretical physics.
- Tutte-Grothendieck invariants. To these invariants belong Tutte, flow, and tension polynomials. For any edge-cut, these invariants are uniquely characterized by minors consisting of the edges of the cut and one of the subgraphs arising after deleting the edges of the cut.
- Interpretations of the Tutte and characteristic polynomials of matroids; Introduced new interpretations of the Tutte polynomial on matroids. This indicates new interpretations of the characteristic polynomial of matroids, thus also interpretations of the flow, chromatic, and tension polynomials on graphs.

Dynamical Systems, Differential Equations, Real and Functional Analysis and Topology

Differential equations, we choose some results with a short description.

- Differential equations with generalized periodic right-hand sides. Periodic solutions for second-order differential equations with generalized forcing modeled by Dirac impulses. Analytical bifurcation results were derived in terms of Melnikov-like conditions for the existence of generalized periodic solutions under Dirac impulsive forcing. Applications to forced harmonic and Duffing oscillators were given. Analytical results were illustrated by numerical computations.
- Gain–loss-driven travelling waves in pt-symmetric nonlinear metamaterials. A one-dimensional model of nonlinear magnetic metamaterials with parity-time-symmetric potentials was considered. Metamaterials will be widely used to produce cheap, high-performance magnets or new types of electromotors in electric cars. Mathematically, these are dynamic systems on grids/lattices, i.e., infinite systems of coupled electronic circuits. The existence of localized traveling waves was obtained by performing a Melnikov function analysis. Direct numerical computations were performed, showing good agreement with the theoretical analysis.
- Weakly fractional differential equations. We compared solutions of q-order fractional differential equations of Caputo type for q near 1 with solutions of the corresponding 1-order ordinary differential equations. By establishing the explicit lower and upper bounds of Mittag-Leffler functions, we obtained effective convergence results. It was shown that the limit cases $q \to 1^+$ and $q \to 1^-$ are different.
- The Poincaré-Adronov-Melnikov method for the existence of grazing impact periodic solutions of differential equations. Perturbative methods are well-established ways to obtain the existence of periodic solutions in smooth differential equations. In the study of non-smooth systems of differential equations, however, one has to face different kinds of periodic solutions. Suppose that the differential equation is non-smooth along a certain surface S. Interesting situation arises when a map M acts on S in such a way that when a solution of the differential equation hits S, then the solution continues immediately at another point on S given by M. Solutions satisfying this property is called impact solutions. Then it is interesting to study the existence of solutions touching S. These solutions are called grazing impact solutions. Such kinds of solutions may be seen as solutions determining a border between solutions hitting S and those that do not hit S at all. We studied the persistence of periodic grazing impact solutions for periodically perturbed differential equations with impacts. An approach of the Poincaré-Adronov-Melnikov method was applied. It is based on introducing and studying the appropriate impact Poincaré mapping. A typical example of an impact non-smooth model is a bouncing ball or swinging pendulum in the bell.

In the real analysis and topology research was focus on

- Minimal usco and minimal cusco maps and compactness. Generalization of the Arzela-Ascoli Theorem from continuous functions to minimal usco/cusco maps into metric spaces.
- Quasicontinuous functions and compactness. Let X be a locally compact space and (Y, d) be a boundedly compact metric space. We characterized compact subsets of the space Q(X, Y) of quasicontinuous functions from X to Y equipped with the topology of uniform convergence on compact sets, and some generalizations.
- Norm continuity of quasicontinuous mappings into $C_p(Y)$. We used a topological game argument to show that, under some restrictions on a topological space X, every quasicontinuous mapping from X to $C_p(Y)$ is norm continuous at points of a dense G_δ subset of X provided that Y is totally countably compact. We improved some old results on joint continuity of separately continuous functions by J. Saint-Raymond.
- USCO and quasicontinuous mappings. Results are contained in the monograph published by DeGruyter (coauthors, D. Holý, Slovakia, W. Moors, New Zealand). Presents two natural generalizations of continuous mappings, namely usco and quasicontinuous mappings. The first class considers set-valued mappings; the second class relaxes the definition of continuity. Both these topological concepts stem naturally from basic mathematical considerations and have numerous applications that are covered in detail. Accessible to researchers who want to learn about Usco and Quasicontinuous mappings. The book presents the most significant results in the field with applications.

Data Processing and Computer Science

The first main topic was the design, implementation, and convergence analysis of efficient serial and parallel algorithms for the eigenvalue decomposition (EVD) and singular value decomposition (SVD) of large, dense matrices. The focus has been on block Jacobi methods with special orderings of subproblems. In 2016-2017, our software was tested on various parallel architectures by the famous group of numerical mathematicians at the University of Tennessee, Knoxville, U.S. The results were published in SIAM Review in 2018. Our parallel block-Jacobi SVD algorithm with dynamic ordering was, for the first time, competing in speed with some algorithms based on the matrix bidiagonalization.

The second main research topic was the analysis of algorithms for the search and exploration of its numerous variants and flavors. The focus has been mostly on search by mobile agents (with various limitations, e.g., on memory and/or communication capabilities) in graphs (often dynamic, time-varying graphs).

The set of all strings accepted by a finite automaton is called a regular language. Such a language can be described by many different finite automata models (deterministic, nondeterministic, alternating, Boolean, unambiguous, self-verifying, etc.). By the descriptional complexity of a regular language in a given automata model, we understand the size of a minimal (with respect to the number of states) automaton describing the given language. A regular operation is an operation on languages such that if operands are regular, then the result is regular as well. The complexity of a regular operation is the function assigning the maximal possible complexity of the resulting language to the sizes of automata describing operands. In the evaluation period (2016 – 2021), we examined the complexity of operations in various automata models, as well as the trade-offs between different automata models. For some operations, we investigated not only the worst-case complexity but rather the range of all possible complexities of the resulting language. We also considered the cases where the operands satisfy some specific property (e.g., are closed under prefixes).

The social impacts of all results also consist of the international cooperation of researchers from several countries.

2. Partial indicators of main activities:

2.1. Research output

2.1.1. Principal types of research output of the institute: basic research/applied research, international/regional (in percentage)

Basic research (2016)

1. **Characterization of some continuous additive generators of triangular subnorms**

The continuous, conditionally cancellative t-subnorms that possess a continuous, additive generator are discussed. While all cancellative continuous t-subnorms possess a continuous additive generator, it is different in the case of conditionally cancellative t-subnorms. Conditions for a continuous, conditionally cancellative t-subnorm to have a continuous, additive generator are described. The monotonicity of such an additive generator is examined. Constructions of corresponding additive generators are also shown.

Author(s): Andrea Mesiarová-Zemánková
Projects: VEGA 2/0049/14, APVV-0178-11 and Program Fellowship of SAS
Reference: A. Mesiarová-Zemánková, *Continuous additive generators of continuous, conditionally cancellative triangular subnorms*, Information Sciences **339** (2016) 53–63.

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2. **Minimal usco and minimal cusco maps**

We proved a non trivial generalization of the Arzela-Ascoli Theorem from continuous functions to minimal upper semicontinuous compact valued maps (usco) and minimal upper semicontinuous compact and convex valued maps (cusco). Such maps are a very important tool in optimization, in functional analysis, in the study of differentiability of Lipschitz functions and in selection theorems.

Authors: Ľ. Holá (MI SAS)**,** D. Holý
Projects: VEGA 2/0018/13, APVV-0269/11
Reference: Ľ. Holá, D. Holý, *Minimal usco and minimal cusco maps*, Journal of Mathematical Analysis and Applications **439** (2016), 737-749.

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3. **All measurements in a probabilistic theory are compatible if and only if the state space is a simplex**

Compatibility of measurements is the theoretical possibility of performing the two measurements simultaneously. In our work we studied the compatibility of measurements on finite-dimensional compact convex state space in the modern framework of general probabilistic theory that generalizes the framework used in quantum information theory. We formulate the necessary and sufficient conditions for two-outcome measurements to be compatible and we use these conditions to show that there exist incompatible measurements whenever the state space is not a simplex.

On one hand, the physical interpretation of the result is that only simplex state space corresponds to a classical system where all measurements are compatible. On the other hand from the mathematical perspective, the result shows the connection between the geometry of the state space and the properties of the general probabilistic theory.

Author: Martin Plávala
Project: VEGA 2/0069/16
Reference: M. Plávala, *All measurements in a probabilistic theory are compatible if and only if the state space is a simplex*, Phys. Rev. A **94** (2016), 042108.

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4. **Operations on unambiguous finite automata**

A nondeterministic finite automaton is unambiguous if it has at most one accepting computation on every input string. The notion of unambiguity is a fundamental notion in the theory of variable-length codes. We investigated the complexity of basic regular operations on languages represented by unambiguous finite automata. We got the exact complexities for intersection, left and right quotients, positive and Kleene closure, shuffle, and concatenation. To prove lower bounds, we used a binary alphabet for intersection and left and right quotients, a ternary alphabet for star and positive closure, a five letter alphabet for shuffle, and a seven-letter alphabet for concatenation. The most interesting result of the paper is obtaining the upper bound $2^{0.79n}$ for the complementation of unambiguous automata.

Authors: J. Jirásek, Jr., **G. Jirásková** (MI SAS)**,** J. Šebej
Projects: VEGA 2/0084/15, APVV-15-0091
Reference: J. Jirásek, Jr., G. Jirásková, J. Šebej, *Operations on unambiguous finite automata*, In: S. Brlek and C. Reutenauer (Eds.): Developments in Language Theory - 20th International Conference, DLT 2016, Montreal, Canada, July 25-28, 2016, Proceedings. Lecture Notes in Computer Science, vol. 9840, pp. 243–255. Springer 2016.
http://dx.doi.org/10.1007/978-3-662-53132-7_20

Applied research (2016)

1. **Analysis and optimization of parallel Jacobi SVD algorithm**

The one-sided block Jacobi (OSBJ) method is known to be an efficient method for computing the singular value decomposition on a parallel computer. We focused on our OSBJ with parallel dynamic ordering and variable blocking. We provided a detailed theoretical analysis of its convergence properties. We identified two performance bottlenecks of the algorithm and proposed new implementations to resolve the problem. Experimental results show that they are effective and can achieve up to 2.0 and 1.5 times speedup of the total execution time on the FX10 and the Altix ICE. As a result, our OSBJ solver outperforms ScaLAPACK PDGESVD in almost all cases when computing the SVD of matrices of order 2160 to 8640 on these machines using 8 to 144 nodes.

Authors: S. Kudo, Y. Yamamoto, **M. Bečka** (MI SAS)**, M. Vajteršic** (MI SAS)
Project: VEGA 2/0026/14
Reference: S. Kudo, Y. Yamamoto, M. Bečka, M. Vajteršic: *Performance analysis and optimization of the parallel one-sided block Jacobi SVD algorithm with dynamic ordering and variable blocking*, Concurrency Computat.: Pract. Exper. (2016), Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/cpe.4059

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2. **Analysis of reliability of automatic shut-off valves**

To increase a safety of a gas transport, automatic shut-off valves are installed along the pipelines. The automatic closing of the valves is activated in an accident when pressure drop during a given time interval exceeds a set threshold. Activation depends on a sensitivity setting, the distance

between a valve and a rupture position, gas transport conditions, and construction parameters of a pipeline. We deal with qualitative and quantitative conditions for a reliable functioning of safety valves. The conditions were determined and verified by a numerical simulation of high transient gas flow after ruptures. Behaviors of safety valves at real ruptures were analyzed as well.

Authors: R. Hajossy, I. Mračka, A. Sedliak, T. Sedláková, T. Žáčik (head)
Project: 1235 Gas transport optimization through transit pipelines

International research (2016)

1. **n-Perfect kite pseudo effect algebras**

Kite pseudo effect algebras were recently introduced as a class of interesting examples of pseudo effect algebras using a po-group, an index set, and two bijections on the index set. We showed when these algebras satisfy some kind of the Riesz Decomposition Property, and we represent kite pseudo effect algebras with a special kind of the Riesz Decomposition Property as an interval in a lexicographic extension of the po-group which solves an open problem on the representation of kites. In addition, we introduce kite n-perfect pseudo effect algebras and we characterize subdirectly irreducible algebras which are building blocks of the theory.

Authors: M. Botur (PřF PU, Olomouc, ČR), **A. Dvurečenskij** (MI SAS)
Projects: APVV-0178-11, grant VEGA No. 2/0059/12 SAV, GAČR 15-15286S.
Reference: M. Botur, A. Dvurečenskij, *On pseudo BL-algebras and pseudo hoops with normal maximal filters*, Soft Computing **20** (2016), 439–448.

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2. **Travelling waves in nonlinear magneto-inductive lattices**

Metamaterials are artificial materials that are engineered to have properties that may not be found in nature. Magnetic metamaterials are non-magnetic materials exhibiting magnetic properties. When the materials composed of magnetic metamaterials are magnetically weakly coupled through their mutual inductances, one obtains magneto-inductive metamaterials, i.e, magneto-inductive lattices. The propagation of electromagnetic waves in magneto-inductive metamaterials has been discussed and modeled by several types of nonlinear equations, such as a nonlinear Klein-Gordon equation, coupled short-pulse equation, higher-order nonlinear Schrödinger equations, and coupled Klein-Gordon equations. We studied coupled Klein–Gordon equations. We considered a lattice equation modeling one-dimensional magneto-inductive metamaterials formed by a discrete array of nonlinearly implicitly coupled nonlinear resonators with periodic forces. The existence and uniqueness results of traveling waves of the system are presented. The resonance condition for the parameter values is also derived leading to multiplicity results. Our analytical results are found to be in good agreement with direct numerical computations.

Authors: M. Agaoglou (Univ. Thessaloniki, GR), **M. Fečkan** (MI SAS), **M. Pospíšil** (MI SAS), V. M. Rothos (Univ. Thessaloniki, GR), H. Susanto (Univ. Essex, UK)
Project: VEGA 2/0029/13
Reference: M. Agaoglou, M. Fečkan, M. Pospíšil, V. M. Rothos, H. Susanto, *Travelling waves in nonlinear magneto-inductive lattices*, Journal of Differential Equations **260** (2016), 1717–1746.
http://www.sciencedirect.com/science/article/pii/S0022039615005112

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3. **On the Dedekind–MacNeille completion and formal concept analysis based on multilattices.**

Formal Concept Analysis is a relatively new mathematical theory of data analysis for the

identification of conceptual structures among data sets. On the other side, the Dedekind–MacNeille completion of a poset P represents the well-known notion and it can be seen as a certain least complete lattice containing P. In this work some results concerning the use of this completion within the framework of Formal Concept Analysis in terms of the poset of concepts associated with a Galois connection between posets are analyzed. Specifically, an interesting property of the Dedekind–MacNeille completion is shown, i.e., that the completion of the concept poset of a Galois connection between posets coincides with the concept lattice of the Galois connection extended to the corresponding completions. Moreover, the specific case when P has a multilattice structure is studied. As one of the main results the corresponding representation theorem is stated and proved.

Authors: J. Medina (Univ. Cadiz, Spain), M. Ojeda-Aciego (Univ. Malaga, Spain), **J. Pócs** (MI SAS), E. Ramírez-Poussa (Univ. Cadiz, Spain)
Project: ESF Fund CZ.1.07/2.3.00/30.0041
Reference: J. Medina, M. Ojeda-Aciego, J. Pócs, E. Ramírez-Poussa, *On the Dedekind–MacNeille completion and formal concept analysis based on multilattices*, Fuzzy Sets and Systems **303** (2016), 1–20.

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4. **Live Exploration of Dynamic Rings**

Almost all the vast literature on graph exploration assumes that the graph is static : its topology does not change during the exploration, except for occasional faults. To date, very little is known for the exploration of dynamic graphs, where the topology is continuously changing. The few studies have been limited to the centralized (or post-mortem) case, assuming complete a priori knowledge of the changes and the times of their occurrence, and have only considered fully synchronous systems. In this paper, we start the study of the decentralized (or live) exploration of dynamic graphs, i.e. when the agents operate in the graph unaware of the location and timing of the changes. We consider dynamic rings under the standard 1-interval-connected restriction, and investigate the feasibility of their exploration, in both the fully synchronous and semi-synchronous cases. When exploration is possible we examine at what cost, focusing on the minimum number of agents capable of exploring the ring. We establish several results highlighting the impact that anonymity and structural knowledge have on the feasibility and complexity of the problem.

Authors: G.A. Di Luna (Univ. of Rome "Sapienza", Taliansko), **S. Dobrev** (MI SAS), P. Flocchini (Univ. of Ottawa, Kanada), Nicola Santoro (Carleton Univ., Kanada)
Project: VEGA 2/0165/16
Reference: G. A. Di Luna, **S. Dobrev**, P. Flocchini, N. Santoro, *Live Exploration of Dynamic Rings*, ICDCS 2016, 570–579.

Basic research (2017)

1. **Reversibility of quantum channels and a Rényi relative entropy for von Neumann algebras**

The family of Rényi relative α-entropies has important applications in information theory, including hypothesis testing and channel coding. Formally, there are many possible extensions to quantum states, but there are two such extensions that play a similar role as the classical counterparts. These are usually called the standard, resp. the sandwiched quantum Rényi relative α-entropies. While the standard version has been known and used for quite a while, the sandwiched entropies for density matrices were introduced relatively recently (M. Müller Lennert et al., J. Math. Phys. 54, 2013). One of the fundamental properties of the relative entropies in information theory is the data processing inequality (DPI), that is, monotonicity under channels. D. Petz (Quart. J. Math. Oxford, 39 (1988), 97-108) proved that preservation of the quantum (Umegaki-Araki) relative entropy for a pair of states is equivalent to the reversibility of the channel with respect to these states. This means

that there is some other channel through which both original states can be recovered. In a series of works [1-3], we extend the sandwiched quantum Rényi relative α-entropies to normal states on von Neumann algebras, using interpolation theory. We prove some of their properties, including DPI and the mentioned equivalence. As a by-product, we prove that a large class of these entropies, including the Umegaki-Araki relative entropy, is monotone under all positive trace-preserving maps. This was formerly known only for factors of type I.

Author: A. Jenčová
Project: VEGA 2/0069/16
References:
[1] A. Jenčová, Preservation of a quantum Renyi relative entropy implies existence of a recovery map, J. Phys. A: Math. Theor. 50 (2017), 085303
[2] A. Jenčová, Rényi relative entropies and noncommutative L_p-spaces, arXiv:1609.08462v3, 2016
[3] A. Jenčová, Rényi relative entropies and noncommutative L_p-spaces II, arxiv:1707.00047, 2017

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2. **On differential equations with generalized periodic right-hand sides**

Periodic solutions are studied for second-order differential equations with generalized forcing modeled by Dirac impulses. Analytical bifurcation results are derived in terms of Melnikov like conditions for the existence of generalized periodic solutions under Dirac impulsive forcing. Applications to forced harmonic and Duffing oscillators are given. Analytical results are illustrated by numerical computations.
Authors: M. Fečkan, M. Pospíšil
Project: VEGA 2/0153/16
Referencia: M. Fečkan, M. Pospíšil: On equations with generalized periodic right-hand side, Ukrainian Mathematical Journal, (Dedicated to the 80th anniversary of the Academician of the NAS of Ukraine Anatoly Mykhailovych Samoilenko), in print 2018.

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3. **On the Betti Numbers of oriented Grassmannians**

We present a complete functional formula expressing the i-th Z_2-Betti number of the oriented Grassmann manifold of oriented 3-dimensional vector subspaces in Euclidean n-space for i from the range determined by the characteristic rank of the canonical oriented 3-dimensional vector bundle over this manifold. The same formula explicitly exhibits the number of linearly independent semi-invariants of degree 3 of a binary form of degree n-3. Using the approach and data presented in this paper, analogous results can be obtained for the oriented Grassmann manifold of oriented 4-dimensional vector subspaces in Euclidean n-space and semi-invariants of degree 4 of a binary form of degree n-4.

Author: J. Korbaš
Project: VEGA 1/0101/17
Reference: J. Korbaš, On the Betti Numbers of oriented Grassmannians and independent semi-invariants of binary forms, Mathematica Slovaca 67 (2017), No. 5, 1263-1268

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4. **Distribution of Sequences: A Theory**

In this book we study distribution functions of the sequence $\xi(3/2)^n$ mod 1, ratio block sequences $X_n = (x_1/x_n),..., (x_n/x_n)$, statistically independent sequences, uniformly maldistributed sequences, the sequence $\varphi(n) / n$, where $\varphi(n)$ is Euler's function, generalized Benford's law, multidimensional

Benford's law, and Gauss-Kuzmin theorem. We also study distribution functions $g(x, y)$, $g(x, 1) = x$, $g(1, y) = y$ called copulas and extremes of $\iint_{[0,1]^2} F(x, y)\mathrm{d}_x\mathrm{d}_y g(x, y)$ over copulas $g(x, y)$.

Author: O. Strauch
Project: VEGA 2/0146/14
Reference: STRAUCH, O.: *Distribution of Sequences: A Theory*, http://www.mat.savba.sk/musav/O Strauch, 2017, p. 510.

Applied research (2017)

1. **Bootstrap distribution of a stochastic sum by numerical inversion of the compounded empririal characteristic function**

We presented a method for evaluation of the bootstrap distribution of a stochastic sum of independent continuous random variables by numerical inversion of its empirical characteristic function. In particular, it is motivated by the classical problems in financial risk management, actuarial science, and/or hydrological modeling. We have presented a non-parametric method for evaluation of the aggregate loss distribution (ALD) based on combining and numerically inverting the empirical characteristic functions (CFs). This approach applied to the compound distribution is based on purely non-parametric considerations, i.e., it is based on using and combining the empirical CFs, which is a new and original contribution. This approach can be, however, naturally generalized to a more complex semi-parametric modeling approach, e.g., by incorporating the generalized Pareto distribution fit of the severity distribution heavy tails, and/or by considering the weighted mixture of the parametric CFs (used to model the expert knowledge) and the empirical CFs (used to incorporate the knowledge based on the historical data - internal and/or external). We have shown a simple and efficient method and algorithms for numerical inversion of the CF, suitable for the evaluation of the ALDs and the associated measures of interest important for applications, such as, e.g., the value at risk (VaR). The presented approach is based on a combination of the Gil-Pelaez inversion formulas for deriving the probability distribution (PDF and CDF) from the compound (empirical) CF and the trapezoidal rule used for numerical integration.

Authors: Viktor Witkovský (IMS SAS), **Gejza Wimmer (MI SAS)**, Tomas Duby
Projects: APVV-15-0295, VEGA 2/0047/15
Reference: submitted for publication in Journal of Computational and Applied Mathematics.

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2. **Localization of gas pipeline rupture using a massive parallelization on GPGPU**

A rupture localization method for transporting gas pipelines was developed. The method, unlike to contemporary rupture localization methods, can be used also in complex nonlinear pipeline networks. Due to a significant acceleration of rupture simulations, using general computing on graphical processing units (GPGPU), an accident position is determined on-line, and therefore it is applicable to real continuous operation. Moreover, the method enables rupture localization also in shut-off conditions of a pipeline subnetwork where only one pressure measurement is available.

Authors: R. Hajossy, I. Mračka, P. Somora, T. Žáčik (head)
Project: 1235 Gas transport optimization through transit pipelines
Reference: I. Mračka, P. Somora, R. Hajossy, T. Žáčik: Localization of Gas Pipeline Rupture Using a Massive Parallelization on GPGPU. In: PSIG 2017, May 9–12, Atlanta, Georgia, USA

International research (2017)

1. **Asymptotic quadratic convergence of the serial block Jacobi algorithm**

Block algorithms for the computation of eigenvalues and eigenvectors of Hermitian matrices are more efficient than their serial variants due to the better utilization of a memory hierarchy in current computers. In the case of scalar algorithms, the proof of the quadratic asymptotic convergence is known for simple and multiple eigenvalues as well as for their clusters, and for some cyclic orderings used for zeroing the off-diagonal matrix elements. For block methods, such a proof is known only for simple eigenvalues, and again only for some cyclic orderings used for zeroing the off-diagonal matrix blocks. We have proved the asymptotic quadratic convergence of the serial block Jacobi algorithm regardless of the distribution of eigenvalues, and for zeroing of two off-diagonal matrix blocks with the largest Frobenius norm in each iteration step. The proof is based on the description of the change of the Frobenius norm for off-diagonal blocks that were not zeroed in a given iteration step. In the proved theorem, there is some un-specified constant. This constant is identified in the next step as a portion of the spectral gap, i.e. the minimal distance between eigenvalues or between the centers of their clusters. The identification is based on the Davis-Kahan lemma, which provides the estimate of the spectral norm of the solution of a certain Sylvester matrix equation. Numerical experiments confirm the developed theory.

Authors: G. Okša (MI SAS), Y. Yamamoto (University of Electro-Communications, Tokyo, Japan), **M. Vajteršic** (MI SAS)
Projects: VEGA 2/0026/14
Reference: Okša, G., Yamamoto, Y, Vajteršic, M.: Asymptotic quadratic convergence of the serial block/Jacobi EVD algorithm for Hermitian matrices, Numerische Mathematik 136, 1071-1095 (2017). DOI: 10.1007/s00211-016-0863-5

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2. **Norm continuity of quasicontinuous mappings into $C_p(Y)$**

We used a topological game argument to show that, under some restrictions on a topological space X every quasicontinuous mapping z X do $C_p(Y)$ is norm continuous at points of a dense G_δ subset of X provided that Y is totally countably compact. We improved some old results on joint continuity of separately continuous functions from the paper J. Saint-Raymond, Jeux topologiques et espaces de Namioka, Proc. Amer. Math. Soc. 87 (1983), 499-504.

Authors: Ľ. Holá (MI SAS), A.K. Mirmostafaee (University of Mashhad, Mashhad, Iran)
Project: VEGA 2/0006/16
Reference: Ľ. Holá, A. K. Mirmostafaee, Norm continuity of quasicontinuous mappings into $C_p(Y)$, accepted in Colloquium Mathematicum

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3. **Optimal Local Buffer Management for Information Gathering with Adversarial Traffic**

We studied the problem of routing on directed paths and trees to a single destination, with rate-limited, adversarial traffic (the rate of incoming packets is limited, but the insertion locations of the packets are determined by an adversary). In particular, we focus on local buffer management algorithms that ensure no packet loss, while minimizing the size of the required buffers. While a centralized algorithm for the problem that uses constant-sized buffers has been recently shown, there is no known local algorithm that achieves a sub-linear buffer size.
We show three main results: A lower bound of $\Omega(c \log n/l)$ for all l-local algorithms on both directed and undirected paths, where c is an upper bound on the link capacity and injection rate.

A surprisingly simple 1-local algorithm for directed paths that uses buffers of size O(log n), when c=1. A natural 2-local extension of this algorithm to directed trees, for c=1, with the same asymptotic bound.

Authors: S. Dobrev (MI SAS), M. Lafond (Universite d'Ottawa, Ottawa, PQ, Canada), L. Narayanan (Concordia University, Montreal, PQ, Canada) , J. Opatrny (Concordia University, Montreal, PQ, Canada)
Project: VEGA 2/0165/16, NSERC grant
Reference: Proceedings of the 29th ACM Symposium on Parallelism in Algorithms and Architectures, pp. 265-274, ACM

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Basic research (2018)

1. **Gain–loss-driven travelling waves in PT-symmetric nonlinear metamaterials**

A one-dimensional model of nonlinear magnetic metamaterials with parity-time-symmetric potentials is considered. Metamaterials will be widely used to produce cheap, high-performance magnets, or new types of electromotors in electric cars. Mathematically, these are dynamic systems on grids/lattices, i.e. infinite systems of coupled electronic circuits. The existence of localized travelling waves is obtained by performing a Melnikov function analysis. Direct numerical computations are performed showing good agreement with the theoretical analysis.

Authors: M. Agaoglou (Aristotle University of Thessaloniki, Greece), **M. Fečkan** (MI SAS)**, M. Pospíšil** (MI SAS), V.M. Rothos (Aristotle University of Thessaloniki, Greece), H. Susanto (Univ. of Essex, UK)
Project: VEGA 2/0153/16
Reference: M. Agaoglou, M. Fečkan, M. Pospíšil, V.M. Rothos, H. Susanto: Gain–loss-driven travelling waves in PT-symmetric nonlinear metamaterials, Wave Motion 76 (2018) 9–18.

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2. **Strong Endomorphism Kernel Property for monounary algebras**

Let *A* be an algebra. An endomorphism *f* of *A* is strong if *f* is compatible with all congruences on the algebra *A*. If every congruence on *A* is a kernel of some strong endomorphism of *A*, then an algebra *A* has a strong endomorphism kernel property (SEKP). Results about this property are published since 2008 for semilattices, distributive lattices, p-algebras, Ockham algebras, and Stone algebras by authors T.S. Blyth, G. Fang, J. Fang, J. Guričan, M. Ploščica, Z.J. Sun, H.J. Silva and L.B.Wang. All algebras with SEKP are described for some types of algebras, e.g. for finite distributive lattices. We were interested in monounary algebras with SEKP and we described all ones with this property. They are monounary algebras that consist of at most two components and their operation is injective on the set *B* which we obtain if we omit one special point from *A*.

Author: E. Halušková
Projekt: VEGA 2/0044/16
Reference: E. Halušková: Strong Endomorphisms Kernel Property for Monounary Algebras, Mathematica Bohemica 143-2 (2018), 161-171.

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3. **Evacuating two robots from multiple unknown exits in a circle**

Distributed on a unit circle are k exits. Two autonomous mobile robots are placed on the circle. Each robot has a maximum speed of 1 and the robots can communicate wirelessly. The robots have a map of the domain, including exits but do not have knowledge of their own initial locations on the domain, rather they only know their relative distance. The goal of the evacuation problem is to give

an algorithm for the robots which minimizes the time required for both robots to reach an exit, in the worst case.

We consider two variations of the problem depending on whether the two robots have control over their initial distance. When the initial distance of the robots is part of the input (i.e. no control), we show that simple algorithms exist which achieve optimal worst-case evacuation times for the cases where: the robots begin colocated with an arbitrary distribution of the exits; and when the exits are either colocated or evenly spaced, with arbitrary starting positions of the robots.

We also give upper and lower bounds on the problem with arbitrary exit distribution and starting positions of the robots. For the problem where robots can choose their initial distance (with knowledge of, but no control over the distribution of exits), we propose a natural family of algorithms parameterized by the maximum distance between any two exits.

Authors: J. Czyzowicz (Uni Quebec, Canada), **S. Dobrev** (MI SAS), K. Georgiou (Uni Ontario, Canada), E. Krankis, F. MacQuarrie (Carleton University, Ontario)
Projects: NSERC Discovery grant
Reference: CZYZOWICZ, J. - DOBREV, S. - GEORGIOU, K. - KRANAKIS, E. - MACQUARRIE, F. Evacuating two robots from multiple unknown exits in a circle. In Theoretical Computer Science, 2018, vol. 709, p. 20-30.

Applied research (2018)

1. **Passive memristor synaptic circuits**

We discovered a new circuit for the simulation of synapses in artificial neural networks. The key advantage of the circuit is that it consists solely of memristor elements which allow for very high integration density using nanotechnologies. The circuit provides for multiple synaptic adaptation mechanisms, namely STDP as well as phase-based coincidence adaptation. The combination of these mechanisms is one possible solution to the problem of catastrophic forgetting in neural networks.

Authors: O. Šuch (MI SAS), M. Klimo (Žilinská univ.), N. T. Kemp (Univ. of Hull, UK), O. Škvarek (Žilinská univ.)
Projects: APVV-14- 0560, VEGA 2/0144/18, and COST Action IC1401 MemoCiS
References: Passive memristor synaptic circuits with multiple timing dependent plasticity mechanisms. In AEU-International Journal of Electronics and Communications, 2018, vol. 96, p. 252-259.

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2. **Generalized Polynomial Comparative Calibration: Parameter Estimation and Applications**

Described were methods and algorithms, based on using the classical regression approach used for the errors-in-variable model and applied it for the comparative calibration with the generalized polynomial calibration function. The present approach is based on strictly metrologist considerations. The input quantities are fully characterized by their state-of-knowledge distributions, derived from the associated uncertainty budgets of the direct measurements from the calibration experiment, which allows to combine of type A and type B methods of evaluation of the measurements, as well as to incorporate correlated measurements with the comparative calibration, i.e. with the situation when both variables entering the calibration experiment are subject to errors (including the used measurement standard). The whole procedure is illustrated on a real example. Appendixes incorporate all needed theoretical results.

Authors: V. Witkovský (IMS SAS), **G. Wimmer** (MI SAS)
Projects: VEGA 2/0047/15, APVV-15-0295
Reference: Witkovský, V., Wimmer, G.**,** *Generalized Polynomial Comparative Calibration: Parameter Estimation and Applications.* In: S. Y. Yurish, *Editor* Advances in Measurements and Instrumentation: Reviews, Book Series, Vol. 1, International Frequency Sensor Association (IFSA) Publishing, 2018, 15-52, ISBN: 978-84-09-07321-4

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3. **Reinforcement learning utilization in the gas transport control**

Reinforcement learning was adapted to the gas transport problems. Reinforcement learning, which is part of machine learning, is currently one of the most active areas of artificial intelligence research. The task is to teach an agent to perform optimal actions based on the characteristics of the environment in which it is currently located. The learning process is based on the evaluation of actions (e.g., compressor revolution setting) using the gas flow simulator in the pipeline network. It turns out that despite the long learning process, if the agent is trained, he is able to make the right decisions autonomously or at least to help the dispatcher.

Authors: T. Žáčik (head), I. Mračka, R. Hajossy, M. Hyčko
Project: 1235 Gas transport optimization through transit pipelines
Reference: T. Žáčik, I. Mračka, R. Hajossy, M. Hyčko: Reinforcement Learning in Gas Transport Control. In: PSIG 2018, May 15–18, Deer Valley, Utah, USA

International research (2018)

1. **Kites and residuated lattice**

We investigated a new construction of an integral residuated lattice starting from an integral residuated lattice and two sets with an injective mapping from one set into the second one. The resulting algebra has a shape of a Chinese cascade kite, therefore, we call this algebra simply a kite. We described subdirectly irreducible kites and we classify them. We show that the variety of integral residuated lattices generated by kites is generated by all finite-dimensional kites. In particular, we described some homomorphisms among kites.

Authors: A. Dvurečenskij (MI SAS), M. Botur (Uni. Olomouc, ČR)
Projects: APVV-16-0073 and VEGA No. 2/0069/16 SAV, GAČR 15-15286S
Reference: M. Botur, A. Dvurečenskij, Kites and residuated lattices, Algebra Universalis **79** (2018), Art. 83. 26 p. DOI: 10.1007/s00012-018-0564-2

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2. **The Ranges of Accepting State Complexities of Languages**

We examined the accepting state complexity, i.e., the minimal number of accepting states of deterministic finite automata for languages resulting from unary and binary operations on languages with accepting state complexity given as a parameter. This is the continuation of the work of J. Dassow: On the number of accepting states of finite automata, J. Autom., Lang. Comb., 21, 2016. We solved most of the open problems mentioned thereof. In particular, we considered the operations of intersection, symmetric difference, right and left quotients, reversal, and permutation (on finite languages), where we have obtained precise ranges of the accepting state complexities. The ranges are: from 0 to mn for intersection, set of natural numbers with zero for symmetric

difference and right and left quotient, set of natural numbers for reversal, and set of natural numbers without one for permutation on finite languages.

Authors: M. Hospodár (MI SAS), M. Holzer (Universität Giessen, DE)
Projects: VEGA 2/0084/15, APVV-15-0091
Reference: M. Hospodár, M. Holzer, The Ranges of Accepting State Complexities of Languages Resulting from Some Operations. In: Proceedings of CIAA 2018, LNCS 10977, Springer, pp. 198-210 (2018) ISBN 978-3-319-94811-9

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3. **On generating sets of the clone of aggregation functions on finite lattices**

In this paper, a general method for constructing generating sets of the aggregation clone on finite lattices is presented. The approach is based on extending of L -valued capacities leading to the so-called full systems of aggregation functions. Several full systems on L are presented and their arities are discussed.

Authors: R. Halaš (Uni. Olomouc, ČR), R. Mesiar (SUT Bratislava), **J. Pócs** (MI SAS)
Projects: GAČR no. 18-06915S, GAČR no. 18-06915S, APVV-16-0073
Reference: R. Halaš, R. Mesiar, J. Pócs: On generating sets of the clone of aggregation functions on finite lattices, Information Sciences **476** (2019), 38–47.

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Basic research (2019)

1. **Note on weakly fractional differential equations**

In this paper, we compare solutions of q-order fractional differential equations of Caputo type for q near 1 with solutions of the corresponding 1-order ordinary differential equations. By establishing the explicit lower and upper bounds of Mittag-Leffler functions, we obtain effective convergence results. It is shown that the limit cases $q \to 1^+$ and $q \to 1^-$ are different. A simple illustrative example is also presented.
Authors: **M. Fečkan** (MI SAS)**, M. Pospíšil** (MI SAS), J. Wang (Guizhou University, Guiyang, China)
Project: VEGA 2/0153/16
Reference: M. Fečkan, M. Pospíšil, J. Wang: *Note on weakly fractional differential equations*, Advances in Difference Equations 2019 (2019), No. 143, 11 pages.

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2. **Distribution function of Sequences: A Theory**

The study of the set of distribution functions of sequences is still unsatisfactory. This book contains: Basic properties of the set of distribution functions, e.g. it is nonempty, closed, and connected, examples of applications of distribution functions, e.g. general scheme of solution of Benford's law, use of statistically independent sequences, statistically convergent sequences, contains the complete solution of a new three-dimensional moment problem, contains computation of distribution functions for special sequences, e.g. 3/2 to the power of n modulo 1 with a functional equation, block sequences, distribution functions of multi-dimensional sequences, e.g. copulas g(x,y), where g(x,1)=x a g(1,y)=y, computation extremes of an integral $\int F(x,y)dg(x,y)$ over g(x,y), and finally the solution of an integral equation "$\int F(x,y)dg(x)dg(y)=0$" over distribution functions g(x) for copositive F(x,y) and a matrix form of the "$\int F(x,y)dg(x)dg(y)$".

Author: O. Strauch (MI SAS)
Project: VEGA grant 2/0109/18
Reference: Strauch, O., *Distribution Function of Sequences: A Theory*. ISBN 978-80-224-1734-1, VEDA, Bratislava, 2019, ISBN 978-80-200-3010-8, Academia, Prague, 2019, pp. 592.

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3. **Nondeterministic complexity in subclasses of convex languages**

We study the nondeterministic state complexity of basic regular operations on the classes of prefix-, suffix-, factor-, and subword-free, -closed, and -convex regular languages and on the classes of right, left, two-sided, and all-sided ideal regular languages. For the operations of concatenation, intersection, union, reversal, star, and complementation, we get tight upper bounds for all considered classes except for complementation on factor- and subword-convex languages. Most of our witnesses are described over optimal alphabets. The description of a proper suffix-convex (i.e., not suffix-free, closed, or ideal) language over a five-letter alphabet meeting the upper bound 2^n for complementation, and obtaining an asymptotically tight bound $\Theta(\sqrt{n})$ for complementation of unary prefix-free languages are among the most interesting results of this paper.

Authors: M. Hospodár (MI SAS)**,** G. Jirásková (MI SAS)**,** P. Mlynárčik (MI SAS)
Projects: VEGA 2/0084/15, APVV-15-0091
Reference: M. Hospodár, G. Jirásková, P. Mlynárčik: *Nondeterministic complexity in subclasses of convex languages*. Theoretical Computer Science 787 (2019), 89-110.
https://doi.org/10.1016/j.tcs.2018.12.027

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Applied research (2019)

1. **Linear comparative calibration**

A new comparative calibration model with a linear calibration function has been proposed, which establishes the relationship between two measuring instruments (measuring systems), each measuring two-dimensional signals. The design of such a two-dimensional calibration model is based on a metrological approach for expressing uncertainty in measurement. The calibration model thus obtained is, from a mathematical-statistical point of view, a model with errors-in-variables (EIV). Thus, the model is nonlinear in parameters. However, after suitable linearization parameters of the calibration function can be estimated using optimal estimation algorithms developed for linear regression models with type II conditions on the parameters. Based on this, an iterative algorithm was developed for optimal estimation of calibration function parameters as well as a method for evaluating uncertainties of the calibrated measuring instrument measurements.

Authors: V. Witkovský (IMS SAS), **G. Wimmer** (MI SAS)
Projects:VEGA 2/2054/18, VEGA 2/0081/19, APVV-15-0295
Reference: **Wimmer, G.,** Witkovský, V., *Two-dimensional linear comparative calibration and measurement uncertainty*, In: MEASUREMENT 2019: Proceedings of The 12th International Conference on Measurement - Bratislava, Slovakia: Institute of Measurement Science, Slovak Academy of Sciences (2019), 66-69 ISBN 978-80-972629-2-

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2. **Optimization of Gas Transport by Evolutionary Calculations on a Small Cluster of Computers**

Dynamic optimization of a large gas transport system is an active area and a highly challenging task, where only a few results were done till now. Our research was focused to utilize distributed

computing (on local resources) to solve such tasks in sufficiently short time to practical usage.

Distributed Evolution Strategy algorithm was implemented to solve transient optimization of the pipeline transport network. Here the transient optimization means optimization of gas consumption during the transition between two steady states by a predefined number of control events determined by the time and revs of each compressor group. We experiment with two concepts of local computer cluster parallelization: 1) Client-Server, where there is a central computer that assigns tasks to other computers, and collects and evaluates the results, and 2) Cooperative Agents, where each work station computes its own instance (agent) of Evolution Strategy and best results are exchanging among agents during computation. We also experiment with external computational resources on the Google cloud platform.

Authors: I. Mračka, T. Žáčik, M. Hyčko, R. Hajossy, P. Somora (Altova, GmbH)
Project: 1235 Gas transport optimization through transit pipelines
Reference: I. Mračka, T. Žáčik, M. Hyčko, R. Hajossy, P. Somora; *Optimization of Gas Transport – Evolutionary Calculations on a Small Cluster of Computers.* In: Pipeline Simulation Interest Group: PSIG–1915, 2019, London, UK

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3. **Symmetric nonlinear functional differential equations at resonance**

The purpose of these investigations is to present several conditions ensuring that a functional differential equation possessing a certain symmetry-invariance property has a symmetric solution. It was shown that the solvability of a problem concerning a class of symmetric solutions to scalar nonlinear functional differential equations at resonance with deviations from R into R can be investigated using the theory of boundary-value problems. Furthermore, we establish conditions on the unique solvability of scalar nonlinear functional differential equations with symmetries in a general form. Several examples illustrate our theory. Equations with functional perturbations are interesting from many points of view, because they describe the variable models in medicine, sociology (for example, Lotka-Volterra model), electrical engineering, and many other sciences.

Authors: N. Dilna (MI SAS), **M. Fečkan** (MI SAS), M. Solovyov (IEE SAS), J.R. Wang (Guizhou University, Guizhou, China), A. Ronto (MÚ AV ČR, Brno)
Projects: VEGA No. 2/0153/16, and 1/0078/17, APVV-16-0418
References:
1. N. Dilna, M. Fečkan, M. Solovyov, JR. Wang: *Symmetric nonlinear functional differential equations at resonance*. In Electronic Journal of Qualitative Theory of Differential Equations, 2019, no. 76, p. 1-16.
2. **N.** Dilna, M. Fečkan, A. Rontó: *On a class of functional differential equations with symmetries*. In SYMMETRY-BASEL, 2019, vol. 11. , 1456.

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International research (2019)

1. **No-free-information principle in general probabilistic theories**

We have introduced a new information-theoretic principle, the no-free-information principle, and compared it to the previously known no-broadcasting and no-information-without-disturbance principles. The no-broadcasting principle states that information cannot be copied, whilst the no-information-without-disturbance principle states that if we want to obtain any information about a system, we must disturb it, thereby changing the state it is in. The newly introduced no-free-information principle states that there is no type of information that can be learned about a system every time we perform any other measurement on it.

By noting that in quantum theory these principles hold, we formalized this problem in a more general class of theories, namely within the framework of general probabilistic theories (GPTs). These theories form a wide class of operational theories where many of the key features of quantum theory can be formulated more generally. By including both quantum and classical theories, as well as countless toy theories, the study of GPTs allows us to compare these theories based on their respective features and quantify their properties.
In the manuscript, we have mathematically characterized the principles in question within the GPT framework, tying our work with previously known results. We show that all three principles are strictly different, which means that there are operationally valid theories (even non-classical theories) where one can obtain non-trivial information about a system without disturbing it and where one can always choose to get some fixed non-trivial information when performing any other measurement.
The published manuscript is published in an open-access journal, i.e., it is freely available online.

Authors: T. Heinosaari, L. Leppäjärvi (Univ. Turku, Turku, Finland), **M. Plávala** (MI SAS)
Projects: VEGA 2/0069/16, APVV-16-0073
Reference: T. Heinosaari, L. Leppäjärvi, M. Plávala, *No-free-information principle in general probabilistic theories*, Quantum 3, 157 (2019).

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2. **Theory of EMV-algebras and states on them**

We have introduced EMV-algebras generalizing both MV-algebras and generalized Boolean algebras, where the top element is not assumed. These algebras resemble local properties of MV-algebras, however, in global, they are not necessarily equivalent to MV-algebras. We have shown that if an EMV-algebra has a top element, then it is equivalent to an MV-algebra, and if it has no top element, then it can be embedded into an EMV-algebra N with top element as a maximal ideal of N. We have shown a categorical equivalence of the category of EMV-algebras without top element with some category of MV-algebras.
We have studied states on EMV-algebras, and their topological properties. We have shown that the state space of an EMV-algebra without top element is locally compact and its one-point compactification is the state space of the representing EMV-algebra with top element.

Authors: A. Dvurečenskij (MI SAS), O. Zahiri (Tehran, Iran)
Projects: APVV-16-0073, VEGA No. 2/0069/16 SAV
References:
1. A. Dvurečenskij, O. Zahiri, *On EMV-algebras*, Fuzzy Sets and Systems **373** (2019), 116–148.
2. A. Dvurečenskij, O. Zahiri, *States on EMV-algebras*, Soft Computing **23** (2019), 7513--7536. DOI: 10.1007/s00500-018-03738-x

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3. **On the cardinality of the manifold set**

We study the following general problem in the theory of classification of high-dimensional manifolds. The structure set S(M) of M in the sense of surgery theory is a set in which all homotopy equivalences from all manifolds to M are organized together. The group of homotopy equivalences of M acts on it by post-composition and we call the quotient set the manifold set of M and denote it by M(M), it consists precisely of homeomorphism classes of manifolds homotopy equivalent to M. The task is, given a manifold M to investigate the behavior of this action with respect to cardinality.

The structure set has been studied extensively by surgery theory, but not much was known in general about M(M) and its relation to S(M). One result is that if the fundamental group contains torsion then M(M) is infinite and there are some calculations in special cases.

We show that for M simply connected, S(M) is infinite if and only if M(M) is infinite. In other words, we show that if there are infinitely many non-equivalent homotopy equivalences from some manifolds to M then among these manifolds are already infinitely many homeomorphism classes. We give a counter-example to this statement in the non-simply connected case and we also discuss some other more technical results in the non-simply connected case.
Potential applications of this result include the proof that M(M) is infinite in cases where this was not known before.

Authors: D. Crowley (University of Melbourne), **T. Macko** (MI SAS)
Projects: VEGA 1/0101/17 and APVV-16-0053
Reference: Crowley, D., Macko, T.: *On the cardinality of the manifold set*. In: Geometriae Dedicata (2019), Vol. 200 (1), pp 265-285, https://doi.org/10.1007/s10711-018-0370-1 (SCOPUS)

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Basic research (2020)

1. **Characterization of n-uninorms with continuous underlying functions and introduction of z-ordinal sum.**

The most important result was the introduction of the z-ordinal sum construction based on a partially ordered set of semigroups, which extends Clifford's ordinal sum based on a linearly ordered set of semigroups. Using this construction we were able to completely characterize n-uninorms with continuous underlying functions. In successive steps, we have shown the connection between these n-uninorms (first for idempotent and then also in general) and partial orders on the unit interval, which have the structure of a (binary) tree. Using this partial order we have shown that each n-uninorm with continuous underlying functions can be expressed as a z-ordinal sum of a countable number of semigroups related to continuous Archimedean and idempotent t-norms, t-conorms and uninorms, with respect to the set A, which corresponds to the division points $z_1,\ldots,z_{n-1}$. We have also shown that the set of points of discontinuity of such an n-uninorm is covered by graphs of its characterizing set-valued functions and we have discussed their properties.

Author: A. Mesiarová-Zemánková
Projects: VEGA 1/0006/19, APVV-16-0073 and Program Fellowship of SAS.
References:
1. A. Mesiarová-Zemánková, *The n-uninorms with continuous underlying t-norms and t-conorms*, International Journal of General Systems. doi: 10.1080/03081079.2020.1863395
2. A. Mesiarová-Zemánková, *Characterization of idempotent n-uninorms*, Fuzzy Sets and Systems. doi: 10.1016/j.fss.2020.12.019

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2. **When is the space of minimal usco/cusco maps a topological vector space**

We defined a vector structure on the space of minimal usco (resp. cusco) maps from a Baire space X to a Stegall locally convex space Y. We proved that for a bornology B on X with an open base the space of bounded minimal usco (resp. cusco) maps equipped with the topology of uniform convergence on B is a locally convex topological vector space. If X is locally compact, then the space of minimal usco (resp. cusco) maps from X to Y equipped with the topology of uniform convergence on compacta is a locally convex topological vector space.

Authors: Ľ. Holá, B. Novotný
Project: VEGA 2/0006/16
Reference: Ľ. Holá, B. Novotný, *When is the space of minimal usco/cusco maps a topological vector space*, J. Math. Anal. Appl. **489** (2020), artno. 124125

3. **Divisible extension of probability**

We outline the transition from classical probability space (Ω, A, p) to its "divisible" extension, where the σ -field A of Boolean random events is extended to the class M(A) of all measurable functions into [0,1] and the σ-additive probability measure p on A is extended to the probability integral ∫(.)dp on M(A). The resulting extension of (Ω, A, p) can be described as an epireflection reflecting A to M(A) and p to ∫(.)dp

The transition from A to M(A), resembling the transition from whole numbers to real numbers, is characterized by the extension of two-valued Boolean logic on A to multivalued Lukasiewicz logic on M(A) and the divisibility of random events: for each random event u is an element of M(A) and each positive natural number n we have u/n is an element of M(A) and ∫(u/n)dp = (1/n) ∫u dp. From the viewpoint of category theory, objects are of the form M(A), morphisms are observables from one object into another one and serve as channels through which stochastic information is conveyed.

We study joint random experiments and asymmetrical stochastic dependence/independence of one constituent experiment on the other one. We present a canonical construction of conditional probability so that observables can be viewed as conditional probabilities.

In the present paper, we utilize various published results related to "quantum and fuzzy" generalizations of the classical theory, but our ultimate goal is to stress mathematical (categorical) aspects of the transition from classical to what we call divisible probability.

Authors: R. Frič, P. Eliaš, M. Papčo
Project: APVV-16-0073
Reference: *Divisible extension of probability*, Math. Slovaca **70** (2020), 1445–1456. doi: 10.1515/ms-2017-0441

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Applied research (2020)

1. **Advanced statistical and computational methods for measurement and metrology**

We suggested a non-parametric method and algorithm for estimating the probability distribution of a stochastic sum of independent identically distributed continuous random variables, based on combining and numerically inverting the associated empirical characteristic function (CF) derived from the observed data. This is motivated by classical problems in financial risk management, actuarial science, and hydrological modelling. This approach can be naturally generalized to more complex semi-parametric modelling and estimating approaches, e.g., by incorporating the generalized Pareto distribution fit for modelling heavy tails of the considered continuous random variables, or by considering the weighted mixture of the parametric CFs (used to incorporate the expert knowledge) and the empirical CFs (used to incorporate the knowledge based on the observed or historical data). The suggested numerical approach is based on a combination of the Gil-Pelaez inversion formulae for deriving the probability distribution (PDF and CDF) from the associated CF and the trapezoidal quadrature rule used for the required numerical integration. The presented non-parametric estimation method is related to the bootstrap estimation approach, and thus, it shares similar properties. The applicability of the proposed estimation procedure is illustrated by estimating the aggregate loss distribution from the well-known Danish fire losses data.

Authors: V. Witkovský (IMS SAS), **G. Wimmer** (MI SAS), T. Duby (OAA Computing Ltd, Bicester, UK)
Projects: APVV-15-0295, VEGA 2/0054/18
Reference: V. Witkovský, **G. Wimmer**, T. Duby, *Estimating the distribution of a stochastic sum of IID random variables*, Mathematica Slovaca, **70** (2020), 759–774.

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2. **Analysis and modelling of COVID-19 spreading**

Due to the SARS-CoV2 epidemiological situation in 2020, we worked on several mathematical models of pandemic development.

The first model is designed to monitor the spread of epidemiological waves between different geographical regions (countries). The situation in the region and the transport between regions are governed by local epidemiological measures. The model can be used to analyze and design epidemiological semaphores.

The second model is based on an agent approach and describes the spread of COVID-19 within one region (e.g., district, province, the whole territory of Slovakia). The model takes into account RT-PCR testing, screening of infected and mass area-wide testing using rapid tests (e.g.,antigen tests). The model is able to reconstruct the observed daily increment of infected, which allows the calibration of the model parameters. In addition, the model allows the analysis of various nationwide testing strategies.

Authors: I. Mračka, T. Žáčik, J. Bogár, R. Hajossy, I. Odrobina, M. Hyčko (MI SAS) + inter-institutional consultations with other colleagues of SAS (Institute of Geography of the SAS, Institute of Physics of the SAS, Institute for Sociology of the SAS, Institute of Virology – Biomedical Research Centre of the SAS).
Project: Institutional project.

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International research (2020)

1. **The ranges of accepting state complexities of languages resulting from some operators**

We examine the accepting state complexity, i.e., the minimal number of accepting states of deterministic finite automata for languages resulting from unary and binary operations on languages with accepting state complexity given as a parameter. This is a continuation of the work of [J. Dassow: On the number of accepting states of finite automata, J. Autom., Lang. Comb., 21, 2016]. We solve most of the open problems mentioned thereof. In particular, we consider the operations of intersection, symmetric difference, right and left quotients, reversal, and permutation, where we obtain precise ranges of accepting state complexities. We also consider symmetric differences on unary finite languages where we obtain a non-contiguous range of accepting state complexities.

Authors: M. Holzer (Universität Giessen), **M. Hospodár** (MI SAS)
Projects: VEGA 2/0132/19, APVV-15-0091, DAAD short-term grant ID 57314022
Reference: M. Holzer, **M. Hospodár**, *The ranges of accepting state complexities of languages resulting from some operations*, International Journal of Foundations of Computer Science, **31** no. 8 (2020), 1159–1177. doi: 10.1142/S0129054120420083

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2. **The periodic structure of a quantum channel**

Quantum channels are fundamental objects in quantum theory. They are widely used to represent the evolution of an open quantum system in discrete-time models. In finite dimensions, the structure of quantum channels is already quite well understood. We study the infinite-dimensional case under the assumption of the existence of a faithful invariant state, when the system can be decomposed into a “stable” and a “reversible” part with respect to the evolution (the Jacobs-DeLeeuw-Glicksberg type decomposition). We show that the reversible part coincides with the largest

subalgebra where the channel acts as an endomorphism. Consequently, this subalgebra is the range of a normal conditional expectation. Using this result, we give a precise description of the structure of the channel, its fixed points, and invariant states, as well as the periodic behaviour of the induced discrete-time semigroup. Similar results hold also in the continuous-time case.
Authors: A. Jenčová (MI SAS), R. Carbone (Univ. Di Pavia, Italy)
Projects: VEGA 2/0069/16, APVV-16-0073
Reference: R. Carbone, A. Jenčová, *On period, cycles and fixed points of a quantum channel*, Annales Henri Poincaré **21** (2020), 155–188.

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3. **On the Poincaré-Adronov-Melnikov method for the existence of grazing impact periodic solutions of differential equations**

Perturbative methods are well-established ways to obtain the existence of periodic solutions in smooth differential equations. In the study of non-smooth systems of differential equations, however, one has to face with different kinds of periodic solutions. Suppose that the differential equation is non-smooth along a certain surface S. Interesting situation arises when a map M acts on S in such a way that when a solution of the differential equation hits S, then the solution continues immediately at another point on S given by M. Solutions satisfying this property are called impact solutions. Then it is interesting to study the existence of solutions touching S. These solutions are called grazing impact solutions. Such kinds of solutions may be seen as solutions determining a border between solutions hitting S and those that do not hit S at all. This paper studies the persistence of periodic grazing impact solutions for periodically perturbed differential equations with impacts. An approach of the Poincaré-Adronov-Melnikov method is applied. It is based on introducing and studying the appropriate impact of Poincaré mapping. A typical example of an impact non-smooth model is a bouncing ball or swinging pendulum in a bell.

Autori: F. Battelli (Univ. Ancona, Italy), **M. Fečkan** (MI SAS)
Projekt: VEGA 2/0153/16
Referencia: F. Battelli, M. Fečkan: *On the Poincaré-Adronov-Melnikov method for the existence of grazing impact periodic solutions of differential equations*, Journal of Differential Equations **268** (2020), 3725–3748.

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Basic research (2021)

1. **Spectrality in order unit spaces.**

An important property of any mathematical model of quantum mechanics is spectrality, that is, the existence of spectral resolutions of effects that allows an integral expression in terms of some special elements called projections. In the operational approach to the foundations of quantum mechanics, a physical system is described by an order unit space in duality with a base normed space, where elements of the distinguished base correspond to states of the system and the order unit space describes its measurements. Perhaps the best well-known extension of spectrality to this setting is due to Alfsen and Shultz, based on the geometry of the dual pair. In the paper, we compare this definition to a purely algebraic approach due to Foulis, based on properties of compressions on ordered groups with an order unit. We show that the latter approach is strictly more general. In particular, we study in detail the case of JB-algebras, where Alfsen-Shultz spectrality holds for JBW-algebras and spectrality in Foulis sense is proved equivalent to the Rickart property, and the order unit spaces obtained from reflexive Banach spaces, where Foulis spectrality is equivalent to the smoothness of the norm, whereas the Alfsen-Shultz spectrality requires also strict convexity.

Authors: A. Jenčová, **S. Pulmannová** (MI SAS)**,**
Projects: VEGA 2/0142/20, APVV-16-0073.
Reference: A. Jenčová, S. Pulmannová, *Geometric and algebraic aspects of spectrality in order unit spaces: a comparison*, Journal of Mathematical Analysis and Applications, **504** (2021), Art. Num. 125360. DOI: 10.1016/j.jmaa.2021.125360

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2. **Characterization of some classed of functions constructed via (z-)ordinal sum and further properties of (z-)ordinal sum**

We characterized all functions that can be obtained as a z-ordinal sum of semigroups related to continuous t-norms, t-conorms, representable uninorms and idempotent semigroups. We showed that this class of functions corresponds to the class of all commutative, associative, and non-decreasing binary functions on the unit interval, which have continuous Archimedean components and are continuous on the diagonal. We presented sufficient and necessary conditions for obtaining uninorms as an ordinal sum of semigroups with carriers that are non-empty subintervals of the unit interval. We proved that each uninorm locally internal on $A(e)$ can be decomposed into an ordinal sum of such semigroups. Further, we studied the uniqueness of the link between t-norms or t-conorms, and related Archimedean components, we showed infinite ordinal sums of aggregation functions covered by one type of ordinal sums both t-norms and t-conorms ordinal sums, and studied the cardinality of the corresponding index sets. For z-ordinal sum we showed that it can always be expressed in the reduced basic form, i.e., that it is enough to assume only trivial semigroups in the branching set and we can always remove semigroups with duplicate carriers. We also investigated the cardinality of the minimal branching set.

Author: A. Mesiarová-Zemánková (MI SAS)
Projects: VEGA 1/0006/19, APVV-16-0073, APVV-20-0069 and Program Fellowship of SAS.
References:
1. A. Mesiarová-Zemánková, R. Mesiar, Y. Su, Ordinal sum constructions for aggregation functions on the real unit interval, Iranian Journal of Fuzzy Systems, DOI: 10.22111/ijfs.2022.6553
2. A. Mesiarová-Zemánková, Commutative, associative and non-decreasing functions continuous around diagonal, Iranian Journal of Fuzzy Systems, DOI: 10.22111/ijfs.2022.6786
3. A. Mesiarová-Zemánková, A note on the simplification of the z-ordinal sum construction, Fuzzy Sets and Systems, DOI: 10.1016/j.fss.2022.05.011
4. Y. Su, W. Zong, A. Mesiarová-Zemánková, Constructing uninorms via ordinal sums in the sense of A. H. Clifford, Semigroup Forum, DOI: 10.1007/s00233-022-10287-1

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3. **Quantum Rényi relative entropies**

The concept of a relative entropy (or divergence) as a measure of informational dissimilarity of states is fundamental in information theory. The classical Rényi relative entropies were introduced by an axiomatic approach as the unique family of divergences satisfying certain natural properties. As it turned out, these quantities play a central role in many information-theoretic tasks. However, there are many possible versions of Rényi relative entropies for quantum systems. One of the versions that have proved useful for finite dimensional systems is the family of sandwiched Rényi relative entropies. In this paper, we extend this family to normal states of an arbitrary von Neumann algebra and show some of its properties, in particular that it can be obtained from an interpolating family of noncommutative Kosaki L_p-spaces. We also show that for all relevant values of the parameter, these quantities are nonincreasing under unital normal positive maps, which was

previously known only for completely positive maps or for restricted values of the parameter. We also discuss the situation when some of these quantities is preserved under such a map for a given pair of states and show that if the map is 2-positive, this happens if and only if the two states can be fully recovered.

Authors: A. Jenčová (MI SAS)
Projects: VEGA 2/0142/20, APVV-16-0073
Reference: A. Jenčová, *Rényi relative entropies an noncommutative L_p-spaces* II, Annales Henri Poincaré **22** (2021), 3235–3254. DOI: 10.1007/s00023-021-01074-9

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4. **On Mahler's conjecture**

Mahler's conjecture states that there exists no such real number ξ that the fractional parts of the sequence $\{x(n)\} = \{(3/2)^n\}$ satisfy $x(n) < 1/2$ for all $n = 0,1,2,\ldots$. If such ξ exists, then $x(n)$ has an asymptotic distribution function $g(x) = 1$ for $x \in (0,1]$. For the proof we used the integral explicit formula of integral

$$\iint F(x,y)\, dg(x)\, dg(y)$$

for distribution function $g(x)$, where $F(x,y)$ is the sum of absolute values of fractional parts $F(x,y) = |\{2x\} - \{3y\}| + |\{2y\} - \{3x\}| - |\{2x\} - \{2y\}| - |\{3x\} - \{3y\}|$. For proof we used Young's integral.

Authors: O. Strauch (MI SAS)
Projects: VEGA 2/0109/18
Reference: O. Strauch, *Mahler's conjecture on $\xi(3/2)^n$ mod 1*, Unif. Distrib. Theory **16** (2021), no. 2, 49–70. DOI: 10.2478/udt-2021-0007

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Applied research (2021)

1. **Two-dimensional linear comparative calibration**

We propose a new calibration model with linear calibration function, based on comparison of two measuring devices – each is measuring two-dimensional measurements with normally distributed errors. From statistical point of view, the considered calibration model is nonlinear in the model parameters, however, after proper linearization, the model can be represented by the linear errors-in-variables (EIV) model. Based on that, we suggest an iterative algorithm for estimating the parameters of the linear calibration function and present the approximate confidence region for the calibration function parameters. Finally, we describe the measuring process and statistical properties of the measurements realized with the calibrated device.

Authors: G. Wimmer (MI SAS), V. Witkovský (IMS SAS), K. Žáková Myšková (Masarykova univ., Brno, Czech. Rep.)
Projects: APVV-15-0295, VEGA 2/0054/18 , VEGA 2/0081/19
Reference: G. Wimmer, V. Witkovský, K. Žáková Myšková, *Two-dimensional linear comparative calibration*, Perspectives in Measurement, Modeling and Interpretation. - Bratislava, Slovakia: Vysoká škola manažmentu/City University of Seattle programs, 2020, 15–37. ISBN 978-80-89306-54-1.

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2. **Description of the exponential phase of the COVID-19 epidemic**

We used a population matrix model to describe the exponential phase of the COVID-19 epidemic. We have extended this model so that it can also describe periodic mass testing followed by isolation of positive cases. We researched and compared different ways of distributing tests over time: for example, testing the entire population on one day of the week only versus evenly distributed testing throughout the whole week, and so on. In addition, we looked for a distribution of those tested in which the maximum or minimum effect on the course of the epidemic was achieved. Due to the isolation of the positive tested, this effect results in a reduction of the current reproductive number. We have shown that with the sensitivity of the available rapid antigen tests and the real achievable participation of the population in different types of periodic testing, a difference in reproductive numbers of no more than 2 % is achieved. It follows that in the practical implementation of mass testing, it is reasonable to prioritize less logistically demanding continuous testing throughout the week over bulk testing on weekends.

Authors: I. Mračka, **M. Hyčko**, **R. Hajossy**, **I. Odrobina**, **T. Žáčik** (MI SAS)
Project: Institutional project.

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3. **Applications of mathematical statistics in analyses of language and text**

New results were obtained in the diachronic analysis of Czech. Using statistical analyses, language properties that undergo a change in diachronic development were distinguished from those which remain invariant. Based on cluster analysis of the most frequent words, some progress was achieved in solving the problem of the authorship of older Czech manuscripts. A synergetic language model was developed also for lower language units (syllables, morphemes). Relations between their properties and the properties of words were investigated.

Authors: J. Mačutek (MI SAS), R. Čech (Univ. Ostrava), P. Kosek,
O. Navrátilová (Masaryk Univ. Brno), etc.
Project: VEGA 2/0096/21
References:

1. R. Čech, P. Kosek, **J. Mačutek**, M. Nogolová, *(Ne)stabilita českého biblického překladu: diachronní stylometrická analýza*, Clavibus unitis, 2021, vol. 10, no. 2, 151–156. ISSN 1803-7747.
2. R. Čech, **J. Mačutek,** P. Kosek, *Czech translations of the Gospel of Matthew from the diachronic point of view – Plus ça change…*, Jazykovedný časopis, 2021, roč. 72, č. 2, s. 656–666. DOI: 10.2478/jazcas-2021-0059
3. R. Čech, P. Kosek, O. Navrátilová, **J. Mačutek**, *On the impact of the initial phrase length on the position of enclitics in Old Czech*, Language and Text: Data, models, information and applications. - Amsterdam : John Benjamin Publishing Company, 2021, p. 9–20. ISBN 978-90-272-1010-4. DOI: 10.1075/cilt.356.01cec
4. M. Kubát, Š. Netolická, R. Čech, **J. Mačutek,** *Martin of Cochem´s Golden Key of Heaven and its Czech Relatives: Quantitative Analysis of Baroque Prayers*,Bohemistyka, 2021, vol. 3, p. 283–294. ISSN 1642-9893. DOI: 10.14746/bo.2021.3.
5. K. Pelegrinová, **J. Mačutek,** R. Čech, *The Menzerath-Altmann law as therelationbetween lengths of words and morphemes in Czech*, In Jazykovedný časopis, 2021, roč. 72, č. 2, s. 405–414. ISSN 0021-5597. DOI: 10.2478/jazcas-2021-0037
6. B. Rujevic, M. Kaplar, S. Kaplar, R. Stankovic, I. Obradovic, **J. Mačutek,** *Quantitative analysis of syllable properties in Croatian, Serbian, Russian, and Ukrainian* Language and Text: Data, models, information and applications. - Amsterdam: John Benjamin Publishing Company, 2021, p. 55–67. ISBN 978-90-272-1010-4. DOI: 10.1075/cilt.356.04ruj

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International research (2021)

1. **On convergence of eigenvalues and eigenvectors in the block-Jacobi EVD algorithm with dynamic ordering**

In the block version of the classical two-sided Jacobi method for the Hermitian eigenvalue problem, the off-diagonal elements of iterated matrix converge to zero. However, this fact alone does not necessarily guarantee that this matrix converges to a fixed diagonal matrix. The same is true for the matrix of accumulated unitary transformations. We have proved that under certain assumptions the iterated matrix indeed converges to a fixed diagonal matrix, whose diagonal elements are the eigenvalues of the input matrix. Next it was shown that for a simple eigenvalue the corresponding column of the accumulated matrix of unitary transformations converges to the corresponding eigenvector. For a multiple eigenvalue or a cluster of eigenvalues, we proved that the orthogonal projectors constructed from the corresponding columns of the accumulated matrix of unitary transformations converge to the orthogonal projector onto the eigenspace corresponding to those eigenvalues. Moreover, the appropriate convergence bounds were obtained for all discussed cases. Convergence results are also valid for the parallel block-Jacobi method with dynamic ordering. The developed theory is illustrated by numerical example.

Authors: Y. Yamamoto (University of Electro-Communications, Tokyo), **G. Okša** (MI SAS), **M. Vajteršic** (MI SAS)
Projects: VEGA 2/0015/20
Reference:

Y. Yamamoto, G. Okša, M. Vajteršic, *On convergence to eigenvalues and eigenvectors in the block-Jacobi EVD algorithm with dynamic ordering*, Linear Algebra and Its Applications **622** (2021), 19–45. DOI: 10.1016/j.laa.2021.03.027

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2. **Power, positive, closure, and quotients on convex languages**

We study the state complexity and nondeterministic state complexity of the k-th power, positive closure, right quotient, and left quotient on the classes of prefix-, suffix-, factor-, and subword-free, -closed, and -convex regular languages, and on the classes of right, left, two-sided, and all-sided ideal languages. We show that the nondeterministic state complexity of the k-th power is kn for closed and convex languages, and $k(n-1)+1$ in the remaining classes, while its state complexity is $n+(k-1)2^n-2$ for right ideals, $k(n-1)+1$ for other ideals and factor- and subword-closed languages, and $k(n-2)+2$ for prefix-, factor-, and subword-free languages. We next prove that the nondeterministic state complexity of positive closure is 1 for factor- and subword-closed languages and n for all other classes, while its state complexity is 2 for factor- and subword-closed languages, $2^{n-2}+1$ for prefix-closed and suffix-free languages, and n for all other considered classes. Besides, we study quotients on convex languages.

Authors: M. Hospodár (MÚ SAV), M. Holzer (Universität Giessen)
Projects: VEGA 2/0132/19, APVV-15-0091
Reference:

M. Holzer, M. Hospodár, *Power, positive closure, and quotients on convex languages*, Theoretical Computer Science, **870** (2021), 53–74. DOI: 10.1016/j.tcs.2021.02.002

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3. **USCO and Quasicontinuous Mappings**

This book presents two natural generalizations of continuous mappings, namely usco and quasicontinuous mappings. The first class considers set-valued mappings, the second class relaxes the definition of continuity. Both these topological concepts stem naturally from basic mathematical considerations and have numerous applications that are covered in detail.

It is accessible to researchers who want to learn about Usco and Quasicontinuous mappings.

The book presents the most significant results in the field with applications.

Autors: Ľ. Holá (MI SAS), D. Holý (Univ. Trnava) and W. Moors (Univ. Auckland, New Zeeland)
Project: VEGA 2/0006/16
Reference:

Ľ. Holá, D. Holý, W. Moors, *USCO and Quasicontinuous Mappings*, De Gruyter Studies in Matematics Volume **81**, 2021, pp. viii + 298 pages. ISBN 978-31-1075-015-7, ebook 978-31-1075-018-8. DOI: 10.1515/9783110750188

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4. **Periodic solutions in slowly varying discontinuous differential equations**

We study persistence of periodic solutions of perturbed discontinuous differential equations with a small parameter, which represents a slowly varying variable. We assume that the unperturbed (frozen) differential equation has a non-singular periodic solution. The results of this paper are motivated by two dimensional systems of Hamiltonian differential equations depending on a scalar variable, which is the solution of a singularly perturbed equation. Sufficient conditions for bifurcations of periodic solutions are derived. The theory is applied to a specific 3-dimensional system of differential equations with a discontinuous component modeling the so-called dry friction or switching.

Autors: M. Fečkan (MI SAS), F. Battelli (Polytech., Ancona, Italy)
Projects: APVV-18-0308, VEGA 1/0358/20, VEGA 2/0127/20
Reference:

F. Battelli, M. Fečkan, *Periodic solutions in slowly varying discontinuous differential equations: the generic case*, Mathematics **9** (19) (2021), Art. Num. 2449.
DOI: 10.3390/math9192449

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2.1.2 List of selected publications documenting the most important results of basic research. The total number of publications should not exceed the number of average FTE researchers per year. The principal research outputs (max. 10% of the total number of selected publications, including Digital Object Identifier – DOI if available) should be underlined. Authors from the evaluated organizations should be underlined.

2016

1. DI LUNA, G. - DOBREV, Stefan - FLOCCHINI, Paola - SANTORO, Nicola. Live exploration of dynamic rings. In ICDCS 2016: IEEE 36th International Conference on Distributed Computing Systems : proceedings. - Danvers : IEEE, 2016, p. 570-579. ISBN 978-1-5090-1482-8 https://doi.org/10.1109/ICDCS.2016.59
2. HALAŠ, Radomír - PÓCS, Jozef. On the clone of aggregation functions on bounded lattices. In Information Sciences, 2016, vol. 329, p. 381-389. (3.364 - IF2015). (2016 -

Current Contents). ISSN 0020-0255. https://doi.org/10.1016/j.ins.2015.09.038

3. JENČOVÁ, Anna. On the convex structure of process positive operator valued measures. In Journal of Mathematical Physics, 2016, vol. 57, no. 1, 14 p. (1.234 - IF2015). (2016 - Current Contents). ISSN 0022-2488. https://doi.org/10.1063/1.4935072
4. MESIAROVÁ-ZEMÁNKOVÁ, Andrea. A note on decomposition of idempotent uninorms into an ordinal sum of singleton semigroups. In Fuzzy Sets and Systems, 2016, vol. 299, p. 140-145. (2.098 - IF2015). (2016 - Current Contents). ISSN 0165-0114. https://doi.org/10.1016/j.fss.2016.04.007
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2017

1. DOBREV, Stefan - EDMONDS, Jeff - KOMM, Dennis - KRÁLOVIČ, Rastislav - KRÁLOVIČ, Richard - KRUG, Sacha - MÖMKE, Tobias. Improved analysis of the online set cover problem with advice. In Theoretical Computer Science, 2017, vol. 689, p. 96-107. (0.698 - IF2016). (2017 - Current Contents). ISSN 0304-3975. https://doi.org/10.1016/j.tcs.2017.05.029
2. DVUREČENSKIJ, Anatolij - HYČKO, Marek. Hyper effect algebras. In Fuzzy Sets and Systems, 2017, vol. 326, p. 34-51. (2.718 - IF2016). (2017 - Current Contents). ISSN 0165-0114. https://doi.org/10.1016/j.fss.2016.12.012
3. JENČOVÁ, Anna - PLÁVALA, Martin. Conditions on the existence of maximally incompatible two-outcome measurements in general probabilistic theory. In Physical Review A, 2017, vol. 96, no. 2, art. no. 022113, p. [1-7]. (2016: 2.925 - IF, Q1 - JCR, 1.482 - SJR, Q1 - SJR CCC). (2017 - Current Contents). ISSN 1050-2947. https://doi.org/10.1103
4. OKŠA, Gabriel - YAMAMOTO, Yusaku - VAJTERŠIC, Marián. Asymptotic quadratic convergence of the serial block-Jacobi EVD algorithm for Hermitian matrices. In Numerische Mathematik, 2017, vol. 136, no. 4, p. 1071-1095. (2.152 - IF2016). (2017 - Current Contents). ISSN 0029-599X. DOI 10.1007/s00211-016-0863-5
5. PLÁVALA, Martin. Conditions for the compatibility of channels in general probabilistic theory and their connection to steering and Bell nonlocality. In Physical Review A, 2017, vol. 96, no. 5, art. no. 052127, [p. 1-17]. (2.925 - IF2016). (2017 - Current Contents). ISSN 1050-2947. DOI:https://doi.org/10.1103/PhysRevA.96.052127
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2018

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2. BOTUR, Michal - HALAŠ, Radomír - MESIAR, Radko - PÓCS, Jozef. On generating of idempotent aggregation functions on finite lattices. In Information Sciences, 2018, vol. 430-431, p. 39-45. (4.305 - IF2017). (2018 - Current Contents). ISSN 0020-0255. https://doi.org/10.1016/j.ins.2017.11.031
3. BORZOOEI, R.A. - DVUREČENSKIJ, Anatolij - SHARAFI, A.H. Material implications in lattice effect algebras. In Information Sciences, 2018, vol. 433-434, p. 233-240. (4.305 - IF2017). (2018 - Current Contents). ISSN 0020-0255. DOI: https://doi.org/10.1016/j.ins.2017.12.049

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5. MESIAROVÁ-ZEMÁNKOVÁ, Andrea. Characterization of uninorms with continuous underlying T-norm and T-conorm by their set of discontinuity points. In IEEE Transactions on Fuzzy Systems, 2018, vol. 26, no. 2, p. 705-714. (2017: 8.415 - IF, Q1 - JCR, 4.024 - SJR, Q1 - SJR, karentované - CCC). (2018 - Current Contents). ISSN 1063-6706. https://doi.org/10.1109/TFUZZ.2017.2688346
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2019

1. STRAUCH, Oto. Distribution of sequences: A theory. Bratislava : Veda ; Praha : Academia, 2019. 591 p. ISBN 978-80-224-1734-1
2. DVUREČENSKIJ, Anatolij - ZAHIRI, Omid. States on EMV-algebras. In Soft Computing, 2019, vol. 23, no. 17, p. 7513-7536. (2018: 2.784 - IF, Q2 - JCR, 0.617 - SJR, Q2 - SJR, CCC). (2019 - Current Contents). ISSN 1432-7643. https://doi.org/10.1007/s00500-018-03738-x
3. FEČKAN, Michal - POSPÍŠIL, Michal - WANG, JinRong. Note on weakly fractional differential equations. In Advances in Difference Equations, 2019, vol. 143, p. [1-11]. (2018: 1.510 - IF, Q1 - JCR, 0.525 - SJR, Q2 - SJR, CCC). (2019 - Current Contents). ISSN 1687-1839. https://doi.org/10.1186/s13662-019-2086-4
4. MING, Hao - WANG, JinRong - FEČKAN, Michal. The application of fractional calculus in chinese economic growth models. In Mathematics, 2019, vol. 7, no. 8. (2018: 1.105 - IF, Q1 - JCR, 0.244 - SJR, Q3 - SJR, karentované - CCC). (2019 - Current Contents). ISSN 2227-7390. https://doi.org/10.3390/math7080665
5. OKŠA, Gabriel - YAMAMOTO, Yusaku - BEČKA, Martin - VAJTERŠIC, Marián. Asymptotic quadratic convergence of the two-sided serial and parallel block-Jacobi SVD algorithm. In Siam Journal on Matrix Analysis and Applications, 2019, vol. 40, no. 2, p. 639-671. (2018: 1.912 - IF, Q1 - JCR, 1.248 - SJR, Q1 - SJR, CCC). (2019 - Current Contents). ISSN 1095-7162. https://doi.org/10.1137/18M1222727
6. ŠUCH, Ondrej - KLIMO, Martin - KEMP, N.T. - ŠKVAREK, Ondrej. Passive memristor synaptic circuits with multiple timing dependent plasticity mechanisms. In AEU-International Journal of Electronics and Communications, 2018, vol. 96, p. 252-259. (2017: 2.115 - IF, Q2 - JCR, 0.420 - SJR, Q2 - SJR, CCC). (2018 - Current Contents). ISSN 1434-8411. https://doi.org/10.1016/j.aeue.2018.09.025

2020

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2. CARBONE, Raffaella - JENČOVÁ, Anna. On period, cycles and fixed points of a quantum channel. In Annales Henri Poincare, 2020, vol. 21, p. 155-188. (2019: 1.489 - IF, Q2 - JCR, 1.214 - SJR, Q1 - SJR, CCC). (2020 - Current Contents). ISSN 1424-0637. https://doi.org/10.1007/s00023-019-00861-9

3. SHER, Muhammad - SHAH, Kamal - FEČKAN, Michal - RAHMAT ALI, Khan. Qualitative analysis of multi-terms fractional order delay differential equations via the topological degree theory. In Mathematics, 2020, vol. 8, no. 218, p. 1-13. (2019: 1.747 - IF, Q1 - JCR, 0.299 - SJR, Q3 SJR, CCC). (2020 Current Contents). ISSN 2227-7390. https://doi.org/10.3390/math8020218
4. DVUREČENSKIJ, Anatolij - LACHMAN, Dominik. Spectral resolutions and observables in n-perfect MV-algebras. In Soft Computing, 2020, vol. 24, p. 843-860. (2019: 3.050 - IF, Q2 - JCR, 0.705 - SJR, Q2 - SJR, CCC). (2020 - Current Contents). ISSN 1432-7643. DOI: 10.1007/s00500-019-04543-w
5. HOLÁ, Ľubica - NOVOTNÝ, Branislav. When is the space of minimal usco/cusco maps a topological vector space. In Journal of Mathematical Analysis and Applications, 2020, vol. 489, p. 1-15. (2019: 1.220 - IF, Q1 - JCR, 1.021 - SJR, Q1 - SJR, CCC). (2020 - Current Contents). ISSN 0022-247X. https://doi.org/10.1016/j.jmaa.2020.124125
6. JENČOVÁ, Anna - PLÁVALA, Martin. Structure of quantum and classical implementations of the Popescu-Rohrlich box. In Physical Review A, 2020, vol. 102, art.no. 042208, p. 1-14. (2019: 2.777 - IF, Q2 - JCR, 1.416 - SJR, Q1 - SJR, CCC). (2020 - Current Contents). ISSN 1050-2947. https://doi.org/10.1103/PhysRevA.102.042208

2021

1. HOLÁ, Ľubica - HOLÝ, Dušan - MOORS, Warren. USCO and Quasicontinuous Mappings. Berlin : Walter de Gruyter, 2021. 295 p. Studies in Mathematics, 81. https://doi.org/10.1515/9783110750188-201. ISBN 978-3-11-075015-7
2. AUBRUN, Guillaume - LAMI, Ludovico - PALAZUELOS, Carlos - PLÁVALA, Martin. Entangleability of Cones. In Geometric and functional analysis, 2021, vol. 31, no. 1, p. 1-25. (2020: 2.148 - IF, Q1 - JCR, 3.952 - SJR, Q1 - SJR, CCC). (2021 - Current Contents). ISSN 1016-443X. https://doi.org/10.1007/s00039-021-00565-5
3. DI NOLA, Antonio - DVUREČENSKIJ, Anatolij - LAPENTA, Serafina. An approach to stochastic processes via non-classical logic. In Annals of Pure and Applied Logic, 2021, vol. 172, art. no. 103012. (2020: 0.678 - IF, Q2 - JCR, 0.943 - SJR, Q1 - SJR, CCC). (2021 - Current Contents). ISSN 0168-0072. https://doi.org/10.1016/j.apal.2021.103012
4. FEČKAN, Michal - WANG, JinRong - ZHAO, Hou Yu. Maximal and minimal nondecreasing bounded solutions of iterative functional differential equations. In Applied Mathematics Letters, 2021, vol. 113, p. 1-7. (2020: 4.055 - IF, Q1 - JCR, 1.439 - SJR, Q1 - SJR, CCC). (2021 - Current Contents). ISSN 0893-9659. https://doi.org/10.1016/j.aml.2020.106886
5. JADLOVSKÁ, Irena - CHATZARAKIS, George E. - DŽURINA, Jozef - GRACE, Said R. On Sharp Oscillation Criteria for General Third-Order Delay Differential Equations. In Mathematics, 2021, vol. 9, no. 14, art. nr. 1675, p. 1-18. (2020: 2.258 - IF, Q1 - JCR, 0.495 - SJR, Q2 - SJR, CCC). (2021 - Current Contents). ISSN 2227-7390. https://doi.org/10.3390/math9141675
6. MESIAROVÁ-ZEMÁNKOVÁ, Andrea. Convex combinations of uninorms and triangular subnorms. In Fuzzy Sets and Systems, 2021, vol. 423, p. 55-73. (2020: 3.343 - IF, Q1 - JCR, 0.902 - SJR, Q1 - SJR, CCC). (2021 - Current Contents). ISSN 0165-0114. https://doi.org/10.1016/j.fss.2020.10.011

2.1.3 List of monographs/books published abroad

1. FEČKAN, Michal - POSPÍŠIL, Michal. Poincaré-Andronov-Melnikov Analysis for Non-Smooth Systems. Amsterdam : Elsevier, 2016. 244 p. ISBN 978-0-12-804294-6

2. BARTKOVÁ, Renáta - RIEČAN, Beloslav - TIRPÁKOVÁ, Anna. Probability theory for fuzzy quantum spaces with statistical applications. Sharjah, UAE : Bentham Science Publishers, 2017. 190 p. ISBN 978-1-68108-539-5.
3. FEČKAN, Michal - WANG, JinRong - POSPÍŠIL, Michal. Fractional-order equations and inclusions. Berlin : Walter de Gruyter, 2017. 366 p. Fractional Calculus in Applied Sciences and Engineering, vol. 3. ISBN 978-3-11-052138-2.

4. PAŠTÉKA, Milan. Density and Related Topics. Praha: Academia; Bratislava: Veda, 2017. 238 p. ISBN 978-80-200-2725-2.
5. STRAUCH, Oto. Distribution of sequences: A theory. Bratislava : Veda ; Praha : Academia, 2019. 591 p. ISBN 978-80-224-1734-1
6. HOLÁ, Ľubica - HOLÝ, Dušan - MOORS, Warren. USCO and Quasicontinuous Mappings. Berlin : Walter de Gruyter, 2021. 295 p. Studies in Mathematics, 81.ISBN 978-3-11-075015-7

2.1.4. List of monographs/books published in Slovakia

1. PALENČÁR, R. - WIMMER, Gejza - PALENČÁR, J. - WITKOVSKÝ, Viktor. Navrhovanie a vyhodnocovanie meraní 1. vydanie. (Design and Evaluation of Measurements, 1st ed.) Bratislava: Slovenská technická univerzita v Bratislave (Slovak University of Technology in Bratislava), 2021. 160 s. ISBN 978-80-227-5080-6

2.1.5. List of other scientific outputs specifically important for the institute, max. 10 items for institute with less than 50 average FTE researchers per year, 20 for institutes with 50 – 100 average FTE researchers per year and so on

1. Collaboration with eustream a.s. Nitra on Applications of mathematical methods to solve of gas transport optimization – more than 20 years of collaboration.

2. Development, computer-aided implementation and placing in praxis of algorithms for pipe gas leaks, contract with CSE-Control, Nitra.

3. Cryptographic methods in public administration. Collaboration with National Security Bureau of SR.

4. IT security – collaboration with Ministry of Defence of the Slovak Republic.

5. Mathematical Institute SAS is the headquarter of the Slovak Association for the Club of Rome.

6. Mathematical methods of diagnostic systems in the primary circuit of nuclear power plants. We design and implement the algorithms for digital signal processing which are used in the diagnostic systems installed in the nuclear power plants (Slovak Republic, Czech Republic).

7. Mathematical modelling of COVID-19 epidemic spreading.

8. Mathematical institute SAS publishes three scientific journals: Mathematica Slovaca (since 1951, Q3), Tatra Mountains Mathematical Publications, Uniform Distribution Theory.

2.1.6. List of patents, patent applications, and other intellectual property rights registered abroad

No patents.

2.1.7. List of patents, patent applications, and other intellectual property rights registered in Slovakia

No patents.

2.1.8. Narrative on the most important research outputs of the institute – especially focused on their importance for society (3-5 pages)

2.1.9. Table of research outputs

Papers from international collaborations in large-scale scientific projects (Dwarf team, ALICE Collaboration, ATLAS collaboration, CD Collaboration, H1 Collaboration, HADES Collaboration, and STAR Collaboration) have to be listed separately

Scientific publications	2016			2017			2018			2019			2020			2021			total			
	number	No. / FTE researches	No. / one million total salary budget	number	No. / FTE researches	No. / one million total salary budget	number	No. / FTE researches	No. / one million total salary budget	number	No. / FTE researches	No. / one million total salary budget	number	No. / FTE researches	No. / one million total salary budget	number	No. / FTE researches	No. /1 million total salary budget	number	averaged number per year	av. No. / FTE researches	av. No. / one million total salary budget
Scientific monographs and monographic studies in journals and proceedings published abroad *(AAA, ABA)*	2	0,069	2,564	3	0,105	3,722	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	1	0,032	0,707	**6**	**1,000**	**0,034**	**0,992**
Scientific monographs and monographic studies in journals and proceedings published in Slovakia *(AAB, ABB)*	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	1	0,036	0,987	0	0,000	0,000	0	0,000	0,000	**1**	**0,167**	**0,006**	**0,165**
Chapters in scientific monographs published abroad *(ABC)*	3	0,103	3,846	1	0,035	1,241	**1**	0,036	1,157	2	0,072	1,974	0	0,000	0,000	0	0,000	0,000	**7**	**1,167**	**0,040**	**1,158**
Chapters in scientific monographs published in Slovakia *(ABD)*	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	**0**	**0,000**	**0,000**	**0,000**
Scientific papers published in journals registered in Current Contents Connect *(ADCA, ADCB, ADDA, ADDB)*	52	1,789	66,667	42	1,464	52,109	43	1,534	49,769	41	1,479	40,474	43	1,453	36,784	46	1,467	32,509	**267**	**44,500**	**1,531**	**44,154**
Scientific papers published in journals registered in Web of Science Core Collection and SCOPUS not listed above *(ADMA, ADMB, ADNA, ADNB)*	16	0,551	20,513	23	0,802	28,536	29	1,035	33,565	44	1,587	43,435	26	0,879	22,241	21	0,670	14,841	**159**	**26,500**	**0,911**	**26,294**
Scientific papers published in other foreign journals (not listed above) *(ADEA, ADEB)*	9	0,310	11,538	6	0,209	7,444	6	0,214	6,944	9	0,325	8,885	8	0,270	6,843	11	0,351	7,774	**49**	**8,167**	**0,281**	**8,103**
Scientific papers published in other domestic journals (not listed above) *(ADFA, ADFB)*	0	0,000	0,000	2	0,070	2,481	0	0,000	0,000	0	0,000	0,000	1	0,034	0,855	2	0,064	1,413	**5**	**0,833**	**0,029**	**0,827**
Scientific papers published in foreign peer-reviewed proceedings *(AECA)*	2	0,069	2,564	0	0,000	0,000	3	0,107	3,472	2	0,072	1,974	3	0,101	2,566	5	0,159	3,534	**15**	**2,500**	**0,086**	**2,481**
Scientific papers published in domestic peer-reviewed proceedings *(AEDA)*	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	0	0,000	0,000	1	0,032	0,707	**1**	**0,167**	**0,006**	**0,165**
Published papers (full text) from foreign scientific conferences *(AFA, AFC)*	18	0,619	23,077	0	0,000	0,000	1	0,036	1,157	0	0,000	0,000	0	0,000	0,000	3	0,096	2,120	**22**	**3,667**	**0,126**	**3,638**
Published papers (full text) from domestic scientific conferences *(AFB, AFD)*	0	0,000	0,000	0	0,000	0,000	2	0,071	2,315	0	0,000	0,000	0	0,000	0,000	0	0	0	**2**	**0**	**0**	**0**

2.2. Measures of research outputs (citations, etc.)

2.2.1. Table with citations per annum (without self-citations)

Citations of papers from international collaborations in large-scale scientific projects (Dwarf team, ALICE Collaboration, ATLAS collaboration, CD Collaboration, H1 Collaboration, HADES Collaboration, and STAR Collaboration) are listed separately

Citations, reviews	2015		2016		2017		2018		2019		2020		total		
	number	No. / FTE researchers	number	No. / FTE researchers	number	No. / FTE researchers	number	No. / FTE researchers	number	No. / FTE researchers	number	No. / FTE researchers	number	averaged number per year	av. No. / FTE researchers
Citations in Web of Science Core Collection (1.1, 2.1)	649	22,33	866	30,20	986	35,18	1 046	37,72	840	28,39	859	27,39	**5 246**	**874,33**	**30,07**
Citations in SCOPUS (1.2, 2.2) if not listed above	159	5,47	66	2,30	89	3,18	168	6,06	184	6,22	217	6,92	**883**	**147,17**	**5,06**
Citations in other citation indexes and databases (not listed above) (3.2,4.2)	1	0,03	0	0,00	0	0,00	0	0,00	0	0,00	0	0,00	**1**	**0,17**	**0,01**
Other citations (not listed above) (3.1, 4.1)	56	1,93	25	0,87	23	0,82	26	0,94	14	0,47	19	0,61	**163**	**27,17**	**0,93**
Reviews (5,6)	0	0,00	0	0,00	0	0,00	0	0,00	1	0,03	0	0,00	**1**	**0,17**	**0,01**

2.2.2. List of 10 most-cited publications published any time with the address of the institute, with number of citations in the assessment period (2015 – 2020)

1. DVUREČENSKIJ, Anatolij - PULMANNOVÁ, Sylvia. *New Trends in Quantum Structures*. Dordrecht : Kluwer Academic ; Bratislava : Ister Science, 2000. 541+xvi pp. ISBN 0-7923-6471-6.

   175 citations in WOS/SCOPUS

2. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. Ulam's type stability of impulsive ordinary differential equations. In *Journal of Mathematical Analysis and Applications*, 2012, vol. 395, no. 1, p. 258-264. (2011: 1.001 - IF, Q1 - JCR, 1.578 - SJR, Q1 - SJR, CCC). (2012 - Current Contents). ISSN 0022-247X. https://doi.org/10.1016/j.jmaa.2012.05.040.

   92 citations in WOS/SCOPUS

3. WANG, J. - ZHOU, Y. - FEČKAN, Michal. Nonlinear impulsive problems for fractional differential equations and Ulam stability. In *Computers & Mathematics with Applications*, 2012, vol. 64, no. 10, p. 3389-3405. (2011: 1.747 - IF, Q1 - JCR, 1.162 - SJR, Q1 - SJR, CCC). (2012 - Current Contents). ISSN 0898-1221.: https://doi.org/10.1016/j.camwa.2012.02.021

   92 citations in WOS/SCOPUS

4. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. A survey on impulsive fractional differential equations. In *Fractional Calculus and Applied Analysis*, 2016, vol. 19, no. 4, p. 806-831. (2015: 2.246 - IF, Q1 - JCR, 1.551 - SJR, Q1 - SJR, - CCC). (2016 – Current Contents). ISSN 1311-0454. https://doi.org/10.1515/fca-2016-0044.

   75 citations in WOS/SCOPUS

5. WAWER, Mathias J. - LI, Kejie - GUSTAFSDOTTIR, Sigrun M. - LJOSA, Vebjorn – BODYCOMBE, Nicole E. - MARTON, Melissa A. - SOKOLNICKI, Katherine L. - BRAY, Mark-Anthony - KEMP, Melissa M. - WINCHESTER, Ellen - TAYLOR, Bradley - GRANT, George B. - HON, Suk-Yee C. - DUVALL, Jeremy - WILSON, Anthony J. - BITTKER, Joshua A. - DANČÍK, Vladimír - NARAYAN, Rajiv - SUBRAMANIAN, Aravind – WINCKLER, Wendy - GOLUB, Todd R. - CARPENTER, Anne E. - SHAMJI, Alykhan F. – SCHREIBER, Stuart L. - CLEMONS, Paul A. Toward performance-diverse small-molecule libraries for cell-based phenotypic screening using multiplexed high-dimensional profiling. In *Proceedings of the National Academy of Sciences of the United States of America*, 2014, vol. 111, no. 30, p. 10911-10916. (2013: 9.809 - IF, Q1 - JCR, 6.989 - SJR, Q1 - - CCC). (2014 - Current Contents). ISSN 0027-8424. https://doi.org/10.1073/pnas.1410933111.

   70 citations in WOS/SCOPUS

6. FEČKAN, Michal - WANG, JinRong - ZHOU, Yong. Controllability of fractional functional evolution equations of Sobolev type via characteristic solution operators. In *Journal of Optimization Theory and Applications*, 2013, vol. 156, no. 1, p. 79-95. (2012: 1.423 - IF, Q1 – JCR, 1.240 - SJR, Q1 - SJR, - CCC). (2013 - Current Contents). ISSN 0022-3239. https://doi.org/10.1007/s10957-012-0174-7.

   51 citations in WOS/SCOPUS

7. PTÁK, Pavel - PULMANNOVÁ, Sylvia. *Orthomodular Structures as Quantum Logics*. Dordrecht : Kluwer Academic Publishers ; Bratislava : VEDA, 1991. 244 s. ISBN 0-7923-1207-4.

   44 citations in WOS/SCOPUS

8. WANG, JinRong - ZHOU, Yong - FEČKAN, Michal. Abstract Cauchy problem for fractional differential equations. In *Nonlinear Dynamics*, 2013, vol. 71, no. 4, p. 685-700. (2012: 3.009 IF, Q1 - JCR, 0.873 - SJR, Q1 - SJR, - CCC). (2013 - Current Contents). ISSN 0924-090X., https://doi.org/10.1007/s11071-012-0452-9.

   42 citations in WOS/SCOPUS

9. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. Relaxed controls for nonlinear fractional impulsive evolution equations. In *Journal of Optimization Theory and Applications*, 2013, vol. 156, no. 1, p. 13-32. (2012: 1.423 - IF, Q1 - JCR, 1.240 - SJR, Q1 - SJR, CCC). (2013 - Current Contents). ISSN 0022-3239. https://doi.org/10.1007/s10957-012-0170-y.

   37 citations in WOS/SCOPUS

10. RIEČAN, Beloslav - NEUBRUNN, Tibor. *Integral, measure, and ordering*. Dordrecht : Kluwer Academic Publishers, 1997. ISBN 80–88683–18–1.

    36 citations in WOS/SCOPUS

2.2.3. List of 10 most-cited publications published any time with the address of the institute, with number of citations obtained until 2020

1. DVUREČENSKIJ, Anatolij - PULMANNOVÁ, Sylvia. *New Trends in Quantum Structures*. Dordrecht : Kluwer Academic ; Bratislava : Ister Science, 2000. 541+xvi pp. ISBN 0-7923-6471-6.

   513 citations in WOS/SCOPUS

2. DANČÍK, Vladimír - ADDONA, T.A. - CLAUSER, K.R. - VATH, J.E. - PEVZNER, P.A. De novo peptide sequencing via tandem mass spectrometry. In *Journal of Computational Biology*, 1999, vol. 6, no. 3-4, p. 327-342. ISSN 1066-5277.

   281 citations in WOS/SCOPUS

3. DVUREČENSKIJ, Anatolij. Pseudo MV-algebras are intervals in l-groups. In *Journal of the Australian Mathematical Society*, 2002, vol. 72, p. 427-445. ISSN 1446-7887.

   111 citations in WOS/SCOPUS, **151** all citations

4. WANG, J. - ZHOU, Y. - FEČKAN, Michal. On recent developments in the theory ofboundary value problems for impulsive fractional differential equations. In *Computers & Mathematics with Applications*, 2012, vol. 64, no. 10, p. 3008-3020. (2011: 1.747 - IF, Q1 – JCR, 1.162 - SJR, Q1 - SJR, - CCC). (2012 - Current Contents). ISSN 0898-1221. https://doi.org/10.1016/j.camwa.2011.12.064.

   105 citations in WOS/SCOPUS

5. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. Ulam's type stability of impulsive ordinary differential equations. In *Journal of Mathematical Analysis and Applications*, 2012,

vol. 395, no. 1, p. 258-264. (2011: 1.001 - IF, Q1 - JCR, 1.578 - SJR, Q1 - SJR,– CCC). (2012 - Current Contents). ISSN 0022-247X. https://doi.org/10.1016/j.jmaa.2012.05.040.

104 citations in WOS/SCOPUS

6. WANG, J. - ZHOU, Y. - FEČKAN, Michal. Nonlinear impulsive problems for fractional differential equations and Ulam stability. In *Computers & Mathematics with Applications*, 2012, vol. 64, no. 10, p. 3389-3405. (2011: 1.747 - IF, Q1 - JCR, 1.162 - SJR, Q1 - SJR, CCC). (2012 - Current Contents). ISSN 0898-1221.: https://doi.org/10.1016/j.camwa.2012.02.021

   100 citations in WOS/SCOPUS

7. DVUREČENSKIJ, Anatolij. States on pseudo MV-algebras. In *Studia Logica*, 2001, vol. 68, p. 301-327.

   81 citations in WOS/SCOPUS

8. DVUREČENSKIJ, Anatolij - VETTERLEIN, Thomas. Pseudoeffect Algebras. I. Basic properties. In *International Journal of Theoretical Physics*, 2001, vol. 40, p. 685-701. ISSN 0020-7748.

   77 citations in WOS/SCOPUS, **128** all citations

9. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. A survey on impulsive fractional differential equations. In *Fractional Calculus and Applied Analysis*, 2016, vol. 19, no. 4, p. 806-831. (2015: 2.246 - IF, Q1 - JCR, 1.551 - SJR, Q1 - SJR, - CCC). (2016 – Current Contents). ISSN 1311-0454. https://doi.org/10.1515/fca-2016-0044.

   75 citations in WOS/SCOPUS

10. RIEČAN, Beloslav - NEUBRUNN, Tibor. *Integral, measure, and ordering*. Dordrecht : Kluwer Academic Publishers, 1997. ISBN 80–88683–18–1.

    70 citations in WOS/SCOPUS

2.2.4. List of 10 most-cited publications published during the evaluation period (2016-2021) with the address of the Institute, with number of citations obtained until 2021

1. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. A survey on impulsive fractional differential equations. In Fractional Calculus and Applied Analysis, 2016, vol. 19, no. 4, p. 806-831. (2.246 - IF2015). (2016 - Current Contents). ISSN 1311-0454. https://doi.org/10.1515/fca-2016-0044,

   96 citations in WOS/SCOPUS

2. WANG, JinRong - FEČKAN, Michal - TIAN, Ying. Stability analysis for a general class of non-instantaneous impulsive differential equations. In *Mediterranean Journal of Mathematics*, 2017, vol. 14, no. 2, art. no. 46. (2016: 0.868 - IF, Q2 - JCR, 0.655 - SJR, Q2 - SJR, karentované - CCC). (2017 - Current Contents). ISSN 1660-5446. Dostupné na: https://doi.org/10.1007/s00009-017-0867-0

   32 citations in WOS/SCOPUS

3. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. Fractional order differential switched systems with coupled nonlocal initial and impulsive conditions. In *Bulletin des sciences mathématiques*, 2017, vol. 141, no. 7, p. 727-746. (2016: 0.750 - IF, Q3 - JCR, 0.738 - SJR, Q2 - SJR, CCC). (2017 - Current Contents). ISSN 0007-4497. https://doi.org/10.1016/j.bulsci.2017.07.007

   25 citations in WOS/SCOPUS

4. DANCA, Marius-F.- FEČKAN, Michal - KUZNETSOV, Nikolay V. - CHEN, Guanrong. Complex dynamics, hidden attractors and continuous approximation of a fractional-order hyperchaotic PWC system. In *Nonlinear Dynamics*, 2018, vol. 91, no. 4, p. 2523-2540. (2017: 4.339 - IF, Q1 - JCR, 1.468 - SJR, Q1 - SJR, CCC). (2018 - Current Contents). ISSN 0924-090X. https://doi.org/10.1007/s11071-017-4029-5

   22 citations in WOS/SCOPUS

5. LIU, Kui - FEČKAN, Michal - O'REGAN, D. - WANG, JinRong. Hyers-Ulam stability and existence of solutions for differential equations with Caputo-Fabrizio fractional derivative. In *Mathematics*, 2019, vol. 7, no. 4, art. no. 333. (2018: 1.105 - IF, Q1 - JCR, 0.244 - SJR, Q3 - SJR, - CCC). (2019 - Current Contents). ISSN 2227-7390. https://doi.org/10.3390/math7040333

   21 citations in WOS/SCOPUS

6. MESIAROVÁ-ZEMÁNKOVÁ, Andrea. Characterization of uninorms with continuous underlying T-norm and T-conorm by their set of discontinuity points. In *IEEE Transactions on Fuzzy Systems*, 2018, vol. 26, no. 2, p. 705-714. (2017: 8.415 - IF, Q1 - JCR, 4.024 - SJR, Q1 - SJR, - CCC). (2018 - Current Contents). ISSN 1063-6706.https://doi.org/10.1109/TFUZZ.2017.2688346

   19 citations in WOS/SCOPUS

7. WANG, JinRong - FEČKAN, Michal - ZHOU, Yong. Approximate controllability of Sobolev type fractional evolution systems with nonlocal conditions. In *Evolution Equations and Control Theory*, 2017, vol. 6, no. 3, p. 471-486. (2016: 0.826 - IF, Q2 - JCR, 0.999 - SJR, Q1 - SJR, - CCC). (2017 - Current Contents). ISSN 2163-2480. https://doi.org/10.3934/eect.2017024

   18 citations in WOS/SCOPUS

8. PLÁVALA, Martin. All measurements in a probabilistic theory are compatible if and only if the state space is a simplex. In *Physical Review A*, 2016, vol. 94, no. 4, art. no. 042108. (2015: 2.765 - IF, Q1 - JCR, 1.747 - SJR, Q1 - SJR, - CCC). (2016 - Current Contents). ISSN 1050-2947. https://doi.org/10.1103/PhysRevA.94.042108

   16 citations in WOS/SCOPUS

9. FEČKAN, Michal - WANG, JinRong. Periodic impulsive fractional differential equations. In *Advances in Nonlinear Analysis*, 2019, vol. 8, no. 1, p. 482-496. (2018: 6.636 - IF, Q1 - JCR, 3.215 - SJR, Q1 - SJR, - CCC). (2019 - Current Contents). ISSN 2191-9496. https://doi.org/10.1515/anona-2017-0015

   16 citations in WOS/SCOPUS

10. DANCA, Marius-F. - FEČKAN, Michal - KUZNETSOV, Nikolay V. - CHEN, Guanrong. Looking more closely at the Rabinovich-Fabrikant system. In *International Journal of Bifurcation and Chaos*, 2016, vol. 26, no. 2, art. no. 1650038 p. [1-21]. (2015: 1.355 - IF, Q2 -

JCR, 0.752 - SJR, Q1 - SJR, - CCC). (2016 - Current Contents). ISSN 0218-1274. https://doi.org/10.1142/S0218127416500383

16 citations in WOS/SCOPUS

2.2.5. List of most-cited authors from the Institute (at most 10 % of average FTE researchers per year) and their number of citations in the assessment period (2015– 2020). The cited papers must bear the address of the institute

1. Michal Fečkan – **1 426** citations
2. Anatolij Dvurečenskij – **888** citations
3. Sylvia Pulmannová – **426** citations
4. Andrea Zemánková – **224** citations

2.2.6. List of most-cited authors from the Institute (at most 10 % of average FTE researchers per year) and their number of citations obtained until 2020. The cited papers must bear the address of the Institute

1. Anatolij Dvurečenskij – **2 104** citations
2. Michal Fečkan – **1 576** citations
3. Sylvia Pulmannová – **1 009** citations

2.2.7. List of most-cited authors from the Institute (at most 10 % of average FTE researchers per year) and their number of citations obtained until 2021 of their papers published during the evaluation period (2016– 2021). The cited papers must bear the address of the Institute

1. Michal Fečkan – **497** citations
2. Andrea Mesiarová-Zemánková – **98** citations
3. Anna Jenčová – **67** citations

2.3. Research status of the institute in international and national context

- **International/European position of the institute**

2.3.1. List of the most important research activities demonstrating the international relevance of the research performed by the institute, incl. major projects (details of projects should be supplied under Indicator 2.4). Max. 10 items for institute with less than 50 average FTE researchers per year, max. 20 for institutes with 50 – 100 average FTE researchers per year and so on

The international relevance of the research performed by the institute can be demonstrated by the broad international cooperation of all research parts of the institute and their position in the corresponding research network. It can be shown by

- many joint publications with authors from abroad,

- citations on our papers,

- awards of our researchers as highly cited authors, and authors of the best publications,
- real cooperation with many researchers from abroad,
- invited lectures on international conferences,
- membership in editorial boards of international journals, scientific boards,
- publishing of three international scientific journals,
- activities for NATO in scientific questions,
- organizing international scientific conferences,
- membership in the program committee of international conferences,
- projects ERA-NET and intenational cooperation projects of SAS and APVV,
- many other indicators as are published in this report.

2.3.2. List of international conferences (co)organised by the institute

2016

1. 32nd Spring conference on Computer Graphics, Smolenice
2. 16th Central European Conference on Cryptology (CECC), Piešťany
3. 30th International Summer School on Real Functions Theory, Stará Lesná
4. Parallel Numerical Computing and Its Applications, Smolenice
5. Workshop - Algebraic Graph Theory, Plzeň, ČR
6. Globálne existenciálne riziká 2016 (Global Existential Risks), Bratislava

2017

1. EuroHCC 2017 (European Historical Ciphers Colloquium), Smolenice
2. CryptArchi Workshop 2017 (modern logic devices in cryptography), Smolenice
3. Graph Embeddings and Maps on Surfaces (GEMS 2017), Podbanské
4. Globálne existenciálne riziká 2017 (Global Existential Risks), Bratislava

2018

1. 18th Central European Conference on Cryptology, Smolenice
2. 10th International Workshop on Non-Classical Models of Automata and Applications (NCMA), Košice
3. 32th International Summer Conference on Real Functions Theory

2019

1. 21th Descriptional Complexity of Formal Systems (DCFS 2019), Košice
2. 24th International Conference on Implementation and Application of Automata (CIAA 2019), Košice
3. 33rd International Summer Conference on Real Functions Theory, Ustka, Poland
4. Set-theoretic methods in topology and real functions theory (dedicated to 80th birthday of Lev Bukovský), Košice

2020

1. 58th Summer School on Algebra and Ordered Sets (SSAOS 2020), Smižany (cancelled due to epidemic situation)
2. 34th International Summer Conference on Real Functions Theory (Virtual ISFRT 2020) - online
3. Conference to NATO projects - Science for Peace and Security (SPS) G5448 “Secure Commu-nication in the Quantum Era”, Smolenice

2021

1. Summer School on Algebra and Ordered Sets (SSAOS 2021), Smižany

2.3.3. List of edited proceedings from international scientific conferences

2017

1. PARNUM 2017 : Book of abstracts. Eds. D. Bartuschat, U. Rüde, M. Vajteršic. Erlangen, Germany, 2017. 31 p.
2. Norwegian-Slovakian Workshop in Crypto : Proceedings. O. Grošek, T. Helleseth, A. Kholosha, K. Nemoga, I. Semaev, P. Zajac (eds.). Bratislava : Slovak University of Technology, 2016. 78 p. ISBN 978-80-227-4541-3.
3. Euro HCC 2017 : Proceedings. J. von zur Gathen, O. Grošek, K. Nemoga, A. Wacker (eds.). Bratislava : SPEKTRUM STU, 2017. 77 p. ISBN 978-80-227-4687-8.

2018

1. Tenth Workshop on Non-Classical Models of Automata and Applications (NCMA 2018). Rudolf Freund, Michal Hospodár, Galina Jirásková, Giovanni Pighizzini (eds.). Wien : Österreichische Computer Gesellschaft, 2018. ISBN 978-3-903035-21-8.

2019

1. Descriptional complexity of formal systems : Proceedings LNCS 11612. Michal Hospodár, Galina Jirásková, Stavros Konstantinidis (eds.). Springer International Publishing, 2019. Lecture Notes in Computer Science, 11612. ISBN 978-3-030-23246-7. ISSN 0302-9743.
2. Implementation and application of automata : Proceedings LNCS 11601. Michal Hospodár, Galina Jirásková (eds.). Springer International Publishing, 2019. Lecture Notes in Computer Science, 11601. ISBN 978-3-030-23678-6. ISSN 0302-9743.
3. Uniform Distribution Theory vol. 14 no. 1 (2019). The 6th International Conference on Uniform Distribution Theory (UDT 2018), CIRM, Luminy, Marseiles, France, October 1-5, 2018. Scientific Board members: O. Strauch, V. Baláž. Vienna : University of Natural Resources and Life Sciences. Bratislava: Matematický ústav SAV. ISSN 1336-913X.

2020

1. Descriptional Complexity of Formal Systems : Proceedings LNCS 12442. Eds. Galina Jiraskova, Giovanni Pighizzini. Springer International Publishing, 2020. 245 p. LNCS, 12442. Available: https://doi.org/10.1007/978-3-030-62536-8 . ISBN 978-3-030-62536-8.

2.3.4. List of journals edited/published by the institute and information on their indexing in WOS, SCOPUS, other database or no database, incl. impact factor and other metrics of journals in each year of the assessment period

- **Mathematica Slovaca**
  ISSN: 1337-2211, ISSN (print): 0139-9918, indexed WOS / SCOPUS
  2016: IF2015: 0.366
  2017: IF2016: 0.346
  2018: IF2017: 0.314
  2019: IF2018: 0.490
  2020: IF2019: 0.654
  2021: IF2020: 0.770
- **Tatra Mountains Mathematical Publications**
  ISSN: 1338-9750, ISSN (print): 1210-3195,
  indexed WOS (selected volumes as conference proceedings) / SCOPUS
  2016: SJR2015: 0.212, Q3
  2017: SJR2016: 0.309, Q3
  2018: SJR2017: 0.360, Q3
  2019: SJR2018: 0.226, Q4
  2020: SJR2019: 0.217, Q4
  2021: SJR2020: 0.230, Q4

- **Uniform Distribution Theory**
  ISSN: 1336-913X – covered by Zentralblatt MATH, Mathematical Reviews.
  (Since 2016 distributed by De Gruyter Open.)
  (Cooperation with BOKU–University, Vienna and Manchester University.)

  Top journal, very narrow subject, two isseus per year, not interesting for abstracting services.

- **National position of the institute**

2.3.5. List of selected activities of national importance

- Cooperation with the Ministry of Defense on problems conserning security of communication.
- Activities on Slovak Action Plan on Cyber Security.
- Publishing of the mathematical journal Tatra Mountains Mathematical Publications, which publishes monothematic volumes with the broad cooperation with the Slovak universities.

2.3.6. List of journals (published only in the Slovak language) edited/published by the institute and information on their indexing in WOS, SCOPUS, other database or no database, incl. impact factor and other metrics of journals in each year of the assessment period

- **Obzory matematiky, fyziky a informatiky (Horizons of Mathematics, Physics and Computer Sciences)**
  ISSN: 1335-4981 – Until December 31, 2019 it was indexed in MathEduc Database by FIZ Karlsruhe – Leibniz Institut für Informationsinfrastruktur.
  (Copublished with JSMF (The Union of Slovak Mathematicians and Physicists), MI SAS and Institute of Informatics of the SAS.)

- **Position of individual researchers in the international context**

2.3.7. List of invited/keynote presentations at international conferences, as documented by programme or invitation letter

2016

1. **HALUŠKA, J.**: *Dobrakov integration versus Kurzweil integration, Seminář o diferenciálních rovnicích a teorii integrálu,* Special session in honor of the 90th birthday of Jaroslav Kurzweil, Praha, ČR, 11.5.-13.5.2016.
2. **JENČOVÁ, A.**: *Quantum divergences and interpolation, Workshop on Quantum Information Theory and Mathematical Physics,* Budapest, Maďarsko, 16.9.-19.9.2016.
3. **JIRÁSKOVÁ, G.**: *Self-verifying finite automata and descriptional complexity, Descriptional Complexity of Formal Systems - 18th IFIP WG 1.2 International Conference, DCFS 2016,* Bucharest, Rumunsko, 5.7.-8.7.2016.
4. **MONTEIRO, G.A.**: *Extremal solutions of measure differential equations and applications to impulsive systems,* O.D.E. 60, Brno, ČR, 6.6.-8.6.2016.
5. **MONTEIRO, G.A.**: *Convergence results for the abstract Kurzweil-Stieltjes integral, Seminář o diferenciálních rovnicích a teorii integrálu,* Special session in honor of the 90th birthday of Jaroslav Kurzweil, Praha, ČR, 11.5.-13.5.2016.
6. **NEDELA, R.**: *Hamilton cycles in graphs embedded into surfaces, Graphs, Groups, Spectra and Symmetries (15.8.-28.8.2016),* Novosibirsk State University, Novosibirsk, Rusko, 28.8.2016.
7. **VAJTERŠIC, M.**: *Parallelization approaches for NMF and SVD,* University of Erlangen-Nuernberg, Kolloqium, Erlangen, Nemecko, 25.5.2016.

8. **VAJTERŠIC, M.**: *MIMD parallelization of matrix decomposition methods,* University of Malaga, Seminar, Malaga, Španielsko, 4.5.2016.

2017

1. **FRIČ, R.**: *A dialogue/monologue on probability,* XXIVth Czech-Polish-Slovak Mathematical Conference, Ružomberok, 6.- 9. 6. 2017
2. **JENČOVÁ, A.**: *Rényi relative entropies and noncommutative Lp-spaces,* Quantum Information Theory and Mathematical Physics, Budapest, 30.8. - 2. 9. 2017
3. **NEDELA, R.**: *Complete regular dessins and skew-morphisms of cyclic groups,* Maps and Dessins, Banff International Research Station, Canada, 24. - 29. 9. 2017
4. **NEMOGA, K.**: *Practical Cyber Security. Quantum Technology,* NATO Cyber Defence Cluster Workshop I, NATO HQ Brussels, Belgicko, 7. - 8. 6.2017
5. **NEMOGA, K.**: *Current Trends in Cyber Defense,* NATO Cyber Defence Cluster Workshop II, NATO HQ Brussels, Belgicko, 11. - 12. 12. 2017

2018

1. **DVUREČENSKIJ, A.**: *EMV-algebras-Extended MV-algebras,* Algebra and Substructural Logics, Take 6 (AsubL Take 6), Cagliari, Taliansko, 10.–15. 6. 2018.
2. **JENČOVÁ, A.**: *Incompatible measurements in general probabilistic theories,* Quantum Information Theory and Mathematical Physics 2018, Budapešť, Maďarsko, 20.–23. 9. 2018.
3. **JENČOVÁ, A.**: *Rényi relative entropies and sufficiency of quantum channels,* Beyond I.I.D. in Information Theory, Isaac Newton Institute, Cambridge, UK, 23.–27. 7. 2018.
4. **JENČOVÁ, A.**: *A geometric view on quantum incompatibility,* Three Days in quantum Mechanics, Genoa, Taliansko, 6.–8. 6. 2018.
5. **JIRÁSKOVÁ, G.**: *Deterministic blow-ups of nondeterministic finite automata,* International Workshop DLT's Satellite Workshop 2018, Kyoto, Japan, 5.–7. 9. 2018.
6. **NEDELA, R.**: *Skew morphisms of cyclic groups and regular dessins,* International Workshop on Symmetries of Graph and Networks 2018 (IWSGN 2018), Sanya, China, 25. 1.–3. 2. 2018.
7. **NEDELA, R.**: *Skew morphisms of groups, regular maps and dessins,* The Third International Conference On Group Actions and Transitive Graphs, SUSTECH, Shenzen, China, 12.–14. 10. 2018.
8. **NEDELA, R.**: *Graph coverings and harmonic morphisms on graphs,* 30th Workshop on Topological Graph Theory, Yokohama, Japonsko, 24.–26. 10. 2018.
9. **PLÁVALA, M.**: *Incompatibility in general probabilistic theory,* IQSA Quantum Structures 2018, Kazaň, Rusko, 16.–20. 7. 2018.

2019

1. **FRIČ, R.**: *Measures on product and stochastic independence,* Institute of Mathematics, Czech Academy of Sciences, Praha, Czech Republic, Seminar on Differential Equations and Integral Theory, Praha, Czech Republic, 5. 12. 2019.
2. **JENČOVÁ, A.**: *Incompatible measurements in GPT,* Workshop GPT&QIT Generalized Probabilistic Theories and Quantum Information, Lyon, 1.–2. 7. 2019.
3. **JENČOVÁ, A.**: *Randomization theorems for bipartite quantum channels,* BIRS Workshop: Algebraic and Statistical ways into Quantum Resource Theories, Banff, Canada, 22.–26. 7. 2019.

2021

1. **DVUREČENSKIJ, A.**: *What are weak pseudo EMV-algebras?* (online), Boolean Algebras, Lattices, Universal Algebra, Set Theory, Topology (BLAST 2021), Las Cruces, NM, USA, 9. 6.–13. 6. 2021.
2. BLUHM, A.— **JENČOVÁ, A.**— NECHITA, I.: *Incompatibility in GPTs, generalized spectrahedra, and tensor norms* (online), 18th International Conference on Quantum Physics and Logic (QPL 2021), Gdansk, Poland, 7.6.2021–11.6.2021.

3. **JENČOVÁ, A.**: *Renyi relative entropies and noncommutative L_p-spaces* (online), Conference in Operator Algebras and Related Topics (Opalg2021), Istanbul, Turkey, 8. 6.–10. 6. 2021.

2.3.8. List of researchers who served as members of the organising and/or programme committees

2016

1. Borsík Ján
2. Jirásková Galina
3. Karabáš Ján
4. Nedela Roman
5. Nemoga Karol
6. Vajteršic Marian
7. Zemánková Andrea

2017

1. Frič Roman
2. Jirásková Galina
3. Nedela Roman
4. Nemoga Karol
5. Ploščica Miroslav
6. Vajteršic Marian
7. Wimmer Gejza

2018

1. Borsík Ján
2. Hospodár Michal
3. Jirásková Galina
4. Krajňáková Ivana
5. Mlynárčik Peter
6. Nedela Roman
7. Nemoga Karol
8. Novotný Branislav
9. Okša Gabriel
10. Palmovský Matúš
11. Vajteršic Marian

2019

1. Borsík Ján
2. Eliaš Peter
3. Frič Roman
4. Hospodár Michal
5. Jirásková Galina
6. Krajňáková Ivana
7. Mlynárčik Peter
8. Nemoga Karol
9. Palmovský Matúš
10. Pócs Jozef
11. Repický Miroslav
12. Vajteršic Marian

2020

1. Eliaš Peter
2. Frič Roman

3. Holá Ľubica
4. Jirásková Galina
5. Nemoga Karol
6. Novotný Branislav
7. Zemánková Andrea

2021

1. Holá Ľubica
2. Jenčová Anna
3. Nemogý Karol
4. Wimmer Gejza
5. Zemánková Andrea

2.3.9. List of researchers who received an international scientific award

2016

1. Vajteršic Marian (Dank und Anerkennung – Univ. Salzburg, Austria)

2019

1. Michal Fečkan (Highly Cited Researcher – Clarivate) - he was recorded among 1 % of the most cited mathematicians of the world in his area

2021

1. Michal Fečkan (Highly Cited Researcher – Clarivate) - he was recorded among 1 % of the most cited mathematicians of the world in his area

- **Position of individual researchers in the national context**

2.3.10. List of invited/keynote presentations at national conferences, as documented by programme or invitation letter

2017

1. **PÓCS, J.**: *Formálna konceptová analýza (Formal Concept Analysis)*, 49. Konferencia slovenských matematikov (49th Conference of Slovak Mathematicians), Jasná pod Chopkom, 23. - 26. 11. 2017

2019

1. **FRIČ, R.**: *Poznámky o kategoriálnej pravdepodobnosti (Notes on Categorical Probability),* Matematický ústav SAV (MI SAS), Seminar on Quantum Logics, Center of Excellence QUTE, Bratislava, 3. 12. 2019.

2.3.11. List of researchers who served as members of organising and programme committees of national conferences

2016

1. Gyürki Štefan

2018

1. Čunderlíková Katarína

2019

1. Čunderlíková Katarína

2.3.12. List of researchers who received a national scientific award

2016

1. Nemoga Karol
   - *Plaque of Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava awarded on the occasion of the 75th anniversary of founding the Faculty*
2. Vajteršic Marian
   - *Commemorative letter of merit for development the computer science and information technologies awarded by* Institute of Informatics SAS.

2017

1. Dvurečenskij Anatolij
   - *Award for the publication in SAS with an extraordinarily high number of citations:* A. Dvurečenskij, S. Pulmannová: New Trends in Quantum Structures, Kluwer Academic, Dordrecht, Ister Science, Bratislave, 2000 *awarded by* Presidium of the SAS.
2. Fečkan Michal
   - *Best publication of the year in SAS:* M. Fečkan, and M. Pospíšil. Poincaré-Andronov-Melnikov Analysis for Non-Smooth Systems. Amsterdam : Elsevier, 2016. 244 s. ISBN 978-0-12-804294-6 *awarded by* SAS.
   - *Award of the rector of the Comenius University on the occasion of 17. November.*
   - *Award of the Literary fund.*
3. Pulmannová Sylvia
   - *Award for the publication in SAS with an extraordinarily high number of citations:* A. Dvurečenskij, S. Pulmannová: New Trends in Quantum Structures, Kluwer Academic, Dordrecht, Ister Science, Bratislave, 2000 *awarded by* Presidium of the SAS.
4. Vajteršic Marian
   - *Honorary Plaque of J. Hronec for merits in Mathematics awarded by* Presidium of the SAS.

2018

1. Fečkan Michal
   - *Researcher of the Year awarded by* CVTI SR (Centre for Scientific and Technological Information of Slovak Republic), SAS and ZSVTS (The Union of Slovak Scientific-Technological Organizations).
2. Nemoga Karol
   - *Honorary Plaque of J. Hronec for merits in Mathematics awarded by* Presidium of the SAS.

2019

1. Dvurečenskij Anatolij
   - *Gold medal of the SAS for lifetime achievement awarded by* Scientific Board of the SAS.
   - *Gold medal of the Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava.*
2. Nemoga Karol
   - *Commemorative medal for the 25 years of establishment of Military Intelligence awarded by the Order of the Minister of Defense SR.*
   - *Material gift for the 25 years of establishment of Military Intelligence awarded by the Order of the Minister of Defense SR.*
3. Pulmannová Sylvia
   - *Medal of the Slovak Academy of Sciences for the support of science.*

2021

1. Fečkan Michal
   - *Honorary Plaque of J. Hronec for merits in Mathematics awarded by* Presidium of the SAS.
2. Vajteršic Marian
   - *An Important Personality of the SAS awarded by* Presidium of the SAS.

2.4. Research grants and other funding resources

(List type of project, title, grant number, duration, total funding and funding for the institute, responsible person in the institute and his/her status in the project, e.g. coordinator "C", work package leader "W", investigator "I". Add information on the projects which are interdisciplinary, and also on the joint projects with several participating SAS institutes)

- **International projects**

2.4.1. List of major projects of Framework Programmes of the EU (which pilar), NATO, COST, etc.

1. **Inter Acadamic Accord: Slovakia – Ukraine (2014-2016)**
   Vector valued measures and integration in polarized vector spaces
   (Haluška Ján**)**
2. **Inter Acadamic Accord: Slovakia – Poland (2019-2022)**
   Mathematical models of uncertainty and their applications
   (Michalíková Alžbeta)
3. **Inter Acadamic Accord: Slovakia – Bulgaria (2021-2022)**
   Generation and applications of probabilistic and intuitionistic fuzzy models of uncertainty
   (Čunderlíková Katarína)
4. **Inter Governmental Agreements: Slovakia – Austria (2021-2022)**
   Frequency and declensional morphology in Slavic languages (Russian, Slovak and Slovene)
   (Mačutek Ján)

Add information on your activities in international networks

Important international networking activity is membership of Karol Nemoga in NATO ISEG, the committee of NATO for approving international scientific projects and evaluating running projects. As a member of the committee, he is evaluating some running projejts and visiting universities in Israel, France, Malta, Montenegro, Spain, etc.

This membership also includes the help to Slovak teams to submit NATO project.

- **National projects, incl. international projects with only national funding**

2.4.2. List of ERA-NET projects funded from SAS budget

1. Data-Driven Drug Discovery For Wound Healing (2018-2021)
(Novotný Branislav, Bokes Pavol)
Coordinator: Biodonostia Health Research Institute, Computational Biology and Systems Biomedicine Group

Multilateral project, cooperation of institutions from Slovakia, Slovenia (2x), Austria, Germany (2x), Great Britain.

2.4.3. List of projects of the Slovak Research and Development Agency, APVV

1. APVV-15-0091: Effective algorithms, automata and data structures (2016-2020)
   Coordinator: Pavol Jozef Šafárik University, Košice
   (Jirásková Galina)
   Funding: 2016: 8953, 2017: 14 407, 2018: 14407, 2019: 13782, 2020: 6 217
   Total Funding in assesed period: 57 766
2. APVV-15-0220: Algebraic, topological and combinatorial methods in the study of discrete structures (2016-2020)
   Coordinator: Faculty of Mathematics, Physics and Informatics, Commenius University, Bratislava
   (Nedela Roman)
   Funding: 2016: 5000, 2017: 14 000, 2018: 10 000, 2019: 10 000, 2020: 5 000
   Total Funding in assesed period: 44 000
3. APVV-0219-12: Automatic processing of traceology objects (2013-2017)
   Coordinator: Faculty of Natural Sciences, Matej Bel University, Banská Bystrica
   (Šuch Ondrej)
   Funding: 2016: 7 664, 2017: 6 483
   Total Funding in assesed period: 14 147
4. APVV-15-0295: Advanced statistical and computational methods for measurement and metrology (2016-2020)
   Coordinator: MI SAS
   (Wimmer Gejza)
   Funding: 2016: 3750, 2017: 7 500, 2018: 7 500, 2019: 7 500, 2020: 3 750
   Total Funding in assesed period: 30 000
5. APVV-16-0073: Probabilistic, algebraic a quantum-mechanical aspect of uncertainty (2017-2021)
   Coordinator: MI SAS
   (Dvurečenskij Anatolij)
   Funding: 2017: 8 163, 2018: 36 278, 2019: 29 407, 2020: 31 907, 2021: 5 625
   Total Funding in assesed period: 111 380
6. APVV-16-0053: Topology and Geometry of Manifolds (2017-2021)
   Coordinator: Faculty of Mathematics, Physics and Informatics, Commenius University, Bratislava
   (Macko Tibor)
   Funding: 2017: 540, 2018: 3 210, 2019: 2 500, 2020: 2 500, 2021: 1 250
   Total Funding in assesed period: 10 000
7. APVV-18-0066: Development of innovative methods for primary metrology torque forces by force effects of the conventional standards (2019-2022)
   Coordinator: Slovenská legálna metrológia, n.o.
   (Wimmer Gejza)
   Funding: 2019: 1 820, 2020: 4 439, 2021: 4 439
   Total Funding in assesed period: 10 698
8. APVV-19-0308: Exceptional structures in discrete mathematics (2020-2024)
   Coordinator: Faculty of Mathematics, Physics and Informatics, Commenius University, Bratislava
   (Nedela Roman)
   Funding: 2020: 1 800, 2021: 3 600
   Total Funding in assesed period: 5 400

9. APVV-19-0220: Ontological representation for security of information systems (2020-2024)
   Coordinator: Faculty of Electrical Engineering and Information Technology, Slovak University for Technology, Bratislava
   (Nemoga Karol)
   Funding: 2020: 3 100, 2021: 7 200,
   Total Funding in assesed period: 10 300

2.4.4. List of projects of the Scientific Grant Agency of the Slovak Academy of Sciences and the Ministry of Education, VEGA (for funding specify only total sum obtained from all VEGA grants in particular year)

Vega Funding (total in EUR, for organisation):

2016: 78 694 (16 projects)
2017: 71 566 (17 projects)
2018: 76 097 (16 projects)
2019: 61 828 (15 projects)
2020: 61 889 (13 projects)
2021: 62 517 (13 projects)

Projects:

1. VEGA 2/0050/15: Generalization of Comtinuity of Functions (2015-2018)
   Coordinator: MI SAS
   (Borsík Ján)
2. VEGA 2/0165/16: Computations with Incomplete Information - Algorithms, Models, Networks (2019-2019)
   Coordinator: MI SAS
   (Dobrev Stefan)
3. VEGA 2/0153/16: Qualitative properties and bifurcations of differential equations and dynamical systems (2016-2019)
   Coordinator: MI SAS
   (Fečkan Michal)
4. VEGA 2/0031/15: Partial operations, quantum structures, and categorical methods in probability (2015-2017)
   Coordinator: MI SAS
   (Frič Roman)
5. VEGA 2/0178/14: Some questions concerning functional, harmonic and stochastic analysis (2014-2017)
   Coordinator: MI SAS
   (Haluška Ján)
6. VEGA 2/0006/16: Topologies on Functional Spaces and Hyperspaces (2016-2019)
   Coordinator: MI SAS
   (Holá Ľubica)
7. VEGA 2/0146/14: Algebraic and probabilistic number theory and their applications (2014-2017)
   Coordinator: MI SAS
   (Jakubec Stanislav)
8. VEGA 2/0069/16: Probabilistic and Categorial Aspects of Modelling Quantum Structures (2016-2019)
   Coordinator: MI SAS
   (Jenčová Anna)

9. VEGA 2/0084/15: Descriptional Complexity of Formal Systems (2015-2018)
   Coordinator: MI SAS
   (Jirásková Galina)
10. VEGA 1/0676/16: Research of Microstructures, Electrical and Optical Properties of Nanotextured Semiconductor Interfaces (2016-2018)
    Coordinator: Faculty of Electrical Engineering and Information Technology, University of Žilina, Žilina
    (Jurečková Mária)
11. VEGA 2/0017/14: Flow and chromatic problems in combinatoris (2014-2017)
    Coordinator: MI SAS
    (Kochol Martin)
12. VEGA 2/0026/14: Parallel block algorithms for the canonical decomposition of tensors (2014-2016)
    Coordinator: MI SAS
    (Okša Gabriel)
13. VEGA 2/0044/16: Selected Problems in Universal Algebra and Lattice Theory (2016-2018)
    Coordinator: MI SAS
    (Ploščica Miroslav)
14. VEGA 1/0097/16: Set-Theoretic Methods in Topology and Real Functions Theory (2016-2019)
    Coordinator: Faculty of Science, Pavol Jozef Šafárik University, Košice
    (Repický Miroslav)
15. VEGA 2/0047/15: Discrete and continuous probabilistic models and their applications (2015-2017)
    Coordinator: MI SAS
    (Wimmer Gejza)
16. VEGA 2/0049/14: Aggregation of inputs from multiple competitive and/or cooperative categories (2014-2017)
    Coordinator: MI SAS
    (Zemánková Andrea)
17. VEGA 1/0101/17: Global and Local Properties of Spaces and Mappings (2017-2019)
    Coordinator: Faculty of Mathematics, Physics and Informatics, Commenius University, Bratislava
    (Macko Tibor)
18. VEGA 2/0004/17: Parallel Block Algorithms for Canonical Decomposition of Matrices (2017-2019)
    Coordinator: MI SAS
    (Okša Gabriel)
19. VEGA 1/0487/17: Algorithms on graphs and on algebraich structures (2017-2019)
    Coordinator: Faculty of Natural Sciences, Matej Bel University, Banská Bystrica
    (Nedela Roman)
20. VEGA 2/0109/18: Number Theory and Its Applications (2018-2021)
    Coordinator: MI SAS
    (Jakubec Stanislav)
21. VEGA 2/0024/18: Chromatic Problems in Combinatorics (2018-2021)
    Coordinator: MI SAS
    (Kochol Martin)
22. VEGA 2/0144/18: Multiclass Speech Segments Classification Using Parallel Classifiers (2018-2021)
    Coordinator: MI SAS
    (Šuch Ondrej)

23. VEGA 2/0054/18: New statistical methods for special classes of probability distributions and their applications (2018-2020)
    Coordinator: MI SAS
    (Wimmer Gejza)
24. VEGA 2/0132/19: Descriptive and Computational Complexity of Formal Languages (2019-2022)
    Coordinator: MI SAS
    (Jirásková Galina)
25. VEGA 2/0106/19: Wooden pipe configuration of historic organ positives in Slovakia (2019-2022)
    Coordinator: Institute of Musicology, SAS
    (Haluška Ján)
26. VEGA 1/0006/19: New Trends in Aggregation Theory and Their Applications (2019-2022)
    Coordinator: Faculty of Civil Engineering, Slovak University of Technology, Bratislava
    (Zemánková Andrea)
27. VEGA 1/0601/20: Models and algorithms for computing with incomplete information (2020-2023)
    Coordinator: Faculty of Mathematics, Physics and Informatics, Commenius University, Bratislava
    (Dobrev Stefan)
28. VEGA 2/0015/20: Convergence of Block Algorithms for Canonical Matrix Decompositions (2020-2023)
    Coordinator: MI SAS
    (Okša Gabriel)
29. VEGA 2/0078/20: Graph invariants, symmetries and labellings (2020-2023)
    Coordinator: MI SAS
    (Nedela Roman)
30. VEGA 2/0097/20: Algebraic and Topological Aspects of Aggregation Functions (2020-2023)
    Coordinator: MI SAS
    (Nedela Roman)
31. VEGA 2/0127/20: Qualitative properties and bifurcations of differential equations and dynamical system (2020-2023)
    Coordinator: MI SAS
    (Fečkan Michal)
32. VEGA 2/0142/20: Mathematical models of non-classical events and uncertainty (2020-2023)
    Coordinator: MI SAS
    (Jenčová Anna)
33. VEGA 2/0048/21: Topological structures on function spaces (2021-2024)
    Coordinator: MI SAS
    (Holá Ľubica)
34. VEGA 2/0096/21: Probability Distributions and their Applications in Modeling and Testing (2021-2023)
    Coordinator: MI SAS
    (Mačutek Ján)

2.4.5. List of projects supported by EU Structural Funds

1. Matematická podpora kvantových technológií
   (Mathematical Support for Quantum Technologies)
   MI SAS
   Operating program: 313000 - Operačný program Výskum a inovácie
   (313000 - Operation Program – Research and Development)
   Announcement code: OPVaI-VA/DP/2018/1.1.3-09
   Application number: NFP313010T683
   Contract number: 016/2019/OPII/VA
   Total funds: 1 001 422.60 € (requested amount: 1 087 916.53 €)

2. Výskum v oblasti analýzy heterogénnych dát za účelom predikcie zmeny zdravotného stavu chronických pacientov
   (Research in the Field of Heterogeneous Data Analysis to Predict Changes in the Health Status of Chronic Patients)
   MI SAS
   Operating program: 313000 - Operačný program Výskum a inovácie
   (313000 - Operation Program – Research and Development)
   Announcement code: OPVaI-VA/DP/2018/1.1.3-10
   Application number: NFP313010T634
   Contract number: 017/2019/OPII/VA
   Total funds: 1 060 712.02 € (requested amount: 1 239 037.76 €)

2.4.6. List of other projects funded from national resources

1. Program Fellowship of SAS (Zemánková Andrea) (2013-2021)
2. SASPRO: Kurzweil's and Dobrakov's approaches to integration (Monteiro Gisselle) (2016-2017)
3. ``Home Return Programme'': Topology of high-dimensional manifolds (Macko Tibor) (2016-2017)
4. ``ŠPVV Programme (State Programme for Research and Development)'': Preparation of national project for quantum Technologies (Nemoga Karol) (Since 2018.) Coordinator: QUTE – Slovak national platform for quantum technologies.

2.4.7. List of projects funded from private funds

No project.

2.4.8. List of projects funded from other competitive funds

1. Internal Project of MI SAS: The optimization model of natural gas transportation (Žáčik Tibor) (Since `90s.)

2.5. PhD studies and educational activities

2.5.1. List of accredited programmes of doctoral studies, period of validity, source of funding

9-1-9 Applied Mathematics (long-term cooperation with FMFI UK, Bratislava), funded from SAS.

2.5.2. Summary table on doctoral studies (number of internal/external PhD students at the end of the year; number of foreign PhD students, number of students who successfully completed their theses during the year, number of PhD students who quit the programme during the year*)*

PhD study	2016			2017			2018			2019			2020			2021		
Number of potential PhD supervisors	13			15			16			18			12			10		
PhD students	number, end of year	defended thesis	students quitted	number, end of year	defended thesis	students quitted	unumber, end of yea	defended thesis	students quitted	number, end of year	defended thesis	students quitted	number, end of year	defended thesis	students quitted	number, end of year	defended thesis	students quitted
Internal total	6	3	1	6	2	0	5	0	1	5	0	0	3	2	0	4	0	0
from which foreign citizens	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
External	1	0	1	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0
Other supervised by the research employees of the institute	10	0	0	5	5	0	5	0	0	1	4	0	1	0	0	2	0	0

2.5.3. PhD carrier path – Information on the next career steps of the PhD graduates who received their degree from the institute

2016

Mgr. Kristína Čevorová – academic sphere – high school/university

Mgr. Tomáš Gregor – commercial company

Mgr. Anton Sedliak – commercial company

2017

Mgr. Peter Mlynárčik – academic sphere – academy / university

Mgr. Matúš Palmovský – academic sphere – academy/ university

2019

Ing. Michal Hospodár – academic sphere – academy

RNDr. Martin Plávala – academic sphere – academy
(currently at Alexander von Humboldt Fellowship)

2020

Mgr. Dušana Babicová – academic sphere – academy

Mgr. Ivana Krajňáková – commercial company

2.5.4. Summary table on educational activities

Teaching	2016	2017	2018	2019	2020	2021
Lectures (hours/year)*	498	702	768	684	651	1 067
Practicum courses (hours/year)*	775	303	579	639	641	388
Supervised diploma and bachelor thesis (in total)	23	12	8	7	3	11
Members in PhD committees (in total)	5	6	6	5	5	2
Members in DrSc. committees (in total)	2	1	1	1	1	3
Members in university/faculty councils (in total)	13	11	11	9	9	6
Members in habilitation/inauguration committees (in total)	6	4	4	2	3	2

2.5.5. List of published university textbooks

1. PALENČÁR, R. - WIMMER, Gejza - PALENČÁR, J. - WITKOVSKÝ, Viktor. Navrhovanie a vyhodnocovanie meraní [Design and Evaluation of Measurements]. Recenzenti [Reviewers]: M. Dovica, D. Janáčová, J. Markovič. 1. Vydanie (1st Edition). Bratislava : Slovenská technická univerzita v Bratislave [Slovak University of Technology], 2021. 160 s. ISBN 978-80-227-5080-6

2.5.6. Number of published academic course books

1 published course book.

2.5.7. List of joint research laboratories/facilities with universities

In 2016 it was founded Institute of Mathematics and Informatics, Banská Bystrica (a detached institute of MI SAS in Banská Bystrica.) (Former joint research facility with Matej Bel University, Banská Bystrica was canceled.)

In 2019 it was established joint research facility of the following institutions

- Faculty of Natural Sciences, Matej Bel University in Banská Bystrica,
- Earth Science Institute of the SAS,
- Mathematical Institute of the SAS,
- Institute of Informatics of the SAS,
- Department of Biodiversity and Botany, Institute of Botany of the SAS.

In 2018 it was established „Quantum Consortium“ with the members

- Institute of Physics, Bratislava, SAS,
- Institute of Mathematics, Bratislava, SAS,
- Institute of Experimental Physics, Košice, SAS,
- Slovak University of Technology, Bratislava,
- Comenius University, Bratislava,
- P. Safarik’s University, Košice.

2.5.8. Supplementary information and/or comments on doctoral studies and educational activities – focused on what changes have occurred since the last evaluation in 2016

2.6. Societal impact

2.6.1. The most important case studies of the research with direct societal impact, max. 4 for institute with up to 50 average FTE researchers per year, 8 for institutes with 50 – 100 average FTE researchers per year and so on. Structure: Summary of the impact; Underpinning research; References to the research; Details of the impact; Sources to corroborate the impact. One page per one case study

No relevant studies.

2.6.2. List of the most important studies and/or other activities commissioned for the decision-making authorities, the government and NGOs, international and foreign institutes (title, name of institution, contract value, purpose (max 20 words))

- Organizing NATO Cyber Security Call 2017, including two seminars at NATO HQ, Brussels. Participation on evaluation process. K. Nemoga.
- Cooperation with the Ministry of Defense SR on security problems. K. Nemoga.

2.6.3. List of contracts and research projects with industrial and other commercial partners, incl. revenues (study title, name of institution, contract value, country of partner, purpose (max 20 words))

1. **Application of mathematical methods for optimization of gas transport**
   Partner: eustream, a.s. Nitra, Slovakia
   Contract value: 2 700,- (2016)
   Purpose: Adjustments to the computation of gas transport optimizations based on the requirements of gas operators.
   Cooperation ended in 2016.

2. **Development and computer implementation and evaluation of algorithms for gas pipeline leak detections**
   Partner: ttc, s.r.o., Nitra, Slovakia
   Contract value: 5 521(2016)
   Purpose: Leak detection system for a transport company in Great Britain operating the gas pipeline Bacton-Great Yarmouth. Since 2017, monitoring and improving algorithms (academic research, without direct commercial gain).
   Cooperation years (in the assessed period): 2016-2021

3. **Mathematica Slovaca**, an international mathematical journal owned by the Institute of Mathematics
   Partner: DeGruyter, International Publishing Company
   Contract value: approximately 5500 EUR yearly
   Purpose: Sole distribution and dissemination of our journal in printed and electronically.

2.6.4.1 List of intangible fixed assets (internally registered IP (confidential know-how), patent applications, patents granted, trademarks registered) denoting background IPR

No registered IP.

2.6.4.2 List of licences sold abroad and in Slovakia, incl. revenues (background IPR identification, name of institution, contract value, country of partner, purpose (max 20 words))

No licences.

2.6.5. Summary of relevant activities, max. 300 words (describe the pipeline of valorization in terms of Number of disclosure, Number of registered IP internally, number of CCR/LIC contracts and their respective summary values, the support you are receiving in specific points internally at the institute, at SAS, externally – also the limitations and drawbacks.

2.7. Popularisation of Science (outreach activities)

2.7.1. List of the most important popularisation activities, max. 20 items

- A. Dvurečenskij, *Fokus – Biele miesta s Danielou Kapitáňovou (Focus – White (Uncharted) Places with D. Kapitáňová)*, Rádio Devín, Interview in radio program, 18. 9. 2016.
- M. Hyčko, *Šťastie má rovnicu (vyjadrenie sa v príspevku) (A Luck Has its Equation – a short comment to a news)*, TV Noviny, TV JOJ, 10. 7. 2016.
- K. Nemoga, *Prečo bola a je matematika dobrá? Potrebujú ju umelci, právnici či lekári? (Why was and is Mathematics Useful? Do Artists, Lawyers or Medical Doctors Need It?)*, Vedecká cukráreň (Scientific Patisserie), CVTI (Centre for Scientific-Technological Information), Public lecture and discussion, 22. 3. 2016
- M. Plávala, *Kvantové počítače pre všetkých (Quantum Computers for Everyone)*, blog post article, https://dennikn.sk/blog/kvantove-pocitace-pre-vsetkych/, 6. 7. 2016.
- E. Halušková, D. Štiberová, P. Mlynárčik, *Mladí vedci plní energie (Youth Full of Energy)*, ZŠ Park Angelium (Grammar School Park Angelium), Košice, Interactive lesson concerning mathematics, 6. 12. 2016.
- A. Dvurečenskij, *They look easy; they can't calculate them. Some problems mathematicians can't solve for decades; for others, you can get a reward of a million dollars. Interview, SME, 15. 5. 2016*
- R. Frič, *Mathematics as a mean for understanding*, Day of Open Door, Košice, public lecture, November 10, 2016.
- E. Halušková, D. Štiberová, P. Mlnarčík, *Young scientists, full of energy*, Basic School, Krosnianska 4, Košice, 11. 11. 2016.
- L. Holá, A. Dvurečenskij, B. Riečan, *T. Neubrun*, chapters in the book, Charles University, Prague.
- T. Macko, The brains from abroad not came, only four, Interview, SME, 31. 10. 2016.
- A. Dvurečenskij, *Grantový program Talenty Novej Európy dnes vstupuje do 11. Ročníka (Grant Programme: Talents for New Europe Enters its 11th Year)*, Press release, 12. 9. 2017.

- R. Hajossy, *Kto, kde, kedy a prečo vymyslel Pytagorovu vetu? (Who, Where, When and Why Pythagorean Theorem was Invented)*, Open Days at MI SAS, Public lecture, 9. 11. 2017.
- K. Nemoga, *Je matematika kráľovnou vied? (Is Mathematics a Queen of Science?)*, Radio interview 9pm-12pm, 21. 1. 2017.
- M. Plávala, *O tom, ako sa niekedy nedajú robiť dve veci naraz a prečo je to zaujímavé pre fyzikov a programátorov (About, How It Is Sometimes Not Possible To Do Two Things At Once And Why It Is Interesting For Physicists And Programmers)*, Open Days at MI SAS, Public lecture, 7 11 2018.
- M. Plávala, *Granty pre študentov pomáhajú slovenskému školstvu. 3 dôvody, prečo sa o ne uchádzať (Grants Help Students in Slovakia, Three Reasons Why To Apply)*, Internet article, https://www.nadaciapontis.sk/clanok/granty-pre-studentov-pomahaju-slovenskemu-skolstvu-3-dovody-preco-sa-o-ne-uchadzat/2572, 12. 1. 2018.
- S. Dobrev, *Ako nájsť poklad keď nemáš pamäť? (Prehľadávanie multi-dimenzionálnych mriežok konečnými automatmi) (How To Find A Treasure If You Do Not Possess Memory? (Search in Multi-dimensional Lattices by a Finite Automata))*, Open Days at MI SAS, Public lecture, 6. 11. 2019.
- K. Nemoga, *Rozhovor o projektoch programu NATO Science for Peace and Security (Interview for Spain TV concerning NATO Peace and Security Programme)*, 23. 9. 2019.
- A. Dvurečenskij, *Matematika je podstatná a nezastupiteľná (Mathematics is Essential and Irreplaceable)*, Published interview with A. Dvurečenskij by S. Ščepán concerning importance of Mathematics, Správy SAV 55 (2), 2019.
- M. Plávala, *Quantum Mystery*, magazine QUARK (for popularization of Science), 2019.
- **K. Nemoga**, O. Grošek, T. Fabšič, *Cryptology and Quantum Computers*, Spotify, 2021, Interview, https://open.spotify.com/episode/1SZ5Y10922qhWSFuM9tonD.

During 2020–2021 popularization activities have ceased due to pandemic situation, however interesting teaching for children (Matematický krúžok (Mathematical Lessons)) by E. Halušková and P. Mlynarčík have continued. K. Nemoga attended several podcasts.

2.7.2. Table of outreach activities according to institute annual reports

Outreach activities	2016	2017	2018	2019	2020	2021	total
Articles in press media/internet popularising results of science, in particular those achieved by the Organization	18	13	20	10	0	2	63
Appearances in telecommunication media popularising results of science, in particular those achieved by the Organization	2	4	0	1	0	0	7
Public popularisation lectures	12	14	13	13	1	0	53

2.8. Background and management. Infrastructure and human resources, incl. support and incentives for young researchers

2.8.1. Summary table of personnel

2.8.1.1. Professional qualification structure (as of 31 December 2021)

	Degree/rank				Research position		
	DrSc./DSc	CSc./PhD.	professor	docent/ assoc. prof.	I.	II.a.	II.b.
Male	8	31	8	14	8	13	18
Female	3	12	0	2	3	3	8

I. – director of research with a degree of doctor of science/DrSc.

II.a – Senior researcher

II.b – PhD holder/Postdoc

2.8.1.2. Age and gender structure of researchers (as of 31 December 2021)

Age structure of researchers	< 31		31-35		36-40		41-45		46-50		51-55		56-60		61-65		> 65	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Male	1,0	1,0	1,0	1,0	3,0	2,2	5,0	3,5	3,0	0,4	4,0	4,0	3,0	1,1	5,0	4,0	9,0	5,5
Female	2,0	1,2	0,0	0,0	1,0	1,0	5,0	3,7	1,0	1,0	2,0	1,0	1,0	1,0	1,0	1,0	2,0	1,1

A – number

B – FTE

2.8.2. Postdoctoral fellowships (list of positions with holder name, starting date, duration. Add brief information about each fellow's career path before and after receiving PhD degree, etc.)

2.8.2.1. MoRePro and SASPRO fellowships

Dr. Giselle Monteiro (Brazil) (SASPRO, 2016-2017)

Dr. O. Zahiri (Iran) (SASPRO2, 2021-, awaiting VISA)

2.8.2.2. Stefan Schwarz fellowships

Dr. Natalia Dilna (Ukraine) (2018-2019, originally acquired 2009, 2 maternal leaves in-between)

2.8.2.3. Postdoctoral positions from other resources (specify)

2.8.3. Important research infrastructure introduced during the evaluation period with the information about the sources of funding *(max. 2 pages)*

- In 2021 we built a new internet network at the building on Štefánikova 49, Bratislava, with 10 Gbits incoming, 1Gbit internal. Four servers plus licenses, incl.

  Covered by institutional source, gained from the previous project from structural funds.

2.9. Supplementary information and/or comments on all items 2.1 – 2.8 (max. 2 pages in total for the whole section)

3. *Implementation of the recommendations from the previous evaluation period*

The multidisciplinary panel recognized the publication outputs of the Mathematical Institute as impressive. But the Multidisciplinary panel addressed the Institute of Mathematics very concrete recommendations (chosen from the Meta-Panel Assessment Report):

- Even though the age structure has improved slightly and only increased 6 months the last four years, it is still a matter of strong concern. The age distribution of the researcher of the IM is problematic.
- way to attract young researchers to Ml is to expand the postdoc program, which at the moment looks rather small. Naturally, that requires other priorities regarding financing.
- The gender distribution is not as bad as in some other mathematical institutions but should be improved. The outreach activities could be extended by having activities aiming at high school teachers, for example as in the Klein Project started by ICMI and IMU.
- Another outreach activity is to invite the high schools to send some of their best math students to the Institute for a weekend of lectures and problem solving (Sonia Kovalevski Days).

Figures of the average age of research fellows in the last ten years are the following

2021	2020	2019	2018	2017	2016	2015	2014	2013	2012
53.7	52.8	51.7	53.0	52.1	52.5	53.0	24.0	57.0	55.0

With some slight variations, the average age of the research fellows is stable. It is connected with the special subject of our research – mathematics. It is possible to do high-level research mathematics at a higher age. Institute has part-time employees over 70, and we agree on their FTE considering their contribution. Frequently researchers at higher ages give higher contributions to the Institute's output.

In the evaluated period, we made contracts (full-time employees) with the following younger colleagues:

- RNDr. Katarína Čunderlíková, PhD.,
- Ing. Michal Hospodár, PhD., our PhD. Student,
- Ing. Irena Jadlovská, PhD.,
- doc. Mgr. Ján Mačutek, PhD.,
- RNDr. Igor Odrobina, CSc.,
- RNDr. Martin Plávala, PhD., our PhD. Student.

We strongly feel that it is necessary to gain further young people, Ph.D. Students and postdocs as well. Especially in mathematical statistics, computer science, and number theory.
Problems are the following:

- Financial problems.
- Little interest in being a Ph.D. Student.

Possible solutions:

- Utilize SASPRO program for postdocs (we have one new SASPRO position).
- Closer cooperation with universities, especially leading good students to master thesis.
- Accept more students from abroad.

Moreover, for high school students, we are organizing the Days of Open Door to present research activities for potential students of mathematics. For university students in the fourth or fifth year of their study, we are giving a possibility to do a one-month research stay at our Institute, and among them, we are looking for prospective new Ph.D. students. Unfortunately, this endeavor was interrupted by COVID period.

Considering gender issues, we do stress that the only role in accepting new people is the ability to "make mathematics." Two new research fellows from the institute, which reached the rank of Doctor of Science, were women, and the next also be a woman. We create excellent working conditions for women. We keep the position at the institute also in the case of maternity leave prolonging.

To gain the attraction of young people to mathematics, we are organizing:

- „Day of Open Doors" for high school students. Typically half day with four lectures in Bratislava, the same in Kosice.
- Talks/courses at primary schools.
- Summer stays for university and high school students under the supervision of one research fellow from the institute.

4. ***Research strategy and future development of the institute for the nextfive years*** (Recommended 3 pages, max. 5 pages)

Research strategy of the institute in the national and international contexts, objectives, and methods (including the information on when the strategy was adopted)

The aim of our Institute for the next five years is to continue in the basic mathematical research in all areas in which we already gained important results. For this aims we will try to apply for projects within the programs of the European Union and national programs. Our goals will be to perform frontier top basic research leading to new fundamental results, collaborate with institutes of the Slovak Academy of Sciences, Slovak universities as well as to collaborate with business and public organizations and applying our results there. This will be done in frames of the Strategy SAS 2030. This will be done in the post covid time and with respect to problems with ongoing war in Ukraine and consecutive possible economic problems in Europe in our country.

1. **Number Theory and cryptography**

Number theory belongs to the traditional mathematical fields. Despite the unexpected applications, the number of scientists working in this field is decreasing, our Institute including. Nevertheless we

will study uniform distributions. Special questions concerning two-dimensional distribution function g(x,y) defined in $[0,1]^2$, which is called copula, if g(x,1)=x and g(1,y)=y for every x,y. Similarly, s-dimensional copula is a distribution function $g(x_1, x_2, ..., x_s)$ such that every k-dimensional face function is uniformly distributed for some but fixed k. In the following we summarize and extend all parts of copulas contained in the books

- O. Strauch, Distribution of Sequences: A Theory published in Veda and Academy, 2019.
- R.B. Nelson, An Introduction to Copulas, Properties and Applications, Lecture Notes in Statistics 139, Springer-Verlag , New York, 1999.1
- N. Balakrishnan and Chin-Diew Lai, Continuous Bivariete Distribution, New York, 2009.

All in the narrow cooperation with our colleagues grouped around the journal Uniform Distribution Theory publishing by our Institute.
In cryptology, except for the fundamental problems of mathematical cryptology, where we closely cooperate with the Faculty of Electrical Engineering and Information Technology, Slovak Technical University (O. Grosek, V. Hromada, T. Fabsic, P. Marak, S. Balogh), we will study applications in protocols with our traditional partners in USA (R. Steinwandt, S. Magliveras), Spain (M. I. G. Vasco), Malta (Ch. Colombo), and others.

2. Uncertainty modeling by statistical methods, quantum structures, and fuzzy sets

Uncertainty modeling is one of the basic problems of measurement in many scientific and technological areas having practical applications in every-day life, or in quantum mechanics or economy. The notion of uncertainty is developed into width and deepness of our knowledge and it is inspired by mathematical foundations of quantum physics.

The discovery of a new physics, which is today called quantum mechanics, initiated a revolution not only in physics but also in mathematics and in philosophical views onto our world. Quantum physics is nowadays a basic tool for new industrial technologies. It is estimated that more than 2/3 of the economical profit of the USA is in a direct or indirect way connected to technologies that have their roots in quantum physic, for example, the discovery of the transistor, superconductivity, laser, etc. It is to be expected that quantum technologies will play a crucial role in both technical and industrial development in the 21st century. There already are industrially exploitable quantum technologies such as a gravimeter based on quantum interference, an atomic clock, or quantum cryptography. It is necessary to mention also a new generation of computers, quantum computers, improvement of computer security, remote Earth research, metrology, and other areas of quantum technology applications. It is clear that in the next 10-15 years, quantum technologies will play a dominant role also in EU countries. We recall that the biggest rivals of the EU, the USA, and China, are making huge investments in commercial usage of technologies based on quantum mechanics. A crucial condition for such development is a high level of basic and applied research in this area. The most developed countries already today invest greatly into “quantum” research. In the field of physics and mathematics, such research exists and is concentrated around quantum computing and quantum cryptography.

Modeling of uncertainty is based on deep theoretical results, mainly mathematical statistics, algebraic structures, functional analysis, Hilbert space and operators, fuzzy and many-valued logics, and topological and categorical views. Mathematical Institute SAS succeeds in top results comparable with the highest world results namely in applications of statistical methods for example in medicine, metrology, and linguistics. Measurement of observables in quantum mechanics is based on results of quantum structures, here MI SAS plays a leading role on a worldwide scale. Today fuzzy theory is the base of many modern fuzzy technologies. The results of our colleagues showed that there is an intimate connection between many-valued logic and algebraic structures, like MV-algebras, pseudo MV-algebras, pseudo effect algebras, and lattice ordered groups. Our

colleagues achieved excellent results in those directions and MI SAS is collaborating with many domestic and foreign centers. Envisaged directions for this area are:

- Statistical methods in measurement, linguistics, medicine, econometrics, insurance, and finance.
- Quantum structures as a mathematical base for quantum mechanics, quantum computing, quantum information, and soft computing.
- Compatibility and non-compatibility in generalized probabilistic theories.
- Quantum convex structures and measurements of quantum-mechanical events.
- Quantum mechanical observables and quantum channels
- Fuzzy sets as a mathematical base for modern fuzzy technologies.
- Study of aggregation operators, aggregation functions, and their algebraic properties.

Many-valued logic, quantum structures will be studied with Profs A. Di Nola and S. Lapenta, Univ. Salerno, Prof. C. Tsinakis, Univ. Vanderbilt, Prof. Ruskai, Uni. Boston, Prof. T. Kowalski, Jagelion Uni. Cracow, Prof. R. Giuntini, Univ. Cagliari, Dr. R. Carbone, Univ. Pavia, Italy, Dr. R.P. Kostecki, Perimeter Inst, Waterloo, Canada, Prof. P. Klement, Univ. Linz, Profs. D. Buhagiar and E. Chetcuti, Univ. Malta, Dr. Xie Y., China, Prof. R.A. Borzooei, Dr. O. Zahiri, Univ. Appl. Sci. Techn. Tehran, Iran, Palacky Univ. Olomouc (Profs. Chajda, Halaš, J. Kuhr, M. Botur), Masaryk Univ. Brno (Prof. J. Paseka), etc.

In quantum information theory and quantum foundations, we will focus on the following directions:

- The study of quantum relative entropies in the general context of von Neumann algebras. Here we follow up on our results on the extension of versions of Rényi relative entropy, with applications in information theory and elsewhere, e.g. in quantum field theory. In particular, we plan to focus on applications to the asymptotic theory of hypothesis testing.
- General probabilistic theories (GPT). Here we continue the study of nonclassical effects (entanglement, incompatibility, nonlocality…) and their relations, in perspective further collaboration with A. Bluhm (University of Copenhagen), I. Nechita (Université de Toulouse) and our former PhD student M. Plávala (Uni Siegen).
- Spectrality in quantum structures. Spectrality is one of the basic features of Hilbert space effects and is necessary for the mathematical description of quantum theory. We proceed with our study of versions of spectrality in effect algebras, order unit spaces, and various special cases.

Considering the problems of quantum structures and cryptology, we are member of the so-called Quantum Consortium together with the Institute of Physics, Institute of Informatics, Faculty of Mathematics, Physics and Informatics, Comenius University, Faculty of Electrical Engineering, and Information Technologies, Slovak University of Technology. Here we open a new direction in the touch of quantum and post-quantum technologies. Starting with the NATO project from 2022 NATO Call.

Special attention we will devote to the aggregation theory.

The roots of the aggregation theory go back to ancient Egypt and Greece, where various particular fusion methods were already known. However, the area of aggregation has seen enormous growth relatively recently. Since the 1980s, aggregation theory has become an established branch of mathematics and computer science, focusing primarily on the design of methods and analysis of real functions at intervals. Aggregation methods aim to extract essential patterns that represent large and complex data sets, creating a clear picture of the whole instead of conceiving reality as a set of individual entities, which are difficult to process and analyze. Mathematical Institute SAS achieved

numerous essential results in this area, including deep study and characterization of several special classes of aggregation functions and related construction methods, non-additive measures and integrals, development of multi-polar aggregation, and their applications.

We plan to focus on the following directions:

- Extension of techniques known for aggregation functions on real intervals to aggregation functions defined on more general lattices and posets.
- Construction and characterization of new classes of aggregation functions on lattices and posets.
- Representation of special classes of aggregation functions on lattices and posets.
- Characterization of various generating sets of aggregation clones on lattices and posets.

We intend to cooperate with Sebastia Massanet, Arnau Mir, UIB Palma de Mallorca, Spain, Michal Holčapek, Antonín Dvořák, OSU Ostrava, Czech Republic, Yong Su, Suzhou University of Science and Technology, China.

3. Discrete Structures, Graph theory, and Algebra

In graph theory and algebra, we shall continue to investigate the following problems in distinguished fields of mathematics:

- Graph isomorphism problem (theoretical computer science)
- Topological graph theory
- Discrete groups actions (Geometry and Algebra)
- Snarks and cubic graphs (graph theory)

More precisely:

The complexity of the graph isomorphism problem is one of the central problems of theoretical computer science. It is known that it lies between P and NP, and it is one of the hot candidates for an intermediate complexity (which would disprove the P=NP conjecture). In late years we were able to prove several interesting approximations of the problem, including an FPT result on chordal graphs of bounded leafage. Here Weisfeiler-Lehman's theory of coherent configurations was employed. We plan to continue the research in this direction. Another interesting problem with many applications is the question of the existence of a subquadratic algorithm solving the simultaneous conjugacy problem.

In late years we achieved many results on maps, hypermaps, and their symmetries. The enumeration results have applications in theoretical physics, namely in gravity models. Recently, a possible application in quantum coding has been discovered. We plan to investigate this relationship in detail.

By the Riemann-Hurwitz equation, there are just finitely many discrete groups of genus >1. Employing results by Zieshang, Lloyd, and others, we were able to derive an algebraic criterion for so-called planar discrete groups, and these are the groups with signature $(0; m_1, \ldots, m_r)$. It allows us to derive and implement an algorithm classifying the actions for small genera. We plan to generalize the achieved results to arbitrary actions. The obtained results are interested in discrete and continuous mathematics, including algebraic geometry, for example.

Snarks are non-three-edge colorable cubic graphs. Investigation of snarks is motivated by the fact that many long-term open problems in graph theory and combinatorics can be reduced to questions on snarks. Our aim is to develop a consistent and systematic theory of snarks that would allow attacking particular problems.

We will cooperate with Martin Škoviera, Edita Máčajová, both Comenius University, Bratislava, Slovakia, Jozef Širáň, Slovak Technical University, Bratislava, Peter Zeman, Lyngby, I. Ponomarenko, Petersburg, M. Seifrtová, Technical University Prague, Kan Hu, Zieshang Ocean University, PRC.

We plan also to study Tutte polynomials and their generalizations, representation of the Tutte polynomial, relations among various representations, and study computational aspects in contexts with the representations, and generalizations of the Tutte polynomials, mainly polynomials with additional parameters.

We will cooperate on studying these problem with the group of researchers dealing with the topic:J. Oxley, New Orleans, USA, J. Nesetril, Prague, Czech Republic, J. Goodal, Prague, Czech Republic, E. Gion, Montpelier, France, J. Makowsky, Haifa, Israel, J. Ellis-Monaghan, Amsterdam, Netherland, I. Moffatt, London, UK, L. Traldi, Lafayette, USA, T. Zaslavsky, Binghampton, USA.

4. Dynamical systems, differential equations, real and functional analysis, and topology

Besides nonlinear differential equations, many real-life phenomena are more precisely modeled using various kinds of non-integer order derivatives and integrals. Due to their nonlocal structure, they are used to describe processes with memory. In recent years, the theory of fractional calculus and its applications is experiencing a great boom, also thanks to the contribution of MI SAS. However, there are still many unresolved problems. We are planning to concentrate on the following directions:

- periodic and asymptotic periodic solutions of nonlinear systems, including fractional derivatives,
- stability and asymptotic properties of solutions,
- the existence of chaos in nonlinear fractional differential equation,
- solvability of the initial value problem for a system of linear fractional functional differential equations

Besides these directions, we also intend to study these problems:

- Existence and bifurcations of solutions of non-smooth differential equations. The classical mechanical problem with such discontinuity is mechanical problems with the so-called dry frictions,
- A qualitative study of implicit differential equations appearing in modeling nonlinear electronic circuits,
- Rather a topical problem at present is the study of traveling waves in infinite systems of coupled differential equations. Such systems model dynamics of atoms in molecules, for instance. Analytical and numerical results are expected on the existence of traveling waves.

We intend to cooperate with Flaviano Battelli and Alessandro Calamai, Dipartimento di Ingegneria Civile, Edile e Architettura, Università Politecnica delle Marche, Ancona, Italy.
Marius-F. Danca, STAR-UBB Institute, Babes-Bolyai University, Cluj-Napoca, Romania.
Matteo Franca, Dipartimento di Matematica, Università degli Studi di Bologna, Italy.
Vassilis Rothos, Department of Mathematics, Faculty of Engineering, University of Thessaloniki, Greece.
András Rontó, Department of Mathematics, Faculty of Education, Masaryk University, Brno, Czech Republic.
JinRong Wang, Department of Mathematics, Guizhou University, Guiyang 550025, Guizhou, China.

Also Július Pačuta, Department of Mathematical Analysis and Numerical Mathematics, Comenius University in Bratislava, Slovakia.

We will continue in a study of spaces of minimal usco and minimal cusco maps equipped with topologies of uniform convergence on bornologies. We will concentrate on metrizability, complete metrizability, and cardinal invariants of these topologies. We also plan to study topological properties of Arg min and Arg max multifunctions.

We will collaborate with Professors R. Ceppitelli, Perugia, Italy, A. K. Mirmostafaee, Mashhad, Iran, D. Holý, Trnava.

5. Computer science and data processing

Automata and Formal Languages: Descriptional and Computational Complexity. In this field we will concentrate on descriptional and computational complexity of formal languages represented by different computational models such as one-way and two-way deterministic and nondeterministic finite automata, alternating and Boolean automata, biautomata, quantum automata, etc. We aim to examine the (accepting) state complexity of languages and operations on the class of regular languages and its subclasses, including the ranges of attainable complexities for respective operations, and the complexity of languages in Kuratowski algebras. For various subclasses of regular and context-free languages, we aim to study their closure properties and the complexity of decision problems concerning these language classes. We also aim to investigate the non-uniform state complexity for some quantum finite automata models and get previously unresolved closure properties.

We intend to cooperate with Jeffrey Shallit, Canada, Ondřej Klíma, Czech Republic, Marek Syzkuła, Poland, Benedek Nagy, Cyprus, Markus Holzer, Germany, Jozef Jirásek jr., Canada, Jozef Jirásek, Slovakia, Juraj Šebej, Slovakia, Alexander Szabari, Slovakia.

We will also studied the problems of distributed computing. Distributed computing requires the analysis of networks, and the "Big Data" science needs very efficient serial and parallel algorithms for the decomposition of large matrices and tensors with the subsequent data compression. Both research fields are essential and include:

- The design, implementation, and testing of efficient parallel block algorithms for canonical decompositions of matrices and tensors.
- Theoretical problems in networks with the accent to linear network decompositions.
- Complexity of regular operations on languages represented by self-verifying automata.
- Theoretical analysis of the convergence (in exact and finite arithmetic) of block Jacobi EVD/SVD algorithms for different orderings of sub-problems (serial and parallel cases).
- Applications of efficient EVD/SVD algorithms in "Big Data" processing (medicine, image processing, machine learning, etc.).

Planned international cooperation in next years:
Yusaku Yamamoto, The University of Electro-Communications, Tokyo, Japan,
Zdeněk Strakoš, Charles University, Prague, Czech Republic,
Miro Rozložník, Institute of Mathematics, Academy of Sciences of Czech Republic, Prague, Czech Republic,
Roman Wyrzykowski, Czestochowa University of Technology, Czestochowa, Poland,
Lata Narayanan, Concordia University, Montreal, Canada,
Nicola Santoro, Evangelos Kranakis: Carleton University, Ottawa, Canada,
Paola Flocchini, Ottawa University, Ottawa, Canada.