Mathematical Institute
Slovak Academy of Sciences, Bratislava

Preprint 18/1999

Beloslav Riečan
Jozefína Petrovičová

Kolmogorov-Sinai theorem for fuzzy T-dynamical systems

August 27, 1999

PREPRINT SERIES

x

KOLMOGOROV-SINAI THEOREM FOR FUZZY T-DYNAMICAL SYSTEMS

BELOSLAV RIEČAN AND JOZEFÍNA PETROVIČOVÁ

Abstract. Recently A. Ban ([1]) successfully introduced entropy of a fuzzy T-dynamical system with respect to a triangular norm T. By the help of the Klement representation of T-measures by Markov kernels we prove some variants of the Kolmogorov-Sinai theorem stating that the entropy of a dynamical system equals the entropy of any generator.

1. Introduction

First let us recall the definition of the Kolmogorov-Sinai invariant. The classical dynamical system is a quadrilup $(X, \mathbb{S}, P, U)$, where $(X, \mathbb{S}, P)$ is a probability space and $U : X \to X$ is a measure preserving map, i.e. $A \in \mathbb{S}$ implies $U^{-1}(A) \in \mathbb{S}$ and $P(U^{-1}(A)) = P(A)$.

The notion of the entropy of a dynamical system can solve in some cases the problem of isomorphism of dynamical systems. Namely, if two dynamical systems are isomorphic, they have the same entropy. Hence, if two dynamical systems have different entropies, they can not be isomorphic.

Let $D = \{A_1, \cdots, A_k\}$ be a measurable partition, hence $A_i \in \mathbb{S} \quad (i = 1, \cdots, k)$. Then the entropy $H(D)$ is defined by the Shannon formula

$$H(D) = \sum_{i=1}^{k} \varphi(P(A_i)),$$

where $\varphi(x) = -x \cdot \log x$ if $x > 0$, $\varphi(0) = 0$. The entropy $h(D, U)$ of a partition D with respect to the dynamical system is the limit

$$h(D, U) = \lim_{n\to\infty} \frac{1}{n} \cdot H\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right),$$

Key words and phrases. Entropy, t-norm, fuzzy T-measure, generator.
This research has been supported by grant VEGA 2/6087/99.

where $\bigvee_{i=0}^{n-1} U^{-i}(D)$ is the least common refinement of the partitions D, $U^{-1}(D)$, $\cdots$, $U^{-(n-1)}(D)$ and $U^{-i}(D) = \{U^{-i}(A_1), \cdots, U^{-i}(A_k)\}$. The entropy of a dynamical system $(X, \mathbb{S}, P, U)$ is the least upper bound

$$h(U) = \sup\{h(D, U); \quad U \text{ is measurable partition of } X\}.$$

It is clear that some computational formulas are useful. The Kolmogorov-Sinai theorem states that the entropy of the dynamical system equals the entropy of any partition being generator of the dynamical system, i.e. such a partition D that the σ-algebra generated by the union $\bigcup_{i=0}^{n-1} U^{-i}(D)$ equals the given σ-algebra $\mathbb{S}$.

Of course, if the entropies of two dynamical systems coincides, they do not distinguish possibly non isomorphic dynamical systems. Therefore it is reasonable to introduce a larger class of invariants. One of possibilities is a notion of an entropy defined by the help of a fuzzy partition instead of a partition. This idea has been realised e.g. in [8], [9], [3], a review can be found in [13].

A further step would be a construction of an entropy by the help of T-fuzzy partition instead of a fuzzy partition, while T is a triangular norm. It was realized successully in [1].

In Section 2 we list all necessary definitions and results from [1]. In Section 3 we formulate and prove some variants of the Kolmogorov-Sinai theorem concerning entropy and generators of dynamical systems.

2. Entropy of fuzzy T-dynamical systems

We start with a t-norm, i.e. a function $T : \langle 0, 1\rangle \times \langle 0, 1\rangle \to \langle 0, 1\rangle$ which is commutative, associative, monotone and such that $T(x, 1) = x$ for any x and the associated t - conorm S defined by the equality $S(x, y) = 1 - T(1 - x, 1 - y)$. It is assumed that the couple (T, S) satisfies the following WH property:

$$T(x, y) = 0 \Rightarrow S(xz, yz) = z \cdot S(x, y),\ T(xz, yz) = z \cdot T(x, y).$$

Recall that any Frank t-norm ([3]) and any Hamacher t-norm satisfy the WH property.

The crutial notion here is the notion of a T-fuzzy partition of a set X, i.e. a finite set $D = \{f_1, \cdots, f_k\}$ of fuzzy subsets of X satisfying the following two conditions:

(i) $\mathrm{S}_{i=1}^{n} f_i = 1_X$;

(ii) $(\mathrm{S}_{i \neq k} f_i)\ T\ f_k = 0_X$ for any k.

If $D = \{f_1, \cdots, f_k\}$, $B = \{g_1, \cdots, g_m\}$ are two T - fuzzy partitions, then

$$D \vee B = \{f_i \cdot g_j\,; \quad i = 1, \cdots, k\,, \quad j = 1, \cdots, m\,\}\,.$$

Here $f_i \cdot g_j$ is the usual product of two functions.

Instead of a dynamical system the following quadrilup (X, F, m, U) is considered:

1. X is a non-empty set.
2. $F \subset \langle 0, 1\rangle^X$ is a T σ-algebra, i.e.
   2.1. $1_X \in F$;
   2.2. $f, g \in F \Rightarrow f \cdot g \in F$, $\max(f - g, 0) \in F$;
   2.3. $f_n \in F$ $(n = 1, 2, \cdots) \Rightarrow S_{n=1}^{\infty} f_n \in F$.
3. $m : F \to \langle 0, 1\rangle$ has the following properties:
   3.1. $m(1_X) = 1$;
   3.2. $(f_n)_{n=1}^{\infty} \in F$, f_n disjoint $\Rightarrow m(S_{n=1}^{\infty} f_n) = \sum_{n=1}^{\infty} m(f_n)$.
4. $U : X \to X$ is such that
   4.1. $f \in F \Rightarrow f \circ U \in F$;
   4.2. $f \in F \Rightarrow m(f \circ U) \in m(f)$.

If $D = \{f_1, \cdots, f_k\}$ is a T - fuzzy partition, then its entropy is defined by the formula

$$H(D) = \sum_{i=1}^{k} \varphi(f_i)\,.$$

Similarly as in the classical case there can be proved the existence of the limit

$$h(D, U) = \lim_{n \to \infty} \frac{1}{n} \cdot H\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right)\,.$$

where $U^{-i}(D) = \{f_1 \circ U^i, \cdots, f_k \circ U^i\}$. The entropy of the T-fuzzy dynamical system (X, F, m, U) is

$$h(U) = \sup\{h(D, U); \quad D \text{ is a } T \text{ - fuzzy partition}\}.$$

We shall use only the following assertion from [1] (Section 5, see also [13]):

Proposition 1. *For any T-fuzzy partitions D, B it is*

$$h(D, U) \leq h(B, U) + H(D|B),$$

where

$$H(D|B) = \sum_{i,j} m(g_j) \cdot \varphi\left(\frac{m(f_i \cdot g_j)}{m(g_j)}\right)$$

(the terms with $m(g_j) = 0$ are omitted).

It is interesting that Proposition 1 can be proved under some weaker conditions ([12], Chapters 6 and 7); instead of a T σ-additivity only T-additivity can be used. On the other hand our variants of the Kolmogorov-Sinai theorem needs some restrictive assumptions. They are listed at the beginning of the next Section.

3. Entropy and generators

First we shall assume that T is a Frank t-norm. Second we shall assume that $m : F \to \langle 0, 1\rangle$ is moreover T-valuation, (i.e.

$$m(T(f, g)) + m(S(f, g)) = m(f) + m(g)$$

for every $f, g \in F$) and it is continuous from below, (i.e.

$$f_n \nearrow f \Rightarrow m(f_n) \nearrow m(f) \text{)}.$$

Particularly m is monotone, i.e.

$$f \leq g \Rightarrow m(f) \leq m(g).$$

We shall say briefly that m is a probability T-measure ([2]).

Finally we shall assume that F is generated σ-T-tribe, i.e. there exists a σ-algebra $\mathbb{S}$ of subsets of X such that F consists of all $\langle 0,1\rangle$ valued $\mathbb{S}$-measurable functions.

Following the Klement representation theorem the function m can be represented by Markov kernel, i.e. such a function $K : X \times B(R) \to R$ that

(i) $K(x,\cdot) : B(R) \to R$ is a probability measure for any $x \in X$;

(ii) $K(\cdot, A) : X \to R$ is $\mathbb{S}$-measurable for any $A \in B(R)$.

Proposition 2. *Let T be a Frank norm, F be a T-tribe generated by a σ-algebra $\mathbb{S}$ of subsets of X, $m : F \to \langle 0,1\rangle$ be a probability T-measure. Then there exists a Markov kernel $K : X \times B(R) \to R$ and a probability measure $P : \mathbb{S} \to \langle 0,1\rangle$ such that*

$$m(f) = \int_X K(x, \langle 0, f(x)\rangle)\, dP(x)$$

for any $f \in F$.

Proof. [2], Corollary 5.9.

In the next text we shall use the notion of a conditional expectation $E(f|\mathbb{S}_0)$ of a random variable $f : (X, \mathbb{S}, P) \to R$ with respect to a sub-σ-algebra $\mathbb{S}_0$ of $\mathbb{S}$, i.e. $E(f|\mathbb{S}_0) : X \to R$ is an $\mathbb{S}_0$-measurable function such that

$$\int_A E(f|\mathbb{S}_0)\, dP = \int_A f\, dP$$

for any $A \in \mathbb{S}_0$. If $\mathbb{S}_0 = \mathbb{S}$, then $E(f|\mathbb{S}_0) = f$. On the other hand, if $\mathbb{S}_0$ is finite with atoms $A_1, \cdots, A_k$, then

$$E(f|\mathbb{S}_0) = \sum_{n=1}^{k} \left(\frac{1}{P(A_i)} \cdot \int_{A_i} f\, dP \right) \cdot \chi_{A_i}$$

P-almost everywhere.

Proposition 3. *Let $D = \{G_1, \cdots, G_k\}$ be a T-fuzzy partition and $B = \{A_1, \cdots, A_t\}$ be a crisp partition. Then*

$$H(D|B) = \sum_{j=1}^{k} \int_X \varphi(E(h_j|\sigma(B)))\, dP\,,$$

where $\sigma(B)$ is the σ-algebra generated by B, $h_j(x) = K(x, \langle 0, G_j(x)\rangle)$.

Proof. By the definition

$$H(D|B) = \sum_i \sum_j m(\chi_{A_i}) \cdot \varphi\left(\frac{m(G_j \cdot \chi_{A_i})}{m(\chi_{A_i})}\right) .$$

But

$$\begin{aligned} m(\chi_{A_i}) &= \int_X K(x, \langle 0, \chi_{A_i}(x)\rangle)\, dP(x) \\ &= \int_X \chi_{A_i}\, dP = P(A_i)\ , \end{aligned}$$

$$\begin{aligned} m(G_j \cdot \chi_{A_i}) &= \int_X K(x, \langle 0, G_j(x) \cdot \chi_{A_i}(x)\rangle)\, dP(x) \\ &= \int_X K(x, \langle 0, G_j(x)\rangle) \cdot \chi_{A_i}(x)\, dP(x) \\ &= \int_{A_i} h_j\, dP\ . \end{aligned}$$

Therefore

$$\begin{aligned} H(D|B) =& \sum_j \sum_i \int_X \varphi\left(\frac{1}{P(A_i)} \cdot \int_{A_i} h_j\, dP\right) \cdot \chi_{A_i}\, dP = \\ =& \sum_j \int_X \varphi\left(\sum_i \left(\frac{1}{P(A_i)} \cdot \int_{A_i} h_j\, dP\right) \cdot \chi_{A_i}\right) dP = \\ =& \sum_j \int_X \varphi(E(h_j|\sigma(B)))\, dP\ . \end{aligned}$$

Proposition 4. *Let $U : X \to X$ be any m-preserving map. Then*

$$h\left(\bigvee_{j=0}^{k} U^{-j}(B), U\right) = h(B, U)\, .$$

Proof. It follows by the equality

$$\frac{1}{n} \cdot H\left(\bigvee_{i=0}^{n-1}\left(\bigvee_{j=0}^{k} U^{-j}(B)\right)\right) =$$

$$= \frac{n+k}{n} \cdot \frac{1}{n+k} \cdot H\left(\bigvee_{i=0}^{n+k-1} U^{-i}(B)\right)\ .$$

Theorem 1. *Let T be a Frank t-norm, F be a T-tribe generated by a σ-algebra $\mathbb{S}$ of subsets of X, $m : F \to \langle 0, 1\rangle$ be a probability T-measure,*

$$m(f) = \int_X K(x, \langle 0, f(x)\rangle)\, dP(x) \quad \text{for } f \in F\,.$$

Let U be an m-preserving map, $B = \{A_1, \cdots, A_k\}$ be a crisp partition generating $\mathbb{S}$, i.e. $\sigma\left(\bigcup_i U^{-i}(B)\right) = \mathbb{S}$. Then for any T-fuzzy partition $D = \{G_1, \cdots, G_k\}$ we have

$$h(D, U) \le h(B, U) + \sum_j \int_X \varphi(h_j)\, dP\,,$$

where $h_j(x) = K(x, \langle 0, G_j(x)\rangle)$ for $x \in X$.

Proof. Put $B_n = \bigvee_{i=0}^{n-1} U^{-i}(D)$. Then $B_n \nearrow \mathbb{S}$. By the Martingal convergence theorem

$$E(h_j|B_n) \nearrow E(h_j|\mathbb{S}) = h_j\,.$$

Since φ is continuous

$$\varphi(E(h_j|B_n)) \to \varphi(h_j)\,.$$

Finally by the Lebesgue convergence theorem

$$H(D|B_n) = \sum_j \int_X \varphi(E(h_j|B_n)) \to \sum_j \int_X \varphi(h_j)\, dP\,.$$

On the other hand

$$\begin{aligned} h(D, U) \le& h(B_n, U) + H(D|B_n) = \\ =& h(B, U) + H(D|B_n) \end{aligned}$$

by Propositions 1 and 4. Therefore

$$\begin{aligned} h(D, U) \le& h(B, U) + \lim_{n\to\infty} H(D|B_n) = \\ =& h(B, U) + \sum_j \int_X \varphi(h_j)\, dP. \end{aligned}$$

Finally we mention the T-modification of the Hudetz entropy. If $D = \{f_1, \cdots, f_k\}$ is a T-fuzzy partition, then

$$\widehat{H}(D) = \sum_{i=1}^k \varphi(f_i) - \sum_{i=1}^k \int_X \varphi(h_i)\, dP\,,$$

where $h_i(x) = (K(x, \langle 0, f_i(x)\rangle)$.

Proposition 5. *For any T-fuzzy partition D there holds*

$$\widehat{H}\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right) = H\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right) - n \cdot \sum_{i=1}^{k} \int_X \varphi(h_i)\, dP\,.$$

Proof. The same as in [13], Prop. 10.3.14.

By Prop. 5 it follows that there exists

$$\lim_{n\to\infty} \frac{1}{n} \cdot \widehat{H}\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right)\,.$$

hence we can define the Hudetz entropy $\widehat{h}(U)$ by

$$\widehat{h}(U) = \sup\{\widehat{h}(D,U)\,; \quad D \text{ is a } T\text{-fuzzy partition}\}\,,$$

where

$$\widehat{h}(D,U) = \lim_{n\to\infty} \frac{1}{n} \cdot \widehat{H}\left(\bigvee_{i=0}^{n-1} U^{-i}(D)\right)\,.$$

Theorem 2. *Let the assumptions of Theorem 1 be satisfied. Then*

$$\widehat{h}(B,U) = \widehat{h}(U)$$

for any crisp generator B.

Proof. Let $B = \{A_1, \cdots, A_k\}$, $D = \{G_1, \cdots, G_t\}$, $h_i(x) = K(x, \langle 0, G_i(x)\rangle)$. By Prop. 5 and Theorem 1

$$\widehat{h}(D,U) = h(D,U) - \sum_i \int_X \varphi(h_i)\, dP \le h(B,U)\,.$$

Moreover, $K(x, \langle 0, \chi_{A_i}(x)\rangle) = \chi_{A_i}(x)$ P-a.e. Therefore

$$\widehat{h}(B,U) = h(B,U) - \sum_i \int_X \varphi(\chi_{A_i})\, dP = h(B,U)\,,$$

hence

$$\widehat{h}(D,U) \le h(B,U) = \widehat{h}(B,U) \le \widehat{h}(U)$$

for any D. Therefore

$$\widehat{h}(U) = \sup \widehat{h}(D,U) \le h(B,U) = \widehat{h}(B,U) \le \widehat{h}(U)\,.$$

References

[1] Ban, A. I., *Entropy of fuzzy T-dynamical systems*, J. Fuzzy Math. **6** (1998), 351–362.

[2] Butnariu, D., Klement, E. P., *Triangular Norm Based Measures and Games with Fuzzy Coalitions*, Kluwer, Dordrecht, 1996.

[3] Dumitrescu, D., *Entropy of fuzzy dynamical systems*, Fuzzy Sets and Systems **70** (1995), 45–57.

[4] Frank, M. J., *On the simultaneous associativity of F(x,y) and x+y-F(x,y)*, Aequationes Math. **19** (1979), 194–226.

[5] Hudetz, T., *Quantum topological entropy: first step of a pedestrian approach*, In: Quantum Probability and Related Topics VIII (Lu Accardi et al., eds.), World Scientific, Singapore, 1993, pp. 237–261.

[6] Klement, E. P., Mesiar, R., Pap, E., *Triangular norms* (to appear).

[7] Kolmogorov, A. N., *A new metric invariant of transitive dynamical systems*, Dokl. Akad. Nauk SSSR **119** (1958), 861–864, (in Russian).

[8] Kuriyama, K., *Entropy of a finite partition of fuzzy sets*, J. Math. Anal. Appl. **94** (1983), 38–43.

[9] Maličký, P., Riečan, B., *On the entropy of dynamical systems*, In: Proc. Conf. Ergodic Theory and Related Topics ll (Georgenthal, 1986), Teubner, Leipzig, 1987, pp. 135–138.

[10] Markechová, D., *A note on the Kolmogorov-Sinaj entropy for the fuzzy case*, Fuzzy Sets and Systems **64** (1994), 87–90.

[11] Mesiar, R., *The Bayes principle and the entropy of fuzzy probability spaces*, Internat. Jour. of General Systems **20** (1991), 67–72.

[12] Petrovičová, J., *Some uncertainty models and their applications*, PhD thesis, Košice, 1999.

[13] Riečan, B., Neubrunn, T., *Integral, Measure, and Ordering*, Kluwer Academic Publishers, Dordrecht and Ister Science, Bratislava, 1997.

[14] Rybárik, J., *The entropy based on pseudo-arithmetical operations*, Tatra Mountains Math. Publ. **6** (1995), 157–164.

Adresses

Beloslav Riečan; Mathematical Institute; Slovak Academy of Sciences; Štefánikova 49; SK - 814 73 Bratislava; Slovakia
e-mail: riecan@mat.savba.sk

Beloslav Riečan; Department of Mathematics; Faculty of Natural Sciences; Matej Bel University; Tajovského 40; SK - 974 01 Banská Bystrica; Slovakia
e-mail: riecan@fpv.umb.sk

Jozefína Petrovičová; Department of Medical Informatics; Faculty of Medicine; P.J. Šafárik University; Trieda SNP 1; SK - 040 66 Košice; Slovakia
e-mail: jpetrov@central.medic.upjs.sk

This preprint series was founded in 1996. Its purpose is to present manuscripts of submitted or unpublished papers and reports of fellows of the Mathematical Institute of the Slovak Academy of Sciences in Bratislava and its Košice branch. The authors are fully responsible for the content of the preprints. Postal addresses:

Mathematical Institute
Slovak Academy of Sciences
Štefánikova 49
SK–814 73 Bratislava
SLOVAKIA

Tel./Fax (+421 7) 5249 7316

Mathematical Institute
Slovak Academy of Sciences
Grešákova 6
SK–040 01 Košice
SLOVAKIA

Tel./Fax (+421 95) 6228291

PostScript files of preprints are available at WWW address:
`http://www.mat.savba.sk/preprints`

Latest preprints of the series:

- **18/1999** *Beloslav Riečan, Jozefína Petrovičová*: Kolmogorov-Sinai theorem for fuzzy T-dynamical systems
- **17/1999** *Beloslav Riečan*: On a Banach space valued observables
- **16/1999** *Michal Fečkan*: Criteria on the nonexistence of invariant submanifolds for dynamical systems
- **15/1999** *Anatolij Dvurečenskij*: On Loomis-Sikorski's theorem for MV-algebras and BCK-algebras
- **14/1999** *Otakar Grošek, Ladislav Satko, Karol Nemoga*: ZKDWCNDPDC...
- **13/1999** *Otakar Grošek, Karol Nemoga, Ladislav Satko*: A remark to Luby-Rackoff and Ueli M. Maurer pseudorandom generators
- **12/1999** *Beloslav Riečan*: On the sum of observables in product MV algebras
- **11/1999** *Peter Eliaš*: A hierarchy of thin sets related to the boundedness of trigonometric series
- **10/1999** *Miloslav Duchoň, Peter Maličký*: On Herglotz theorem in partially ordered vector spaces
- **9/1999** *Ján Jakubík*: Torsion classes of specker lattice ordered groups
- **8/1999** *Ján Jakubík*: Graph automorphisms and cells of lattices
- **7/1999** *Miloslav Duchoň, Peter Maličký*: Positive linear maps on trigonometric polynomials
- **6/1999** *Miloslav Duchoň, Anna Kolesárová*: Fourier transforms of vector-valued measures on a certain compact semigroup
- **5/1999** *Miroslav Ploščica*: Separation properties in congruence lattices of lattices
- **4/1999** *Georges Chevalier, Anatolij Dvurečenskij, Karl Svozil*: Piron's and Bell's geometric lemmas and Gleason's theorem
- **3/1999** *Jarmila Hedlíková*: Relatively orthomodular lattices
- **2/1999** *Anatolij Dvurečenskij, Pekka Lahti, Kari Ylinen*: Some remarks on the moment problem
- **1/1999** *Anatolij Dvurečenskij*: Loomis-Sikorski theorem for σ-complete MV-algebras and ℓ-groups
- **27/1998** *Martin Kochol*: Cubic graphs without a Petersen minor have nowhere-zero 5-flows
- **26/1998** *Anatolij Dvurečenskij, Ferdinand Chovanec, Eva Rybáriková*: D-homomorphisms and atomic σ-complete Boolean D-posets
- **25/1998** *Anatolij Dvurečenskij*: Commutative BCK-algebras with product

(continued on inside back cover)

24/1998 *Anatolij Dvurečenskij, Antonio Di Nola*: Product MV-algebras

23/1998 *Anatolij Dvurečenskij, Maria Gabriella Graziano*: Dedekind complete commutative BCK-algebras

22/1998 *Anatolij Dvurečenskij*: Ideals of commutative BCK-algebras

21/1998 *Anatolij Dvurečenskij, Beloslav Riečan*: Weakly divisible MV-algebras and product

20/1998 *Ivan Korec*: Definability of addition and multiplication from Pascal's triangle modulo n and number-theoretical functions

19/1998 *Roberto Giuntini, Sylvia Pulmannová*: Ideals and congruences in effect algebras and QMV algebras

18/1998 *Ivan Korec*: Definability of exact arithmetical operations from approximate ones

17/1998 *Georges Chevalier, Sylvia Pulmannová*: Some ideal lattices in partial abelian monoids

16/1998 *Anatolij Dvurečenskij*: On categorical equivalences of commutative BCK-algebras

15/1998 *Ivan Korec*: Definability of addition and multiplication from some quadratic forms $ax^2 + cy^2$

14/1998 *Beloslav Riečan*: On the L^p space of observables on product MV algebras

13/1998 *Beloslav Riečan*: On the strong law of large numbers on some ordered structures

12/1998 *Ladislav Mišík, jr., Tibor Žáčik*: A simplified formula for calculation of metric dimension of converging sequences

11/1998 *Beloslav Riečan*: On the joint distribution of observables

10/1998 *Beloslav Riečan*: On the product MV algebras

9/1998 *Roman Frič*: A Stone-type duality and its applications to probability

8/1998 *Roman Frič*: Ring of maps: sequential convergence and completion

7/1998 *Roman Frič, Ján Jakubík*: Sequential convergences on Boolean algebras defined by systems of maximal filters

6/1998 *Roman Frič*: Categorical reflections on fields of events and probability

5/1998 *Beloslav Riečan*: On the probability theory and fuzzy sets

4/1998 *Ivan Korec*: Definability of arithmetical operations from binary quadratic forms

3/1998 *Beloslav Riečan*: On the conditional expectation of observables in MV algebras of fuzzy sets

2/1998 *Beloslav Riečan*: On the conditional probability on MV algebras

1/1998 *Paula B. Cohen, Robin L. Hudson, K. R. Parthasarathy, S. Pulmannová*: Hall's transformation via quantum stochastic calculus

13/1997 *Ivan Korec*: Real-time generation of primes by a one-dimensional cellular automaton with 9 states

12/1997 *Ivan Korec*: Arithmetical operations strongest with respect to the first order definability

11/1997 *Otokar Grošek, Karol Nemoga, Marcel Zanechal*: Why use bijective S-boxes in GOST-algorithm

10/1997 *Beloslav Riečan*: On the L^p space of observables

9/1997 *Ladislav Mišík Jr., Tibor Žáčik*: A formula for calculation of metric dimension of converging sequences

8/1997 *Anatolij Dvurečenskij, Maria Gabriella Graziano*: Commutative BCK-algebras and lattice ordered groups

7/1997 *Anatolij Dvurečenskij, Maria Gabriella Graziano*: On representations of commutative BCK-algebras