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ON THE STRONG LAW OF LARGE NUMBERS FOR WEAK OBSERVABLES IN MV ALGEBRAS

Beloslav Riečan

Abstract. The strong law of large numbers is proved for an independent sequence of weak observables in an MV algebra.

1. Introduction

A typical example of an MV algebra is the set M of all functions $f\colon \Omega \to [0,1]$ endowed with two binary operations $\oplus$ and $\odot$ define by

$$f \oplus g = \min(f+g, 1)$$
$$f \odot g = \max(f+g-1, 0)$$

and a unary operation $*$ define by

$$f^* = 1 - f\,.$$

Generally ([1]) and MV algebra is an algebraic system $(M, \oplus, \odot, *, 0, 1)$ where $\oplus, \odot$ are binary operations, $*$ is a unary operation and $0, 1$ are fixed elements such that the following identities are satisfied: $a \oplus b = b \oplus a$, $a \oplus (b \oplus c) = (a \oplus b) \oplus c$, $a \oplus 0 = a$, $a \oplus 1 = 1$, $(a^*)^* = a$, $0^* = 1$, $a \oplus a^* = 1$, $(a^* \oplus b)^* \oplus b = (a \oplus b^*)^* \oplus a$, $a \odot b = (a^* \oplus b^*)^*$. If we define $a \vee b = (a \odot b^*) \oplus b$, $a \wedge b = (a \oplus b^*) \odot b$ and $a \leq b$, if $a \vee b = b$, then we obtain a distributive lattice.

By the Mundici representation theorem ([5]) there is a commutative lattice ordered group G and a strong unit u in G such that

$$M = [0,u],\ a \oplus b = (a+b) \wedge u\,,$$
$$a \odot b = (a+b-u) \vee 0,\ a^* = u - a\,,$$

and u plays the role 1 in M and the neutral element 0 of G plays the role 0 in M. We may work with elements of an MV algebra as elements of a commutative group. E.g., if $a \leq b$, $a, b \in M$, then there is in G the element $b - a$ (i.e., such element that $a + (b-a) = b$) and, because of $a \leq b$, the element $b - a \in [0,u] = M$. Similarly in the following definition of an observable, if $A \cap B = \emptyset$, then $x(A) + x(B) =$

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$x(A \cup B) \in [0,u]$, hence $x(A) \oplus x(B) = \big(x(A)+x(B)\big) \wedge u = x(A)+x(B)$. So, we can work with the group operation $+$ instead of the MV algebra operation $\oplus$.

A state ([2]) is a mapping $m\colon M \to [0,1]$ such that $m(a) = 1$, $m(a_n) \nearrow m(a)$, whenever $a_n \nearrow a$ and $m(a) = m(b) + m(c)$, whenever $a = b \oplus c$, $b \le c^*$.
An observable is a mapping $x\colon \mathcal{B}(R) \to M$ such that $x(R) = 1$, $x(A_n) \nearrow x(A)$, whenever $A_n \nearrow A$ and $x(A \cup B) = x(A) + x(B)$, whenever $A \cap B = \emptyset$.

In this article we shall consider weak observables replacing the condition $x(R) = 1$ by a weaker one, namely $m\big(x(R)\big) = 1$. This condition was mentioned for the first time probably in [4]. In [6] the sum of independent weak observables was defined. Subsequently the weak law of large numbers has been proved in [3] and the central limit theorem in [7]. All arguments of previous MV algebras probability reuslts are based on the following simple formula $(*)$. If x_1, x_2 are independent weak observables (Definition 2), then

$$(*) \qquad m\big((x_1+x_2)((-\infty,t))\big) = m_{x_1} \times m_{x_2}\big(\{(u,v); u+v<t\}\big)$$

where $m_{x_i}\colon \mathcal{B}(R) \to [0,1]$ is the probability distribution define by $m_{x_i}(A) = m\big(x_i(A)\big)$. From a sequence $(x_n)_n$ of osbervables a probability space (R^N, σ, P) is constructed with the infinite product measure $P = \prod_{i=1}^{\infty} m_{x_i}$. If $\xi_n\colon R^N \to R$ is the coordinate random variable defined by $\xi_n\big((t_i)_i\big) = t_n$, then

$$m\left(\left(\sum_{i=1}^{n} x_i\right)(c,d)\right) = P\left(\left(\sum_{i=1}^{n} \xi_i\right)^{-1}((c,d))\right).$$

This translation formula gives possible to prove the weak Law of Large Numbers (and the Central Limit Theorem as well). Of course, if we want to prove the strong law of large numbers, we need a stronger instrument, namely

$$m\left(\bigwedge_{k=1}^{n}\left(\left(\sum_{i=1}^{k} x_i\right)(c,d)\right)\right) \ge P\left(\bigcap_{k=1}^{n}\left(\left(\sum_{i=1}^{n} \xi_i\right)^{-1}((c,d))\right)\right).$$

It will be proved in Section 3 (Lemma 3). In Section 2 we shall introduce basic definitions, particularly, the definition of the sum of independent weak observables. Also the strong law of large numbers will be formulated there.

2. Independent observables

Definition 1. Let M be a given σ-complete MV algebra (i.e., every sequence $(u_n)_n$ of elements of M has the least upper bound $\bigvee_{n=1}^{\infty} u_n$ and the greatest lower bound $\bigwedge_{n=1}^{\infty} u_n$). Let $m\colon M \to [0,1]$ be a fixed state on M. A weak observable (with respect to m) is a mapping $x\colon \mathcal{B}(R) \to M$ from the σ-algebra of Borel subsets of R into M satisfying the following conditions:

(i) $m\big(x(R)\big) = 1$.

(ii) If $A, B \in \mathcal{B}(R)$, $A \cap B = \emptyset$, then $x(A \cup B) = x(A) + x(B)$.

(iii) If $A_n \in \mathcal{B}(R)$ $(n = 1, 2, \dots)$ and $A_n \nearrow A$, then $x(A_n) \nearrow x(A)$.

For any weak observable $x\colon \mathcal{B}(R) \to M$ we define the probability distribution $m_x\colon \mathcal{B}(R) \to [0,1]$ by the formula $m_x(A) = m\big(x(A)\big)$ and the mean value $E(x)$ by the formula

$$E(x) = \int_R t\, dm_x(t)\,.$$

Definition 2. Weak observables $x_1, \dots, x_n\colon \mathcal{B}(R) \to M$ are independent, if

$$m\big(x_1(A_1) \odot \cdots \odot x_n(A_n)\big) = m\big(x_1(A_1)\big) \cdot \ldots \cdot m\big(x_n(A_n)\big)$$

for any $A_1, \dots A_n \in \mathcal{B}(R)$. A sequence $(x_n)_n$ of weak observables in independent, if $x_1, \dots, x_n$ are independent for every n.

Definition 3. A σ-complete MV algebra M is said to be weakly σ-distributive if for any bounded double sequence (a_{ij}) of element of M such that $a_{ij} \searrow 0$ ($j \to \infty$, $i = 1, 2, \dots$) it holds

$$\bigwedge_{\varphi\colon\ N \to N}\ \bigvee_{i=1}^{\infty} a_{i\varphi(i)} = 0\,.$$

The sum of two weak observables in a weakly σ-distributive MV algebra has been introduced in [6] (Theorem 2). Motivated by the equality

$$(\xi + \eta)^{-1}\big((-\infty, t)\big) = \bigcup_{n=1}^{\infty}\ \bigcup_{(i,j)\in\alpha_n(t)} \xi^{-1}(A_i^n) \cap \eta^{-1}(A_j^n)\,,$$

where

$$\begin{aligned} A_i^n &= \Big[\frac{i-1}{2^n}, \frac{i}{2^n}\Big)\,, \\ \alpha_n(t) &= \big\{(i,j);\ i + j < t2^n\big\}\,, \end{aligned}$$

it is defined by the following way.

Definition 4. Let $x, y\colon \mathcal{B}(R) \to M$ be independent observables, M be weakly σ-distributive MV algebra. Put

$$\begin{aligned} \alpha_1(t) &= \big\{(i,j);\ i + j < 2t\big\}\,, \\ \alpha_n(t) &= \big\{(i,j);\ i + j < 2^n t\big\} \setminus \\ &\qquad \setminus \big\{(i,j);\ \exists\, m < n \exists\, (u,v) \in \alpha_m(t),\ A_i^n \times A_j^n \subset A_n^m \times A_v^m\big\}\,, \\ \Gamma_n(t) &= \sum_{k=1}^{n} \sum \big\{x(A_i^k) \odot y(A_j^k);\ (i,j) \in \alpha_k(t)\big\}\,. \end{aligned}$$

Then $x + y\colon \mathcal{B}(R) \to M$ is the weak observable satisfying the equality

$$(x+y)\big((-\infty, t)\big) = \bigvee_{n=1}^{\infty} \Gamma_n(t)$$

for every $t \in R$.

Definition 5. If $x\colon \mathcal{B}(R) \to M$ is a weak observable, $a, c \in R$, $c \neq 0$, then we define the weak observables xc, $x - a\colon \mathcal{B}(R) \to M$ by the formulas

$$cx(A) = x\big(\frac{1}{c}A\big)\,,$$
$$(x-a)(A) = x(A+c)\,,$$

where $\frac{1}{c}A = \big\{\frac{1}{c}u;\ u \in A\big\}$, $A + a = \{u + a;\ u \in A\}$.

For to define the m-almost everywhere convergence we shall be inspired again by the classical case. A sequence $(\eta_n)_n$ of random variables on a probability space $(\Omega, \mathcal{S}, P)$ converges P-almost everywhere to 0, if

$$P\Big(\bigcap_{p=1}^{\infty}\bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty}\eta_k^{-1}\Big((-\frac{1}{p},\frac{1}{p})\Big)\Big) = 1\,.$$

Definition 6. A sequence $(k_n)_n$ of weak observables converges m-almost everywhere to 0, if

$$m\Big(\bigwedge_{p=1}^{\infty}\bigvee_{n=1}^{\infty}\bigwedge_{k=n}^{\infty}z_k\Big((-\frac{1}{p},\frac{1}{p})\Big)\Big) = 1\,.$$

Now we are able to formulate the strong law of large numbers. It will be proved in the following section.

Theorem. *Let M be a weakly σ-distributive MV algebra. Let $(x_n)_n$ be independent sequence of weak observables of L^2 (i.e., $\int_R t^2 dm_{x_n}(t) < \infty$ for every n). Put $E(x_n) = a_n$ $(n = 1, 2, \dots)$, $\sigma^2(x_n) = \int_R (t - a_n)^2 dm_{x_n}(t)$. Let $\sum_{n=1}^{\infty}\frac{\sigma^2(x_n)}{n^2} < \infty$. Then*

$$\frac{1}{n}\sum_{i=1}^{n}x_i - \frac{1}{n}\sum_{i=1}^{n}a_i \to 0$$

m-almost everywhere.

3. Kolmogorov's construction

For to prove Theorem 1 we shall consider a probability space (R^N, σ, P), where P is the infinite product of the probability distributrions $m_{x_1}, m_{x_2}, m_{x_3}, \dots$. Here σ is the σ-algebra generated by the sets

$$A = \{t \in R^N;\ t_1 \in A_1, \dots, t_n \in A_n\}$$

(where $n \in N$, $A_1, \dots, A_n \in \mathcal{B}(R)$) and P is the measure determined by the equalities

$$P(A) = m_{x_1}(A_1)\cdot\ldots\cdot m_{x_n}(A_n)$$

$(n \in N,\ A_i \in \mathcal{B}(R))$. Further, we shall consider the coordinate random variables $\xi_n\colon R^N \to R$ defined by the equality

$$\xi_n(t) = t_n\,.$$

Lemma 1. *If $(x_n)_n$ is an independent sequence of weak observables, then $(\xi_n)_n$ is an independent sequence of random variables.*

Proof. [7], Lemma 1.

Lemma 2. *Let $a \in R$. Put $z_n = \left(\frac{1}{n}\sum_{i=1}^n x_i\right) - a$, $\zeta_n = \left(\frac{1}{n}\sum_{i=1}^n \xi_i\right) - a$. Then*

$$P \circ \zeta_n^{-1} = m \circ z_n\,,$$

hence the random variable $\frac{1}{n}\sum_{i=1}^n \xi_i - a$ has the same distribution as the observable $\frac{1}{n}\sum_{i=1}^n x_i - a$.

Proof. [7], Theorem 1.

Lemma 3. *Let $c, d \in R$, $c < d$. Then*

$$m\Big(\bigwedge_{k=1}^{s}\Big(\Big(\sum_{i=1}^{k} x_i\Big)((c,d))\Big) \geq P\Big(\bigcap_{k=1}^{s}\Big(\Big(\sum_{i=1}^{k}\xi_i\Big)^{-1}((c,d))\Big)\Big).$$

Proof. For the sake of simplicity we shall realise it only for the case $s = 2$. Put $a_{ij}^k = x_1(A_i^k) \odot x_2(A_j^k)$. Let $a, b \in R$, $a < b$. Then

$$\Gamma_n(b) - \Gamma_n(a) = \sum_{k=1}^{n}\Big(\sum_{(i,j)\in\alpha_k(b)} a_{ij}^k - \sum_{(i,j)\in\alpha_k(a)} a_{ij}^k\Big)\,.$$

Put

$$\beta_k(a) = \left\{(i,j);\ i + j \geq 2^k a\right\}.$$

Then

$$\Gamma_n(b) - \Gamma(a) \geq \sum_{k=1}^{n}\sum\left\{a_{ij};\ (i,j) \in \alpha_k(b) \cap \beta_k(a)\right\}.$$

Finally put

$$\gamma_k(c,d) = \left\{(i,j);\ 2^k c < i + j < 2^k d\right\}.$$

Then

$$\Gamma_n(b) - \Gamma_n(a) \geq \sum_{k=1}^{n}\sum\left\{a_{ij}^k;\ (i,j) \in \alpha_k(b) \cap \beta_k(a) \cap \gamma_k(c,d)\right\}. \tag{1}$$

Evidently

$$x_1\big((c,d)\big) \geq \sum_{k=1}^{n}\sum\left\{a_{ij}^k;\ (i,j) \in \gamma_k(c,d)\right\} \tag{2}$$

The relation (1) implies

$$\begin{aligned}(x_1 + x_2)\big((-\infty, b)\big) - \Gamma_n(a) &\geq \\ &\geq \sum_{k=1}^{n}\sum\left\{a_{ij}^k;\ (i,j) \in \alpha_k(b) \cap \beta_k(a) \cap \gamma_k(c,d)\right\}.\end{aligned} \tag{3}$$

By (2) nad (3) we obtain

$$m\big(x\big((c,d)\big)\wedge\big[(x_1+x_2)\big((-\infty,b)\big)-\Gamma_n(a)\big]\big)\geq$$
$$\geq\sum_{k=1}^{n}\sum\big\{m_{x_1}\times x_{x_2}(A_i^k\times A_j^k);\ (i,j)\in\alpha_k(b)\cap\beta_k\cap\gamma_k(c,d)\big\}\,.$$

Putting $n\to\infty$ in this inequality we obtain

$$\begin{aligned}
&m\big(x_1\big((c,d)\big)\wedge(x_1+x_2)(<a,b))\big)=\\
&=m\big(x_1\big((c,d)\big)\wedge\big((x_1+x_2)\big((-\infty,b)\big)-(x_1+x_2)\big((-\infty,a)\big)\big)\big)=\\
&=\lim_{n\to\infty}m\big(x_1\big((c,d)\big)\wedge\big((x_1+x_2)\big((-\infty,b)\big)-\Gamma_n(a)\big)\big)\geq\\
&\geq m_{x_1}\times m_{x_2}\Big(\bigcup_{k=1}^{\infty}\bigcup\big\{A_i^k\times A_j^k;\ (i,j)\in\alpha_k(b)\cap\beta_k(a)\cap\gamma_k(c,d))\big\}\Big)=\\
&=m_{x_1}\times m_{x_2}\big(\big\{(u,v);\ c<u<d,\ a<u+v<b\big\}\big)=\\
&=P\big(\xi_1^{-1}\big((c,d)\big)\cap(\xi_1+\xi_2)^{-1}\big((a,b)\big)\big)\,.
\end{aligned}$$

Therefore

$$\begin{aligned}
&m\big(x_1\big((c,d)\big)\wedge(x_1+x_2)\big((c,d)\big)\big)=\\
&=\lim_{k\to\infty}m\big(x_1\big((c,d)\big)\wedge(x_1+x_2)(<c+\frac{1}{k},d))\big)\geq\\
&\geq\lim_{k\to\infty}P\big(\xi_1^{-1}\big((c,d)\big)\cap(\xi_1+\xi_2)^{-1}\big((c+\frac{1}{k},d)\big)\big)=\\
&=P\big(\xi_1^{-1}\big((c,d)\big)\cap(\xi_1+\xi_2)^{-1}\big((c,d)\big)\big)\,.
\end{aligned}$$

Lemma 4. *Let $c,d\in R$, $c<d$. Then*

$$m\Big(\bigwedge_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}x_i\Big)\big((c,d)\big)\Big)\geq P\Big(\bigcap_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}\xi_i\Big)^{-1}\big((c,d)\big)\Big).$$

Proof. By the definition and Lemma 3

$$m\Big(\bigwedge_{k=1}^{s}\Big(\frac{1}{k}\sum_{k=1}^{k}x_i\Big)\big((c,d)\big)\Big)=m\Big(\bigwedge_{k=1}^{s}\Big(\sum_{i=1}^{k}x_i\Big)\big((kc,kd)\big)\Big)\geq$$
$$\geq P\Big(\bigcup_{k=1}^{s}\Big(\sum_{i=1}^{k}\xi_i\Big)^{-1}\big((kc,kd)\big)\Big)=P\Big(\bigcap_{k=1}^{s}\Big(\frac{1}{k}\sum_{k=1}^{k}\xi_i\Big)^{-1}\big((c,d)\big)\Big)\,.$$

Lemma 5. *Let $a, c, d \in R$, $c < d$. Then*

$$m\Big(\bigwedge_{k=1}^{s}\Big(\Big(\frac{1}{k}\sum_{i=1}^{k}x_i\Big)-a\Big)(c,d)\Big) \geq P\Big(\bigcap_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}\xi_i-a\Big)^{-1}((c,d))\Big).$$

Proof. By the definition and Lemma 4

$$\begin{aligned}
&m\Big(\bigwedge_{k=1}^{s}\Big(\Big(\frac{1}{k}\sum_{i=1}^{k}x_i\Big)-a\Big)((c,d))\Big) = \\
&= m\Big(\bigwedge_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}x_i\Big)((a+c,a+d))\Big) \geq \\
&\geq P\Big(\bigcap_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}\xi_i\Big)^{-1}((a+c,a+d))\Big) = \\
&= P\Big(\bigcap_{k=1}^{s}\Big(\frac{1}{k}\sum_{i=1}^{k}\xi_i-a\Big)^{-1}((c,d))\Big).
\end{aligned}$$

Proof of Theorem. Evidently $E(x_n) = E(\xi_n)$, $\sigma^2(x_n) = \sigma^2(\xi_n)$. Since $(\xi_n)_n$ are independent we obtain

$$\frac{1}{n}\sum_{i=1}^{n}\xi_i - \frac{1}{n}\sum_{i=1}^{n}E(\xi_i) \to 0$$

P-almost everywhere. Of course, by Lemma 5

$$\begin{aligned}
1 &\geq m\Big(\bigwedge_{p=1}^{\infty}\bigvee_{n=1}^{\infty}\bigwedge_{k=n}^{\infty}\Big(\frac{1}{k}\sum_{i=1}^{k}x_i-\frac{1}{k}\sum_{i=1}^{k}a_i\Big)\Big((-\frac{1}{p},\frac{1}{p})\Big)\Big) = \\
&= \lim_{p\to\infty}\lim_{n\to\infty}\lim_{j\to\infty} m\Big(\bigwedge_{k=n}^{n+j}\Big(\frac{1}{k}\sum_{i=1}^{k}x_i-\frac{1}{k}\sum_{i=1}^{k}a_i\Big)\Big((-\frac{1}{p},\frac{1}{p})\Big)\Big) \geq \\
&\geq P\Big(\bigcap_{p=1}^{\infty},\bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty}\Big(\frac{1}{k}\sum_{i=1}^{k}\xi_i-\frac{1}{k}\sum_{i=1}^{k}a_i\Big)^{-1}\Big((-\frac{1}{p},\frac{1}{p})\Big)\Big) = 1.
\end{aligned}$$

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