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Miloslav Duchoň
Beloslav Riečan

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ON THE KURZWEIL–STIELTJES INTEGRAL IN ORDERED SPACES

Miloslav Duchoň and Beloslav Riečan

Abstract. The paper is in some sense an extension of some results of C. D e b i è v e and M. D u c h o ň in the following three directions. Firstly lattice ordered groups are considered instead of Riesz spaces. Second, instead of operator valued functions a biadditive operator is considered. Third, instead of Riemann–Stieltjes theory the Henstock–Kurzweil construction of the integral can be used, so that a larger class of functions can be taken.

1. Preliminaries

An Abelian lattice ordered group is an Abelian group $(G,+)$ and a lattice such that $a \leqq b$ implies $a+c \leqq b+c$ for $a,b,c \in G$. We shall consider three l-groups X,Y,Z together with a biadditive, positive operator

$$b\colon X \times Y \to Z\,,$$

i.e., a function such that

$$\begin{aligned} b(x_1+x_2,y) &= b(x_1,y)+b(x_2,y)\,,\\ b(x,y_1+y_2) &= b(x,y_1)+b(x,y_2)\,,\\ x \geq 0, y \geq 0 \implies & \ b(x\cdot y) \geq 0\,. \end{aligned}$$

We shall write $x \cdot y$ instead of $b(x,y)$. A very inportant example of l-groups X,Y,Z and of a biadditive operator b is the following.

Let X,Z be Riesz spaces, $Y = L(X,Z)$ be the space of linear mappings from X to Z. Then $b\colon X \times Y \to Z$ defined by

$$b(x,y) = y(x), \quad x \in X, y \in Y\,,$$

satisfies all assumptions stated above.

An l-group G is called complete (σ-complete), if every bounded set (every countable bounded set) has the supremum.

A σ-complete l-group G will be called weakly σ-distributive, if for every bounded double sequence $(a_{ij})_{i,j}$ such that $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1,2,\dots$) it is

$$\bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i\varphi(i)} = 0\,.$$

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For every $a \in G$ put $a^+ = a \vee 0$, $a^- = (-a) \vee 0$, $|a| = a^+ + a^- = a \vee (-a)$. We say that $x_n \xrightarrow{o} x$, if there is $(a_n)_n$ such that $a_n \downarrow 0$ and $|x_n - x| \leqq a_n$.

We shall say that $x_n \xrightarrow{D} x$, if there exists a bounded double sequence $(a_{ij})_{i,j}$ with $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$) such that for every $\varphi\colon N \to N$ there is $n_o \in N$ such that

$$|x_n - x| \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for every $n \geq n_0$.

Proposition 1. *If $x_n \xrightarrow{o} x$, then $x_n \xrightarrow{D} x$. If G is moreover weakly σ-distributive, then $x_n \xrightarrow{D} x$ implies $x_n \xrightarrow{o} x$.*

Proof. Let $x_n \xrightarrow{o} x$, i.e. there exists $a_n \downarrow 0$ such that $|x_n - x| \leqq a_n$. Put $a_{ij} = a_j$ for every i. Let $\varphi\colon N \to N$. If we put $n_0 = \min\{\varphi(n);\ n \in N\}$, then for every $n \geq n_0$

$$|x_n - x| \leqq a_n \leqq a_{n_0} \leqq \bigvee_{n=1}^{\infty} a_{\varphi(n)} = \bigvee_{i=1}^{\infty} a_{i\varphi(i)}\,,$$

hence $x_n \xrightarrow{D} x$.

Let G be weakly σ-distributive and $x_n \xrightarrow{D} x$. Put $a_n = \bigvee_{i=n}^{\infty} |x_i - x|$. Then $|x_n - x| \leqq a_n$ for every n. By the definition to every $\varphi\colon N \to N$ there exists $n_o \in N$ such that

$$|x_n - x| \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for every $n \geq n_0$. It follows

$$a_{n_0} = \bigvee_{n=n_0}^{\infty} |x_n - x| \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}\,.$$

We want to prove that $a_n \downarrow 0$. But

$$\bigwedge_{n=1}^{\infty} a_n \leqq a_{n_0} \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}\,.$$

Since

$$\bigwedge_{n=1}^{\infty} a_n \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for every $\varphi \in N^N$ and G is weakly σ-distributive,

$$0 \leqq \bigwedge_{n=1}^{\infty} a_n \leqq \bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i\varphi(i)} = 0\,.$$

Double sequences $(a_{ij})_{i,j}$ can play the role of $\varepsilon > 0$ in the real-valued case. Namely, if $a_{ij} \in R$ and $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$), then to every $\varepsilon > 0$ and every i there exists $\varphi(i)$ such that $a_{i\varphi(i)} < \varepsilon$. Of course, also $\bigvee_{i=1}^{\infty} a_{i\varphi(i)} \leqq \varepsilon$. (See [Fremlin].)

Let $T = \langle a, b\rangle \subset R$ and $f\colon T \to X$, $g\colon T \to Y$. By a decomposition of T a finite family $D = \{(E_1, s_1), \dots, (E_n, s_n)\}$ will be understood, when E_i are closed non-overlapping intervals such that $\bigcup_{i=1}^{n} E_i = T$ and $s_i \in E_i$ $(i = 1, \dots, n)$. If we denote $E_i = \langle t_{i-1}, t_i\rangle$ $(i = 1, \dots, n)$, then we can define the integral sum

$$S_D^1(f, g) = \sum_{i=1}^{n} \big(f(t_i) - f(t_{i-1})\big) \cdot g(s_i)$$

of the function g with respect to f and the decomposition D, and the integral sum

$$S_D^2(f, g) = \sum_{i=1}^{n} f(s_i) \cdot \big(g(t_i) - g(t_{i-1})\big)$$

of the function f with respect to g and the decomposition D.

An integral can be defined as a limit of integral sums. We shall consider two possibilities.

2. Riemann-Stieltjes and Kurzweil–Stieltjes integral

If $D = \{(E_1, s_1) \dots, (E_n, s_n)\}$ is a decomposition of T, then we denote $p(D) = \max\big(\lambda(E_1), \dots, \lambda(E_n)\big)$, where λ is the Lebesgue measure.

Definition 1. We shall say that there exists the Riemann–Stieltjes integral of the function f with respect to g, if there exists $c \in Z$ and a bounded double sequence $(a_{ij})_{i,j}$ $a_{ij} \downarrow 0$ $(j \to \infty,\ i = 1, 2 \dots)$ such that to every $\varphi\colon N \to N$ there exists $\delta > 0$ such that

$$\big|S_D^2(f, g) - c\big| < \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for every D with $p(D) < \delta$.

If Z is weakly σ-distributive, then the element c is determined uniquely. Namely, if c_1, c_2 are two such elements, then

$$|c_1 - c_2| \leqq \big|c_1 - S_D^2(f, g)\big| + \big|S_D^2(f, g) - c_2\big| < \bigvee_{i=1}^{\infty} a_{i\varphi(i)} + \bigvee_{i=1}^{\infty} a_{i\psi(i)}\,,$$

for every $\varphi\colon N \to N$ and $\psi\colon N \to N$, hence

$$0 \leqq |c_1 - c_2| \leqq \bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i\varphi(i)} + \bigwedge_{\psi \in N^N} \bigvee_{i=1}^{\infty} a_{i\psi(i)} = 0\,.$$

Let us consider now an arbitrary function $\delta\colon T \to (0, \infty)$ and denote by $\mathcal{A}(\delta)$ the set of all decompositions D such that

$$E_i \subset \big(s_i - \delta(s_i), s_i + \delta(s_i)\big) \quad i = 1, \dots, n\,.$$

While in the Riemann construction δ is a constant function, in the Henstock–Kurzweil case δ can be arbitrary.

Definition 2. We shall say that there exists the Kurzweil–Stieltjes integral of the function f with respect to g, if there exists $c \in Z$ and a bounded double sequence $(a_{ij})_{i,j}$ $a_{ij} \downarrow 0$ $(j \to \infty,\ i = 1, 2, \dots)$ such that to every $\varphi\colon N \to N$ there exists $\delta\colon T \to (0, \infty)$ such that

$$\left|S_D^2(f, g) - c\right| < \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for every $D \in \mathcal{A}(\delta)$.

It is easy to see that in weakly σ-distributive l-groups Z the element c is determined uniquely. In the both cases (Riemann as well as Kurzweil) we shall denote it by the same sign $\int_T f\,dg$. Namely, the following proposition can be easily proved.

Proposition 2. *If f is integrable with respect to g in the Riemann sense, then it is integrable in the Kurzweil sense and both integrals coincide.*

Theorem 1. *If either integral exists (in the Riemann–Stieltjes sense, or Kurzweil–Stieltjes sense, respectively), then both exist (in the corresponding sense) and*

$$\int_T f(s)dg(s) = f(b) \cdot g(b) - f(a), g(a) - \int_T g(s)df(s)\,.$$

Proof.

$$\sum_{j=1}^{n} f(t_j)\left[g(s_j) - g(s_{j-1})\right] =$$

$$= f(b)g(b) - f(a)g(a) - \sum_{j=0}^{n} g(s_j)\left[f(t_{j+1}) - f(t_j)\right]$$

Proposition 3. *Let Z be complete. Then a function $f\colon T \to X$ is integrable with respect to $g\colon T \to Y$, if and only if the following condition is satisfied:*

There exists a bounded sequence $(b_{ij})_{i,j}$ of elements of Z such that $b_{ij} \downarrow 0$ $(j \to \infty,\ i = 1, 2, \dots)$ and for every $\varphi\colon N \to N$ there is $\delta\colon T \to (0, \infty)$ with

$$\left|S_{D_1}(f, g) - S_{D_2}(f, g)\right| < \bigvee_i b_{i\varphi(i)}$$

for every $D_1, D_2 \in A(\delta)$.

Proof. Evidently the condition is necessary. Now we prove that it is sufficient. To every $\varphi\colon N \to N$ there is $\delta(\varphi)$ with the property stated above. Put

$$K = \left\{\delta\colon T \to (0, \infty);\ \exists\, \varphi\colon N \to N,\ \delta = \delta(\varphi)\right\}.$$

Then for $\delta \in K$ the set

$$\left\{S_D(f, g);\ D \in A(\delta)\right\}$$

is bounded. Since Z is complete, there exist

$$a_\delta = \bigwedge_{D \in A(\delta)} S_D(f,g), \; b_\delta = \bigvee_{D \in A(\delta)} S_D(f,g).$$

If $\delta_1, \delta_2 \in K$, then $\delta = \min(\delta_1, \delta_2) \leqq \delta_1, \delta_2$, so $A(\delta) \subset A(\delta_1) \cap A(\delta_2)$, hence $\{S_D(f,g);\, D \in A(\delta)\}$ is bounded, too, and

$$\begin{aligned} a_{\delta_1} = \bigwedge_{D \in A(\delta_1)} S_D(f,g) \leqq \bigwedge_{D \in A(\delta)} S_D(f,g) \leqq \\ \leqq \bigvee_{D \in A(\delta)} S_D(f,g) \leqq \bigvee_{D \in A(\delta_2)} S_D(f,g) = b_{\delta_2}. \end{aligned}$$

Therefore

$$\bigvee_{\delta \in K} a_\delta \leqq \bigwedge_{\delta \in K} b_\delta ,$$

hence there exists $c \in Z$ such that

$$a_\delta \leqq c \leqq b_\delta$$

for all $\delta \in K$. Now let $\varphi\colon N \to N$. Then there is $\delta(\varphi)\colon T \to (0,\infty)$ such that

$$S_{D_1}(f,g) \leqq S_{D_2}(f,g) + \bigvee_i b_{i\varphi(i)}$$

for all D_1, $D_2 \in A\bigl(\delta(\varphi)\bigr)$. Fix D_2. Then

$$b_{\delta(\varphi)} \leqq S_{D_2}(f,g) + \bigvee_i b_{i\varphi(i)}.$$

Since the inequality holds for every $D_2 \in A\bigl(\delta(\varphi)\bigr)$, we have

$$b_{\delta(\varphi)} \leqq a_{\delta(\varphi)} + \bigvee_i b_{i\varphi(i)}.$$

By the weak σ-distributivity of Z we obtain $\bigwedge_\varphi \bigvee_i b_{i\varphi(i)} = 0$ and so

$$\bigwedge_\varphi b_{\delta(\varphi)} - \bigvee_\varphi a_{\delta(\varphi)} \leqq 0,$$

hence

$$c = \bigwedge_\varphi b_{\delta(\varphi)} = \bigvee_\varphi a_{\delta(\varphi)}.$$

Then for every $D \in A\bigl(\delta(\varphi)\bigr)$

$$S_D(f,g) - c \leqq b_{\delta(\varphi)} - a_{\delta(\varphi)} \leqq \bigvee_i b_{i\varphi(i)}$$

and similarly

$$c - S_D(f,g) \leqq b_{\delta(\varphi)} - a_{\delta(\varphi)} \leqq \bigvee_i b_{i\varphi(i)}.$$

so that

$$\bigl|S_D(f,g) - c\bigr| \leqq \bigvee_i b_{i\varphi(i)}$$

and the proof is complete.

3. Existence theorems

We shall say that a function $f\colon T \to X$ is continuous in a point t_0 with respect to $(a_{ij})_{i,j}$ ($a_{ij} \downarrow 0$, $j \to \infty$, $i = 1, 2, \dots$), if to every $\varphi\colon N \to N$ there exists $\delta > 0$ such that $\left|f(t) - f(t_0)\right| < \bigvee_i a_{i\varphi(i)}$ whenever $|t - t_0| < \delta$. It is said to be continuous uniformly with respect to $(a_{ij})_{i,j}$ if to every $\varphi\colon N \to N$ there exists $\delta > 0$ such that $\left|f(t_1) - f(t_2)\right| < \bigvee_i a_{i\varphi(i)}$ whenever $|t_1 - t_2| < \delta$.

Let $g\colon T \to Y$, and let $(a_{ij})_{i,j} \subset X$ be a bounded sequence such that $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$). We shall say that g is of D-bounded semi-variation with respect to (a_{ij}) if the following condition is satisfied:

There exists a bounded sequence $(b_{ij})_{i,j} \subset Z$ with $b_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$) such that for every decomposition $(t_0, t_1, \dots, t_n)$ of $< a, b >$ we have for any $\varphi\colon N \to N$

$$\Big|\sum_{i=1}^{n} x_i \cdot \big(g(t_i) - g(t_{i-1})\big)\Big| \leqq \bigvee_{j=1}^{\infty} b_{j\varphi(j)}$$

whenever $x_i \in X$, $|x_i| \leqq \bigvee_{j=1}^{\infty} a_{j\varphi(j)}$ $(i = 1, \dots n)$.

A function $g\colon T \to Y$ is said to be of (o)-bounded semi-variation if there exists an order bounded additive operator $U\colon X \to Z$ such that for any partition $(t_0, t_1, \dots, t_n)$ of T the following inequality holds

$$\big|\sum x_i \cdot \big[g(t_{i+1}) - g(t_i)\big]\big| \leqq U(x)$$

whenever $x_i, x \in X$ are such that $|x_i| \leqq x$.

A simple example of a function of D–bounded semi-variation is every function of (o)-bounded semi-variation. Of course for a_{ij} we have $b_{ij} = U(a_{ij})$.

Theorem 2. *Let Z be a complete weakly σ-distributive l-group. Let $(a_{ij})_{i,j} \subset X$ be a bounded sequence, $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$). Let $f\colon T \to X$ be continuous in every point of T with respect the same $(a_{ij})_{i,j}$. Let $g\colon T \to Y$ be of D-bounded semi-variation with respect to $(a_{ij})_{i,j}$. Then there exists the Kurzweil–Stieltjes integral $\int_T f\,dg$.*

Proof. Since f is continuous in $t_0 \in T$, to every $\varphi\colon N \to N$ there exists $d(t_0) > 0$ such that $|t - t_0| < d(t_0)$ implies $\left|f(t) - f(t_0)\right| < \bigvee_i a_{i\varphi(i)}$. Define $\delta\colon T \to (0, \infty)$ by the formula $\delta(t) = d(t)$. Let $D_1, D_2 \in \mathcal{A}(\delta)$, D_2 being a refinement of D_1.

Let $D_1 = (t_0, t_1, \dots, t_k)$ with intermediate points $s_i (1 \leqq i \leqq k)$ and $D_{2,i}$ the subpartition of $[t_{i-1}, t_i]$ given by D_2 with points $t_{i,j}$ and intermediate points $s_{i,j} (1 \leqq j \leqq k_i)$.

Then we have

$$S_{D_1} - S_{D_2} = \sum_{i=1}^{k} \Big[\big(g(t_i) - g(t_{i-1})\big)\big(f(s_i)\big) - \sum_{j=1}^{k_i} \big(g(t_{i,j}) - g(t_{i,j-1})\big)\big(f(s_{i,j})\big)\Big]$$

As $t_{i,0} = t_{i-1}$ and $t_{i,k_i} = t_i$, we have

$$\begin{aligned} \left|S_{D_1} - S_{D_2}\right| &= \Big|\sum_{i=1}^{k} \sum_{j=1}^{k_i} \big(f(s_i) - f(s_{i,j})\big) \cdot \big(g(t_{i,j}) - g(t_{i,j-1})\big)\Big| \leqq \\ &\leqq \bigvee_{j=1}^{\infty} b_{j\varphi(j)}\,, \end{aligned}$$

since $|s_i - s_{ij}| \leqq \delta(s_i)$, hence $\left|f(s_i) - f(s_{i,j})\right| < \bigvee_{i=1}^{\infty} u_{i\varphi i})$.
Since Z is complete, f is integrable.

Theorem 3. *Let Z be a complete weakly σ-distributive ℓ-group. Let $(a_{ij})_{i,j} \subset X$ be a bounded sequence, $a_{ij} \downarrow 0$ ($j \to \infty$, $i = 1, 2, \dots$). Let $f\colon T \to X$ be uniformly continuous with respect to $(a_{ij})_{i,j}$. Let $g\colon T \to Y$ be of D-bounded semi-variation with respect to $(a_{ij})_{i,j}$. Then there exists the Riemann–Stieltjes integral $\int_T f\,dg$.*

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