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Quantum Logics with the Riesz Interpolation Property

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Abstract

We study the quantum logics which satisfy the Riesz interpolation property. We call them the RIP logics. We observe that the class of RIP logics is considerable large – it contains all lattice quantum logics and, also, many (infinite) non-lattice ones. We then find out that each RIP logic can be enlarged to an RIP logic with a preassigned center. We continue, showing that the "nearly" Boolean RIP logics must be Boolean algebras. In a somewhat surprising contrast to this, we finally show that the attempt for the σ-complete formulation of this result fails: We show by constructing an example that there is a non-Boolean nearly Boolean σ-RIP logic. As a result, there are interesting σ-RIP logics which are intrinsically close to Boolean σ-algebras.

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1 Introduction

The Riesz Interpolation Property (RIP) has found a recognized place in the investigation of partially ordered groups [Goo] and structures closed to MV-algebras ([CDM] and [DvPu 1]). It has also been used, in an implicit form, in orthomodular lattices in connection with pasting constructions [NaRo].

In this note we want to iniciate an investigation of the RIP in the context of quantum logics. Upon calling quantum logics that satisfy the RIP the RIP logics, we find that all lattice logics are RIP, and finite quantum logics are RIP exactly when they are lattices. We then investigate permanence properties of RIP logics, showing, e.g., that the RIP logics are closed under the formation of products and ultraproducts, bounded Boolean powers, etc. In the second part, we ask the question of which class of quantum logics we obtain if we add the RIP to the property of being "nearly Boolean" (see [NaPt]). This question is natural because of the abundance of nearly Boolean non-Boolean logics and because of the significant conceptual value of RIP. We show that nearly Boolean logics with the RIP must already be Boolean. We then see that the finitely additive RIP setup essentially differs from the σ-additive RIP setup (Theorem 2.3 and Theorem 2.4), allowing for a construction of a large class of σ-complete logics with the RIP.

2 Notions and Results

Basic notions on quantum logics are taken from [PtPu]. Let us only review those which we shall mainly use in the sequel.

Definition 2.1 A *quantum logic* is a triple $(L; \leq, ')$, where L is a non-void set endowed with a partial ordering, $\leq$, and a unary operation, $'$, and which is subject to the following requirements $(a, b \in L)$:

(i) L possess a least and a greatest element, $0, 1$,

(ii) if $a \leq b$, then $b' \leq a'$,

(iii) $a'' = a$,

(iv) $a \vee a' = 1$, $a \wedge a' = 0$,

(v) if $a \leq b$, then $a' \wedge b$ exists and we have $\quad b = a \vee (a' \wedge b) \quad$ (the so-called orthomodular law).

Further, a quantum logic $L = (L; \leq, ')$ is said to be *concrete* if it allows for a set representation, i.e., if $L = \Delta$, where $\Delta \subseteq \exp S$ for a non-void set S, and where $\leq$ means the set inclusion $\subseteq$ in S, the operation $'$ means the formation of the set complement in S, and $A \vee B = A \cup B$ provided $A \subseteq B' := S \setminus B$.

Let us now formulate the key definition of this article.

Definition 2.2 Let L be a quantum logic. Let us say that L has the Riesz interpolation property (the RIP), or, alternatively, that L is an *RIP logic*, if the following condition is satisfied: For each $a, b, c, d \in L$ with $a \leq c$, $b \leq c$, $a \leq d$ and $b \leq d$ there is an element $r \in L$ such that $a \leq r$, $b \leq r$, $r \leq c$ and $r \leq d$.

Since we will often use concrete logics, let us note that for the concrete logics the RIP reads as follows: For $L = \Delta$, $\Delta \subseteq \exp S$, we require that for sets (= elements of L) $A, B, C, D \in L$ with $A \cup B \subseteq C \cap D$ there is a set $R \in L$ with $A \cup B \subseteq R \subseteq C \cap D$. Obviously, neither $A \cup B$ nor $C \cap D$ have to belong to L (indeed, $A \cup B$ belongs to L for all $A, B \in L$ (respectively, $C \cap D \in L$ for all $C, D \in L$) exactly when L is a Boolean algebra, see, e.g. [NaPt]).

Our first result collects basic properties of the RIP logics. (Standardly, by the *center* of L we mean the set of all "absolutely compatible " elements in L, see [PtPu]).

Theorem 2.3 *Let L be a quantum logic.*

(i) *If L is a lattice, then L is an RIP logic; if L is finite, then L is an RIP logic exactly if L is a lattice.*

(ii) *There is a (concrete) non-lattice RIP logic; this logic can be required minimal in the sense that it is countable and contains only four elements which do not belong to its center.*

(iii) *The class of RIP logic is closed under the formation of direct products; thus, there are non-concrete non-lattice RIP logics.*

(iv) *The class of RIP logics is closed under the formation of bounded Boolean powers; thus, there are non-Boolean RIP logics with arbitrary centers.*

(v) *The class of RIP logics is closed under the formation of surjective morphisms which are ordering faithful* [2]*; thus, the class of RIP logics is closed under the formation of ultraproducts.*

Proof. (i) If L is a lattice, then we simple take $r = c \wedge d$ in the definition of RIP logic.

(ii) Suppose that L is finite and RIP. Take two elements $c, d \in L$ and consider the set $A = \{a \in L : a \text{ is an atom in } L \text{ and } a \leq c,\ a \leq d\}$. Obviously, A is finite. If $A = \emptyset$, then $c \wedge d = 0$. If $A \neq \emptyset$, then by the RIP extended inductively on a finite number of elements we see that that there exists an element, $r \in L$, such that $a \leq r$ for each $a \in A$ and, moreover, $r \leq c$ and $r \leq d$. It is easy to see that $r = c \wedge d$.

(ii) Let Ω be the set of all integers and let Δ consist of the sets

$$\begin{aligned} C &= \{p \in \Omega : p \geq 0\}, & C' &= \Omega \setminus C, \\ D &= \{p \in \Omega : p \ \text{ is even}\}, & D' &= \Omega \setminus D, \end{aligned}$$

and all sets whose intersection with any of the sets C, C', D, D' is either a finite set in Ω or a cofinite set in Ω. Then $L = \Delta$ is a concrete logic which is RIP. Indeed, $\emptyset \in \Delta$ and if $A \in \Delta$, then $\Omega \setminus A \in \Delta$. Moreover, if $A, B \in \Delta$ and $A \subseteq B' = \Omega \setminus B$, then $A \cap B = \emptyset$, and we easily see that $A \cup B \in \Delta$. We have verified that $L = \Delta$ is a concrete logic. As for the RIP, the only case which needs an argumentation is the quadruple A, B, C, D where A, B are arbitrary (the case of taking C, D' instead of C, D argues similarly). But then both A, B must be finite by our construction and the RIP verifies easily. (The center of L is obviously the Boolean algebra of finite-cofinite subsets of Ω.)

(iii) Since the ordering of the product is coordinatewise, the proof is easy. (For the appendix, it is sufficient to take the product of a non-concrete lattice logic (such logics exist - see [Gre]) with a non-lattice RIP logic.)

(iv) Suppose that B is a Boolean algebra and $\tilde{\Delta} \subseteq S$ is its Stone set-representation on S. Suppose L is a quantum logic. By the bounded Boolean power of L, $B \otimes L$, we mean the sublogic of $\prod_{s \in S} L_s$, where $L_s = L$ for each $s \in S$, which consists of all B-measurable step functions with values in L. The fact that $B \otimes L$ is RIP provided L is RIP then verifies easily. Since the center of the bounded Boolean power $B \otimes L$ of a Boolean algebra B with a

[2] A quantum logic morphism $f : L \to K$ is *ordering faithful* supposing for any $a, c \in K$ with $a \leq c$ there is $\tilde{a}, \tilde{c} \in L$ such that $\tilde{a} \leq \tilde{c}$ and $f(\tilde{a}) = a$ and $f(\tilde{c}) = c$.

logic L is B, and since L is obviously a sublogic of $B \otimes L$ (see [BGHR], for the non-lattice case see [DvPu]), the proof of the appendix is obvious.

(v) The proof is straightforward. $\square$

In connection with the investigation of "nearly Boolean" logics ([NaPt]) and recent results establishing a large class of non-Boolean nearly Boolean logics ([Pta]), a natural question arises of whether there are non-Boolean nearly Boolean logics which enjoy the RIP. Our next result shows that it is not the case. The result can be understood as a characterization of Boolean algebras among concrete quantum logics. Before we formulate the result, let us agree to call a concrete logic $L = \Delta$, $\Delta \subseteq \exp S$, *nearly Boolean* if each two-valued state[3] $s : \Delta \to \{0,1\}$ satisfies the following condition: Whenever $s(C) = s(D) = 1$ for some $C, D \in \Delta$, then there is a set $A \in L$ such that $A \subseteq C \cap D$ and $s(A) = 1$. Obviously, each Boolean algebra can be viewed as a nearly Boolean logic as a consequence of the Stone representation theorem for Boolean algebras and an obvious property of states on Boolean algebras.

Theorem 2.4 *Let L be a nearly Boolean concrete logic satisfying the Riesz interpolation property. Then L is a Boolean algebra.*

Proof. Let $L = \Delta$, $\Delta \subseteq \exp S$. We need to show that if $C, D \in L$, then $C \cap D \in L$. Let us first show that if L is nearly Boolean, then there is a finite family $\{A_1, \dots, A_n\}$ in L such that $\bigcup_{i=1}^n A_i = C \cap D$.

To this end, write $\mathcal{S}_{C,D} := \{s : s$ is a two-valued state on Δ with $s(C) = s(D) = 1\}$. By the standard application of Tychonoff's theorem, we see that $\mathcal{S}_{C,D}$ is a compact space when viewed as a topological subspace of $\{0,1\}^{\Delta}$. For each $A \subseteq C \cap D$, $A \in L$, let $\mathcal{S}_A := \{s \in \mathcal{S}_{C,D} : s(A) = 1\}$. Then each $\mathcal{S}_A$ is open in $\mathcal{S}_{C,D}$ (because $\mathcal{S}_A = \{s \in \mathcal{S}_{C,D} : s(A) > 0\}$) and since L is nearly Boolean, we have

$$\mathcal{S}_{C,D} = \bigcup_{\substack{A \in L \\ A \subseteq C \cap D}} \mathcal{S}_A.$$

By the compactness of $\mathcal{S}_{C,D}$, we infer that there is a finite family $\{A_1, \dots, A_n\}$ of elements of L such that $\mathcal{S}_{C,D} = \bigcup_{i=1}^n \mathcal{S}_{A_i}$.

We claim that $C \cap D = \bigcup_{i=1}^n A_i$. Indeed, if it is not the case, there is a point $p \in (C \cap D) \setminus \bigcup_{i=1}^n A_i$. If we take the state s_p concentrated at p

[3]By a *two-valued* state on L we mean a mapping $s : L \to \{0,1\}$ such that $s(1) = 1$ and $s(a \vee b) = s(a) + s(b)$ provided $a \leq b'$.

($s_p(B) = 1$ if and only if $p \in B$), we see that $s_p \in \mathcal{S}_{C,D} \setminus \bigcup_{i=1}^{n} \mathcal{S}_{A_i}$. This is a contradiction. Hence we have proved the statement we set up at the beginning of the proof.

The rest is straightforward. Since $C \cap D = \bigcup_{i=1}^{n} A_i$, the RIP gives us that there is a set $R \in L$ such that $R \subseteq C \cap D$ and $A_i \subseteq R$ for each $i = 1, \ldots, n$. But then $R = C \cap D$ and therefore $C \cap D \in L$. This implies that L must be Boolean and the proof is complete. $\square$

In the last part of the paper, let us take up the σ-complete form of the Riesz interpolation property in quantum logics. By analogy with Theorem 2.4 (ii), each σ-complete lattice logic is the σ-RIP logic. We want to exhibit a class of σ-complete (non-lattice) logics with the σ-RIP (as a by-product, by doing so we will add to Theorem 2.3(ii)). We will compare the previous theorem with its σ-complete version. Prior to that, let us agree to call a concrete logic L *σ-complete* if it contains the suprema of all countable disjoint families. In other words, if $L = \Delta$, $\Delta \subseteq \exp S$, is a concrete logic, it is σ-complete if for any countable family A_i ($i \in \mathbb{N}$) of mutually disjoint element of L we have $\bigcup_{i=1}^{\infty} A_i \in L$ (see [PtPu] for more details; sometimes the σ-complete concrete logics are called Dynkin systems of sets, see e.g. [Bau] and [Neu]). A σ-complete concrete logic L is said to fulfil the *countable Riesz interpolation property*, or it said to be the *σ-RIP logic*, if for any pair of sequences A_i, C_j ($i, j \in \mathbb{N}$) in L with $A_i \subseteq C_j$ for $i, j \in \mathbb{N}$, there is a set $R \in L$ such that $\bigcup_{i=1}^{\infty} A_i \subseteq R$ and $R \subseteq \bigcap_{j=1}^{\infty} C_j$.

Theorem 2.5 *There is a σ-complete concrete logic L, $L = \Delta$, $\Delta \subseteq \exp S$, such that the following statements hold true:*

(i) *L is nearly σ-Boolean, i.e., for each σ-additive two-valued state s on L, the following condition is fulfilled: If $\{A_i\}$ is a sequence in L such that $s(A_i) = 1$ for each $i \in \mathbb{N}$, then there is a set $C \in L$ such that $C \subseteq A_i$ for each $i \in \mathbb{N}$ and $s(C) = 1$,*

(ii) *L fulfils the countable Riesz interpolation property.*

Proof. We shall apply some of the technique of the paper [BNPW]. Let L_1 be the concrete logic consisting of all subsets of the set $\Omega_1 := \{1, 2, 3, 4\}$ whose cardinality is even. It is easy to see that L_1 is a lattice logic, $\mathrm{card} L_1 = 8$. Further, let us take a set, P_1, of the first uncountable cardinality. Let us consider the Cartesian product $\Omega_1 \times P_1$. Let $\Delta := \{A \subseteq \Omega_1 \times P_1 :$ the set $\{p \in P_1 : (\Omega_1 \times \{p\}) \cap A \in L_1\}$ is co-countable in $P_1\}$. Then $\Delta \subset \exp(\Omega_1 \times P_1)$.

Let $L = \Delta$ and verify that L is a σ-complete concrete logic. Indeed, $\Omega \in \Delta$. Further, since L_1 is a quantum logic, we easily check that $\{p \in P_1 : (\Omega_1 \times \{p\}) \cap A \in L_1\} = \{p \in P_1 : ((\Omega_1 \times \{p\}) \cap ((\Omega_1 \times P_1) \setminus A)) \in L_1\}$. Thus $A \in \Delta$ if, and only if, $(\Omega_1 \times P_1) \setminus A \in \Delta$. Finally, if A_i $(i \in \mathbb{N})$ are mutually disjoint, $A_i \in L$, then so are the sets $(\Omega_1 \times \{p\}) \cap A_i)$ and, therefore, $\bigcup_{i=1}^{\infty} A_i \in \Delta$.

This shows that L is a σ-complete logic. (For a later use, observe that L also contains all singletons and all sets of the form $\Omega_1 \times \{p\}$ for each $p \in P_1$.)

Let us show that $L = \Delta$ is nearly σ-Boolean. Let s be a σ-additive two-valued state on L. Since $\Omega_1 \times P_1 = \bigcup_{p \in P_1} \Omega_1 \times \{p\}$, we see that s can be viewed as a two-valued state on $\exp P_1$. As known (see e.g. [HrJe]), the fact that $\mathrm{card} P_1 = \aleph_1$, guarantees a point $p_1 \in P_1$ such that $s(\Omega_1 \times \{p_1\}) = 1$. Since a singleton $\{\omega\} \times \{p_1\}$ belongs to L, we see that there is a singleton, $\{\omega_1\} \times \{p_1\}$, such that $s(\{\omega_1\} \times \{p_1\}) = 1$. We infer that each two-valued state on our L must be concentrated at a point. As a result, if $s(A) = s(B) = 1$ for some $A, B \in L$, there must be a point $q = (\omega_1, p_1)$, $q \in A \cap B$ such that $s(\{q\}) = 1$. Since each singleton belongs to L, we infer that L is nearly σ-Boolean.

Let us finally check on the countable RIP on L. Let us have sets A_i, C_j $(i, j \in \mathbb{N})$ in L. Let us suppose that we have $A_i \subseteq C_j$ for each $i, j \in \mathbb{N}$. We are looking for a set $R \in L$ such that $\bigcup_{i=1}^{\infty} A_i \subseteq R \subseteq \bigcap_{j=1}^{n} C_j$. In constructing this set R, we will make use of the circumstance that L_1 (= even number subsets of $\{1, 2, 3, 4\}$) is a lattice. Let us define the set R by specifying, for each $p \in P_1$, the set $R_p = (\Omega_1 \times \{p\}) \cap R$.

Let $p \in P_1$. If for this $p \in P_1$ we find that all sets $(\Omega_1 \times \{p\}) \cap A_i$ $(i \in \mathbb{N})$ as well all sets $(\Omega_1 \times \{p\}) \cap C_j$ $(j \in \mathbb{N})$ belong to L_1, we set $R_p = \bigvee_{i \in \mathbb{N}}((\Omega_1 \times \{p\}) \cap A_i) \vee \bigvee_{j \in \mathbb{N}}((\Omega_1 \times \{p\}) \cap C_j)$ (obviously, the supremum is taken in the (complete) lattice L_1). If it is not the latter case, we set $R_p = \bigcup_{i \in \mathbb{N}}((\Omega_1 \times \{p\}) \cap A_i) \cup \bigcup_{j \in \mathbb{N}}((\Omega_1 \times \{p\}) \cap C_j)$. Then $\bigcup_{i=1}^{\infty} A_i \subseteq R \subseteq \bigcap_{j \in \mathbb{N}} C_j$. But the set $\{p \in P_1 : \ (\Omega_1 \times \{p\}) \cap R\}$ is a countable intersection of co-countable sets in P_1 and therefore it is co-countable. This shows that $R \in L$ and the proof is complete. $\square$

If we examine the proof of the previous theorem, we see that we in fact have enlarged an eight-element lattice concrete logic to obtain a σ-complete concrete logic with the σ-RIP. The σ-complete logic constructed this way had in addition exceptionally nice properties in that it resembled a Boolean σ-algebra. As one easily checks, the same construction can be performed if we replace the eight-element lattice concrete logic with an arbitrary lattice concrete σ-complete logic, the proof is only slightly more involved. When replacing the first uncountable cardinality for an arbitrary cardinality, however, one needs to make use of the assumption nonM of the non-existence of measurable cardinals [HrJe]. The general result then reads as follows (by saying nearly Boolean in the result we automatically mean non-Boolean nearly Boolean).

Theorem 2.6 *Each concrete σ-complete lattice logic $L_1 = \Delta$, where $\Delta \subseteq \exp S$ for S countable, can be enlarged in such a way that we obtain a concrete σ-complete logic L with the σ-RIP. Moreover, this L can be required nearly Boolean. Under the assumption of nonM, an arbitrary concrete σ-complete lattice logic can be enlarged to obtain a concrete σ-complete logic L with the σ-RIP. Moreover, the enlarged logic can be required nearly Boolean. A consequence: Under the assumption of nonM, there is a proper class of concrete σ-complete logics which are both σ-RIP and nearly Boolean.*

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