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ON HERGLOTZ THEOREM IN PARTIALLY ORDERED VECTOR SPACES

Miloslav Duchoň — Peter Maličký

Abstract. Herglotz theorem is generalized for a monotone σ-complete partially ordered vector space.

Introduction

The classical Herglotz theorem states that a sequence $(a_k)_{k=-\infty}^{\infty}$ of complex numbers is the sequence of the Fourier-Stieltjes coefficients of a nondecreasing function g defined on the interval $[0, 2\pi]$ if and only if the sequence $(a_k)_{k=-\infty}^{\infty}$ is positive defined. Recall that $a_k = \int_0^{2\pi} e^{-ikt} dg(t)$ is said to be the kth Fourier-Stieltjes coefficient of a function g (of bounded variation defined on the interval $[0, 2\pi]$). A sequence $(a_k)_{k=-\infty}^{\infty}$ of complex numbers is said to be positive define if and only if $0 \le \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_k \, a_{k-j}$ for any finite sequence of complex numbers $(c_j)_{j=-n}^{n}$. A generalization of Herglotz theorem for vector lattices was given in the paper [1]. The present paper generalizes this theorem for sequence $(a_k)_{k=-\infty}^{\infty}$, elements of which belong to the complexification of a monotone σ-copmplete partially ordered vector space.

1. Monotone σ-complete partially ordered vector spaces

In the whole paper the symbol Y denotes a real monotone σ-complete partially ordered vector space and Z the complexification of Y. A real partially ordered vector space is said to be monotone σ-complete if any increasing sequence $(a_n)_{n=1}^{\infty}$, which is bounded above, has a least upper bound $\bigvee_{n=1}^{\infty} a_n$ [3].

Proposition 1.1. *Let Y be a real monotone σ-complete partially ordered vector space.*
Then

(i) *Any decreasing sequence $(a_n)_{n=1}^{\infty}$, which is bounded below, has a greatest lower bound $\bigwedge_{n=1}^{\infty} a_n$ and $\bigvee_{n=1}^{\infty}(-a_n) = -\bigwedge_{n=1}^{\infty} a_n$.*

(ii) *Y is Archimedean, i.e., for any positive $y \in Y$ the sequence $(ny)_{n=1}^{\infty}$ is unbounded above and $\bigwedge_{n=1}^{\infty} \frac{1}{n} y = 0$.*

(iii) *For any positive $y \in Y$ and any decreasing sequence $(\alpha_n)_{n=1}^{\infty}$ of reals with $\lim_{n\to\infty} \alpha_n = 0$ we have $\bigwedge_{n=1}^{\infty} a_n y = 0$.*

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(iv) *For any increasing (decreasing) sequences* $(a_n)_{n=1}^{\infty}$ *and* $(b_n)_{n=1}^{\infty}$*, which are bounded above (below), we have* $\bigvee_{n=1}^{\infty}(a_n + b_n) = \bigvee_{n=1}^{\infty} a_n + \bigvee_{n=1}^{\infty} b_n$, $\left(\bigwedge_{n=1}^{\infty}(a_n + b_n) = \bigwedge_{n=1}^{\infty} a_n + \bigwedge_{n=1}^{\infty} b_n\right)$.

2. Riemann-Stieltjes integral

Take a continuous real function f, nondecreasing Y-valued function g defined on the interval $[a,b]$ and a partition

$$\Pi = \{a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b\}$$

of the interval $[a,b]$. Put

$$s(f,g,\Pi) = \sum_{j=1}^{n} m_j\big(g(x_j) - g(x_{j-1})\big) \quad \text{and} \quad S(f,g,\Pi) = \sum_{j=1}^{n} M_j\big(g(x_j) - g(x_{j-1})\big)\,,$$

where $m_j = \min_{x\in[x_{j-1},x_j]} f(x)$ and $M_j = \max_{x\in[x_{j-1},x_j]} f(x)$.

Proposition 2.1. *There is a unique element* $I \in Y$ *such that*

$$s(f,g,\Pi') \le I \le S(f,g,\Pi)$$

for any partitions Π' *and* Π *of the interval* $[a,b]$.

Proof. The inequality $s(f,g,\Pi') \le S(f,g,\Pi)$ is obvious. For any natural n and $0 \le i \le 2^n$ put $x_{n,i} = a + i(b-a)2^{-n}$ and $\Pi_n = \{x_{n,i}\}_{i=0}^{2^n}$. Obviously,

$$\bigvee_{n=1}^{\infty} s(f,g,\Pi_n) = \bigwedge_{n=1}^{\infty} S(f,g,\Pi_n)\,.$$

Now, the element

$$I = \bigvee_{n=1}^{\infty} s(f,g,\Pi_n) = \bigwedge_{n=1}^{\infty} S(f,g,\Pi_n)$$

has the required properties.

The element I will be denoted by $\int_a^b f(x)dg(x)$ and is said to be the Riemann-Stieltjes integral of f with respect to g on the interval $[a,b]$.

3. Positive linear maps

The symbol $\mathbf{T}$ denotes the quotient space $\mathbf{R}/2\pi\mathbf{Z}$, which may be identified with the unit circle $\{\zeta\colon |\zeta| = 1\}$ of the complex plane. The set of all real continuous functions on the space $\mathbf{T}$ will be denoted by $C(\mathbf{T})$ and the set of all real (complex) continuous 2π-periodic functions on the real line by $C_{2\pi}(\mathbf{R})$ (by $X_{2\pi}(\mathbf{R},\mathbf{C})$). Clearly, the formula $\psi(e^{it}) = f(t)$ defines one-to-one correspondence between the sets $C(\mathbf{T})$ and $C_{2\pi}(\mathbf{R})$. A linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ is said to be positive if $\Phi(f) \ge 0$ for $f \ge 0$. Obviously, any nondecreasing function $g\colon [0,2\pi] \to Y$ defines a positive linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ by the formula $\Phi(f) = \int_0^{2\pi} f(x)dg(x)$. We shall show that it is sufficient to consider only functions g, which are left o-continuous at any $x \in (0, 2\pi\rangle$, i.e., $\bigvee_{n=1}^{\infty} g(x_n) = g(x)$ for any increasing sequence $(x_n)_{n=1}^{\infty}$ converging to $x \in (0, 2\pi\rangle$.

Recall that Y-valued Baire measure m on a compact $\mathbf{X}$ is a Y-valued function defined on Baire subsets of $\mathbf{X}$ such that $m(\emptyset) = 0$, $m(A) \geq 0$ and $m\left(\bigcup_{j=1}^{\infty} A_j\right) = \bigvee_{n=1}^{\infty} \sum_{jk=1}^{n} m(A_j)$ for any sequence $(A_j)_{j=1}^{\infty}$ of pairwise disjoint Baire subsets of $\mathbf{X}$. For more details see [3].

Theorem 3.1. *The formulas*

$$\Phi(f) = \int_0^{2\pi} f(x)dg(x)$$

$$\Phi(f) = \int_{[0,2\pi]} f(t)dm(t) \quad \textit{and}$$

$$g(t) = m\big([0,t)\big) \quad \textit{for} \quad t \in [0, 2\pi]$$

define one-to-one correspondence between the sets of all positive linear maps $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$, Y-valued Baire measures m on the interval $[0, 2\pi)$ and nondecreasing functions $g\colon [0, 2\pi] \to Y$, which are left o-continuous at any $x \in (0, 2\pi]$ and $g(0) = 0$.

Proof. Let $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ be a positive linear map. For $f \in C_{2\pi}(\mathbf{R})$ put $\psi(e^{it}) = f(t)$ and $\Psi(\psi) = \Phi(f)$. Obviously, we have a positive linear map $\Psi : C(\mathbf{T}) \to Y$. By [3, p. 687] for such a map there is a unique Y-valued Baire measure μ on $\mathbf{T}$ such that $\Psi(\psi) = \int_{\mathbf{T}} \psi(\zeta)d\mu(\zeta)$. For a Baire subset $A \subset [0, 2\pi)$ put $m(A) = \mu\big(\{e^{it} \colon t \in A\}\big)$. Then $\Phi(f) = \int_{[0,2\pi]} f(t)dm(t)$ for any $f \in C_{2\pi}(\mathbf{R})$. Now, put $g(t) = m\big([0,t)\big)$, for $t \in [0, 2\pi]$. Then $g(0) = 0$ and the function g is left o-continuous at any $t \in (0, 2\pi\rangle$. Obviously,

$$s(f, g, \Pi') \leq \int_{[2,\pi)} f(t)dm(t) \leq S(f, g, \Pi)$$

for any partitions Π' and Π of the interval $[0, 2\pi]$. It means $\int_{[0,2\pi)} f(t)dm(t) = \int_0^{2\pi} f(t)dg(t) = \Phi(f)$.

Take a function $h\colon [a, b] \to Y$ of the form $h(t) = \sum_{j=1}^{n} \varphi_j(t)y_j$, where $\varphi_j\colon [a, b] \to \mathbf{R}$ are continuous and $y_j \in Y$. Then the integral

$$\int_a^b h(t)dt = \sum_{j=1}^{n} \left(\int_a^b \varphi_j(t)dt\right) y_j$$

is correctly defined and $\int_a^b h(t)dt \geq 0$ whenever $h(t) \geq 0$ for all $t \in [a, b]$.

Clearly, any linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ may be extended onto a linear map $\Phi\colon C_{2\pi}(\mathbf{R}, \mathbf{C}) \to Z$ by the formula $\Phi(f + ig) = \Phi(f) + i\Phi(g)$. For any integer j put $\chi_j(t) = e^{ijt}$. For a linear map $\Phi\colon C_{2\pi}(\mathbf{R}, \mathbf{C}) \to Z$ the element $z_j = \Phi(\chi_j)$ is said to be the jth Fourier coefficient of Φ. For a function $f \in C_{2\pi}(\mathbf{R}, \mathbf{C})$ put $f_s(t) = f(s-t)$ and define a function $\varphi(s) = \Phi(f_s)$. The function φ is a 2π-periodic Y-valued function. It will be denoted by $\Phi * f$ and is said to be a convolution of Φ and f. Obviously, $\Phi * f \geq 0$ whenever $f \geq 0$ and Φ is positive.

4. Fourier coefficients of a positive linear map

Now, we give a characterization of a positive linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ in terms of its Fourier coefficients. The main result, Theorem 4.7, uses only the following easy lemmas and is independent of Theorem 3.1.

Definition 4.1. A sequence $(z_j)_{j=-\infty}^{\infty}$ of elements of Z is said to be positive defined if $0 \le \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_k z_{k-j}$ for any finite sequence $(c_j)_{j=-n}^{n}$ of complex numbers.

Lemma 4.1. *Let $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$ be a positive linear map. Then the sequence $(z_j)_{j=-\infty}^{\infty}$ of the Fourier coefficients of Φ is positive defined and*

$$\sigma_n(Y,t) = \sum_{j=-N}^{N} \Big(1 - \frac{|j|}{N+1}\Big) z_j e^{ijt}$$

are nonnegative for all real t and natural N.

Proof. Let $(c_j)_{j=-n}^{n}$ be a sequence of complex numbers. Put

$$f(t) = \sum_{j=-n}^{n} c_j e^{ijt} \quad \text{and} \quad g(t) = \big|f(t)\big|^2 = \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_k e^{i(j-k)t}.$$

Then

$$0 \le \Phi(g) = \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_k z_{k-j}.$$

So, the sequence $(z_j)_{j=-\infty}^{\infty}$ is positive defined.

Clearly, the Ceasro sum $\sigma_N(Y,t)$ is the convolution of Φ and the Fejer kernel

$$F_N(t) = \sum_{j=-N}^{N} \Big(1 - \frac{|j|}{N+1}\Big) e^{ijt} = \frac{\sin^2\big(\frac{N+1}{2}t\big)}{(N+1)\sin^2\big(\frac{1}{2}t\big)}.$$

Therefore $\sigma_n(Y,t) \ge 0$.

Lemma 4.2. *Let a sequence $(z_j)_{j=-\infty}^{\infty}$ of elements of Z be positive defined. Then*

(i) *$\sigma_n(Z,t) \ge 0$ for all real t and natural N.*

(ii) *$0 \le z_0 \in \mathbf{Y}$, $z_{-n} = \bar{z}_n, \pm \operatorname{Re}(z_n) \le z_0$ and $\pm \operatorname{Im}(z_n) \le z_0$ for any integer n.*

Proof. (i). Put $c_j = e^{-ijt}$ for $0 \le j \le N$. Then

$$0 \le \sum_{j=0}^{N} \sum_{k=0}^{N} c_j \bar{c}_k z_{k-j} = \sum_{j=-N}^{N} (N+1-|j|) z_j e^{ijt} = (N+1)\sigma_N(t).$$

(ii). Put $c_0 = 1$. Then

$$0 \le \sum_{j=0}^{0} \sum_{k=0}^{0} c_j \bar{c}_k z_{k-j} = c_0 z_0 = z_0,$$

i.e., $0 \le z_0 \in Y$. Put $c_0 = c_n = 1$ and $c_j = 0$ for $0 < j < n$. Then

$$0 \le \sum_{j=0}^{n} \sum_{k=0}^{n} c_j \bar{c}_k z_{k-j} = 2z_0 + z_n + z_{-n} \,.$$

It means that $\operatorname{Im}(z_n + z_{-n}) = 0$ and $-\operatorname{Re}(z_n + z_{-n}) \le 2z_0$. Replacing c_n by -1, we obtain $\operatorname{Re}(z_n + z_{-n}) \le 2z_0$. Put $c_n = \pm i$, then we have $\pm \operatorname{Im}(z_n) \le z_0$ and $\operatorname{Re}(z_n - z_{-n}) = 0$. Therefore $z_{-n} = \bar{z}_n, \pm \operatorname{Re}(z_n) \le z_0$ and $\pm \operatorname{Im}(z_n) \le z_0$.

Lemma 4.3. *Let $(z_j)_{j=-\infty}^{\infty}$ be a sequence of elements of Z such that $z_j = 0$ for $|j| > m$. If $\sum_{j=-m}^{m} z_j e^{ijt} \ge 0$ for all t, then the sequence $(z_j)_{j=-\infty}^{\infty}$ is positive defined.*

Proof. Put

$$f(t) = \sum_{j=-n}^{n} c_j e^{ijt} \,,\; g(t) = \bigl|f(t)\bigr|^2 = \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_k e^{i(j-k)t}$$

and $h(t) = \sum_{j=-n}^{n} z_j e^{ijt}$. Then

$$0 \le \frac{1}{2\pi} \int_0^{2\pi} g(t)h(t)dt = \sum_{j=-n}^{n} \sum_{k=-n}^{n} c_j \bar{c}_j z_{k-j} \,.$$

Lemma 4.4. *Let a sequence $(z_j)_{j=-\infty}^{\infty}$ of elements of Z be such that $\sigma_n(Z,t) \ge 0$ for all real t and natural N. Then $\pm \operatorname{Re}(z_j) \le 2z_0$ and $\pm \operatorname{Im}(z_j) \le 2z_0$ for any integer j.*

Proof. For any integer $N \ge 0$ put

$$\zeta_j = \Bigl(1 - \frac{|j|}{N+_1}\Bigr) z_j \quad \text{for} \quad -N-1 < j < N+1 \quad \text{and} \quad \zeta_j = 0$$

otherwise. Since $\sigma_N(Z,t) \ge 0$ for all t, the sequence $(\zeta_j)_{j=-\infty}^{\infty}$ is positive defined by Lemma 4.3. Now, Lemma 4.2 implies $\pm \operatorname{Re}(\zeta_j) \le \zeta_0$ and $\pm \operatorname{Im}(\zeta_j) \le \zeta_0$ for $-N-1 < j < N+1$. It means

$$\pm\Bigl(1 - \frac{|j|}{N+1}\Bigr) \operatorname{Re}(z_j) \le z_0 \quad \text{and} \quad \pm\Bigl(1 - \frac{|j|}{N+1}\Bigr) \operatorname{Im}(z_j) \le z_0$$

for $-N-1 < j < N+1$. Now, for any integer j take $N = 2|j| - 1$. Then

$$\pm \operatorname{Re}(z_j) \le 2z_0 \quad \text{and} \quad \pm \operatorname{Im}(z_j) \le 2z_0$$

for any integer j.

Lemma 4.5. *For any function $f \in C_{2\pi}(\mathbf{R})$ there are decreasing and increasing sequences $(\varphi_n)_{n=1}^{\infty}$ and $(\psi_n)_{n=1}^{\infty}$ of trigonometric polynomials converging unifirmly to the function f.*

Proof. It is well known, see [2, Theorem 2.12], that the convolutions $\frac{1}{2\pi} f * F_N$ are trigonometric polynomials, which converge uniformly to the function f. So, for any natural n there is a trigonometric polynomial f_n such that $f_n - 2^{-n} < f < f_n + 2^{-n}$. Put $\varphi_n = f_n + 3 \cdot 2^{-n}$ and $\psi_n = f_n - 3 \cdot 2^{-n}$. Then

$$\begin{aligned}\varphi_{n+1} &= f_{n+1} + 3 \cdot 2^{-n-1} < f + 4 \cdot 2^{-n-1} < f_n + 2^{-n} + 4 \cdot 2^{-n-1} \\ &= f_n + 3 \cdot 2^{-n} = \varphi_n\end{aligned}$$

and

$$\begin{aligned}\psi_{n+1} &= f_{n+1} - 3 \cdot 2^{-n-1} > f - 4 \cdot 2^{-n-1} > f_n - 2^{-n} - 4 \cdot 2^{-n-1} \\ &= f_n - 3 \cdot 2^{-n} = \psi_n\,.\end{aligned}$$

So, we obtain the required sequences of trigonometric polynomials.

Theorem 4.6. *Let a sequence $(z_j)_{j=-\infty}^{\infty}$ of elements of Z be such that $\sigma_N(Z,t) \geq 0$ for all real t and natural N. Then there is a positive linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$, for which the sequence of the Fourier coefficients is the sequence $(z_j)_{j=-\infty}^{\infty}$.*

Proof. Take a trigonometric polynomial $f(t) = \sum_{j=-n}^{n} c_j e^{ijt}$ and put $\Phi(f) = \sum_{j=-n}^{n} c_j z_{-j}$. Linearity of Φ is obvious, we want to prove positivity of Φ. So, assume $f(t) \geq 0$. We have to prove that $\sum_{j=-n}^{n} c_j z_{-j} \geq 0$. Since $f(t) \geq 0$ and $\sigma_N(Z,t) \geq 0$ for all t, we have

$$0 \leq \frac{1}{2\pi} \int_0^{2\pi} f(t) \sigma_N(Z,t)\,dt = \sum_{j=-n}^{n} c_j \bigl(1 - \frac{|j|}{N+1}\bigr) z_{-j}\,,$$

whenever $n \leq 2N+1$. Denote

$$S = \sum_{j=-n}^{n} c_j z_{-j} \quad \text{and} \quad S_N = \sum_{j=-n}^{n} c_j \bigl(1 - \frac{|j|}{N+1}\bigr) z_{-j}\,.$$

Then

$$S \geq S - S_N = \sum_{j=-n}^{n} c_j \frac{|j|}{N+1} z_{-j} = \sum_{j=1}^{n} 2\,\mathrm{Re}\bigl(c_j \frac{|j|}{N+1} z_{-j}\bigr)\,,$$

because the elements c_j, c_{-j} and z_j, z_{-j} are conjugated. Let $c_j = a_j + ib_j$, where a_j and b_j are real, and $z_j = u_j + iv_j$, where $u_j, v_j \in Y$. Then

$$z_{-j} = u_j - iv_j \quad \text{and} \quad S \geq 2 \sum_{j=1}^{n} \frac{j}{N+1} (a_j u_j + b_j v_j)\,.$$

Lemma 4.4 implies $\pm u_j \le 2z_0$ and $\pm v_j \le 2z_0$. The inequalities $\pm a_j \le c_0$ and $\pm b_j \le c_0$ follow from Lemma 4.2. Therefore

$$S \ge -8\sum_{j=1}^{n} \frac{j}{N+1} c_0 z_0 = -\frac{4n(n+1)}{N+1} c_0 z_0 \,.$$

It means

$$S \ge \bigvee_{N=1}^{\infty} -\frac{4n(n+1)}{N+1} c_0 z_0 = 0 \,.$$

So, Φ is positive. Now, take an arbitrary $f \in C_{2\pi}(\mathbf{R})$. By Lemma 4.5, there exist nonicreasing and nondecreasing sequences $(\varphi_n)_{n=1}^{\infty}$ and $(\psi_n)_{n=1}^{\infty}$ of trigonometric polynomials converging uniformly to the function f. We have

$$\varphi_n \le \varphi_{n+1} \le f \le \psi_{m+1} \le \psi_m$$

for any integers m and n. Therefore

$$\Phi(\varphi_n) \le \Phi(\varphi_{n+1}) \le \Phi(\psi_{m+1}) \le \Phi(\psi_m) \,,$$

which implies $\Phi(\varphi_n) \le \Phi(\psi_m)$ for any integers m and n. It means

$$\bigvee_{n=1}^{\infty} \Phi(\varphi_n) \le \Phi(\psi_m) \quad \text{and} \quad \bigvee_{n=1}^{\infty} \Phi(\varphi_n) \le \bigwedge_{m=1}^{\infty} \Phi(\psi_m) \,.$$

Now, consider the sequence $(\psi_n - \varphi_n)_{n=1}^{\infty}$, which is nonincreasing and converges uniformly to zero. Denote $\alpha_n = \sup_{t\in[0,2\pi}\big(\psi_n(t) - \varphi_n(t)\big)$. Then

$$0 = \bigwedge_{n=1}^{\infty} \alpha_n \Phi(1) = \bigwedge_{n=1}^{\infty} \Phi(\alpha_n) \ge \bigwedge_{n=1}^{\infty} \Phi(\psi_n - \varphi_n) = \bigwedge_{n=1}^{\infty} \Phi(\psi_n) - \bigvee_{n=1}^{\infty} \Phi(\varphi_n) \,,$$

which means

$$\bigvee_{n=1}^{\infty} \Phi(\varphi_n) \ge \bigwedge_{n=1}^{\infty} \Phi(\psi_n) \,.$$

Therefore

$$\bigvee_{n=1}^{\infty} \Phi(\varphi_n) = \bigwedge_{n=1}^{\infty} \Phi(\psi_n) \,.$$

So, neither $\bigvee_{n=1}^{\infty} \Phi(\varphi_n)$ nor $\bigwedge_{n=1}^{\infty} \Phi(\psi_n)$ does not depend on the choice of nonincreasing and nondecreasing sequences $(\varphi_n)_{n=1}^{\infty}$ and $(\psi_n)_{n=1}^{\infty}$ of trigonometric polynomials converging uniformly to the function f. Put

$$\Phi(f) = \bigvee_{n=1}^{\infty} \Phi(\varphi_n) = \bigwedge_{n=1}^{\infty} \Phi(\psi_n) \,.$$

Then we have the required positive linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$.

Theorem 4.7. *For any sequence $(z_j)_{j=-\infty}^{\infty}$ of elements of Z the following properties are equivalent.*

- (i) *There is a positive linear map $\Phi\colon C_{2\pi}(\mathbf{R}) \to Y$, for which $(z_j)_{j=-\infty}^{\infty}$ is the sequence of the Fourier coefficients.*
- (ii) *The sequence $(z_j)_{j=-\infty}^{\infty}$ is positive defined.*
- (iii) *The Ceasro sums $\sigma_N(Z,t) = \sum_{j=-N}^{N} \left(1 - \frac{|j|}{N+1}\right) z_j e^{ijt}$ are nonnegative for all natural N.*

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