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Decomposition formulas for the flow polynomial

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DECOMPOSITION FORMULAS FOR THE FLOW POLYNOMIAL

Martin Kochol

Abstract. The flow polynomial $F_G(k)$ of a graph G evaluates the number of nowhere-zero A-flows in G for any Abelian group A of order k. Suppose that G has an edge cut C that divides G into two subgraphs G_1 and G_2 (having the same components as $G-C$) such that each edge of C has exactly one end in one of them. We give a formula expressing the flow polynomial of G from the flow polynomials of the bridgeless graphs arising from G after deleting or contracting all edges either from G_1, or from G_2, or from $G_1 \cup G_2$.

1. Preliminaries

The graphs considered in this paper are all finite and unoriented. Multiple edges and loops are allowed. If G is a graph, then $V(G)$ and $E(G)$ denote the sets of vertices and edges of G, respectively, and $c(G)$ denotes the number of components (maximal connected subgraphs) of G. If e is an edge of G, then $G-e$ and G/e denote the graphs arising from G after deleting and contracting e, respectively.

Let A be an additive Abelian group of order k. Choose an arbitrary but fixed orientation of G. Then a nowhere-zero A flow in G is a mapping $\varphi : E(G) \to A\backslash\{0\}$ so that for every vertex, the sum of the values of φ on the incoming edges equals the sum on the outcoming ones (see, e. g., [J], [K], [Z]). Clearly, the number of nowhere-zero A-flows in G does not depend on the chosen orientation and, by Tutte [T2], neither on the structure of A, but only on its order k. Thus we can denote this number by $F_G(k)$. By Tutte [T1], $F_G(k)$ is a polynomial function of k. This polynomial is called the *flow polynomial* of G and is fully characterized by the conditions (a)–(c) of the following statement (see, e. g., [BO], [T3]).

Lemma 1. *Let G be a graph. Then the following is satisfied.*

(a) $F_G(k) = 1$ *if* $E(G) = \emptyset$.
(b) $F_G(k) = (k-1)F_{G-e}(k)$ *if e is a loop of G.*
(c) $F_G(k) = F_{G/e}(k) - F_{G-e}(k)$ *if $e \in E(G)$ is not a loop.*
(d) $F_G(k) = 0$ *if G has a bridge.*
(e) $F_G(k)$ *is a polynomial of degree* $|E(G)| - |V(G)| + c(G)$ *if G is bridgeless.*

In this paper we show that the flow polynomial of a graph G can be expressed as a function of flow polynomials of some minors of G.

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2. The general formula

Let G be a graph and $C = \{e_1, \ldots, e_n\}$ be an edge cut of G. Then there are subgraphs I and I' of G so that the components of I and I' are also components of $G - C$ and each edge from C has exactly one end in I and the second end in I' (note that I and I' are uniquely determined if G is connected). Let v_i and v_i' ($i = 1, \ldots, n$) denote the ends of e_i in I and I', respectively. For $i = 1, \ldots, n$, apply the following construction: delete e_i and add vertices u_i, u_i' and edges u_iv_i, $u_i'v_i'$. Let H (H') be the graph arising from I (I') after adding edges $u_1v_1, \ldots, u_nv_n$ ($u_1'v_1', \ldots, u_n'v_n'$).

Let $P = \{Q_1, \ldots, Q_r\}$ be a partition of the set $\{1, \ldots, n\}$ (i. e., $Q_1, \ldots, Q_r$ are pairwise disjoint nonempty sets whose union is $\{1, \ldots, n\}$). For $j = 1, \ldots, r$, identify the set of vertices $\{u_i; i \in Q_j\}$ ($\{u_i'; i \in Q_j\}$) to a new vertex x_j (x_j') and denote the resulting graph by $[H, P]$ ($[H', P]$). If one of $Q_1, \ldots, Q_r$ is a singleton, P is called *trivial*, otherwise it is called *nontrivial*. If P is trivial, then $[H, P]$ and $[H', P]$ have bridges, whence by Lemma 1(d), $F_{[H,P]}(k) = F_{[H',P]}(k) = 0$. Thus we are interested only in the case if P is nontrivial. Let $\mathcal{P}_n = \{P_{n,1}, \ldots, P_{n,p(n)}\}$, $p(n) = |\mathcal{P}_n|$, denote the set of nontrivial partitions of $\{1, \ldots, n\}$. We write $[H, i]$, $[H', i]$, $F_{H,i}(k)$ and $F_{H',i}(k)$ instead of $[H, P_{n,i}]$, $[H', P_{n,i}]$, $F_{[H,P_{n,i}]}(k)$ and $F_{[H',P_{n,i}]}(k)$, respectively.

Take n isolated edges $w_1w_1', \ldots, w_nw_n'$ and partitions $P = \{Q_1, \ldots, Q_r\}$, $P' = \{Q_1', \ldots, Q_{r'}'\}$ of the set $\{1, \ldots, n\}$. For $j = 1, \ldots, r$ and $j' = 1, \ldots, r'$, identify the sets of vertices $\{w_i; i \in Q_j\}$ and $\{w_i'; i \in Q_{j'}'\}$ to new vertices y_j and $y_{j'}'$, respectively, and denote the resulting graph by $[P, P']$. If at least one of P and P' is trivial, then $[P, P']$ has a bridge, whence by Lemma 1(d), $F_{[P,P']}(k) = 0$. Thus we are interested only in the case $P, P' \in \mathcal{P}_n$. We write $[i, j]$ and $F_{i,j}(k)$ instead of $[P_{n,i}, P_{n,j}]$ and $F_{[P_{n,i},P_{n,j}]}(k)$, respectively.

Let $M_n(k)$ be the $p(n) \times p(n)$ matrix so that the (i, j)-entry is $F_{i,j}(k)$, $i, j \in \{1, \ldots, p(n)\}$. Clearly, $M_n(k)$ is a symmetric matrix. Let $M_{G,C}(k)$ be the matrix arising from $M_n(k)$ after adding $(p(n) + 1)$st row and $(p(n) + 1)$st column so that the $(p(n) + 1, p(n) + 1)$-entry is $F_G(k)$, the $(p(n) + 1, i)$-entry is $F_{H,i}(k)$ and the $(i, p(n) + 1)$-entry is $F_{H',i}(k)$, $i \in \{1, \ldots, p(n)\}$. Let $M_{n,i,j}(k)$ denote the matrix arising from $M_n(k)$ after deleting the ith row and the jth column, $i, j \in \{1, \ldots, p(n)\}$. From the well-known properties of determinants we have

$$\begin{aligned} |M_{G,C}(k)| = {} & F_G(k) \cdot |M_n(k)| + \\ & \sum_{i=1}^{p(n)} \sum_{j=1}^{p(n)} (-1)^{2p(n)+1+i+j} \cdot F_{H,i}(k) \cdot F_{H',j}(k) \cdot |M_{n,i,j}(k)|. \end{aligned} \tag{1}$$

Theorem 1. *Let G be a graph and $C = \{e_1, \ldots, e_n\}$ be an edge cut of G. Then $|M_{G,C}(k)| = 0$.*

Proof. We use induction on $|E(G)| \geq n$. If $|E(G)| = n$, then $E(G) = C$ and there exist partitions P and P' of the set $\{1, \ldots, n\}$ so that $G = [P, P']$. Let $p = p(n)$. If P (P') is trivial, then G and $[H, i]$ ($[H', i]$) have bridges for every $i = 1, \ldots, p$, whence by Lemma 1(d), the last row (column) of $M_{G,C}(k)$ is zero and $|M_{G,C}(k)| = 0$. Suppose that $P = P_{n,s}$ and $P' = P_{n,t}$. Then $G = [s, t]$, $[H, i] = [s, i]$ and $[H', i] = [i, t]$ for $i = 1, \ldots, p$, whence the sth and $(p + 1)$st rows (and also the tth and $(p+1)$st columns) of $M_{G,C}(k)$ are equal. Thus $|M_{G,C}(k)| = 0$.

If $|E(G)| > n$, then there exists an edge e of G not belonging to C. Without loss of generality we can suppose that $e \in E(H)$. If e is a loop of G, then e is a loop of $[H, i]$ for every $i = 1, \ldots, p$, whence by Lemma 1(b) and (1), we have

$$|M_{G,C}(k)| = F_G(k) \cdot |M_n(k)| +$$
$$\sum_{i=1}^{p}\sum_{j=1}^{p}(-1)^{2p+1+i+j} \cdot F_{H,i}(k) \cdot F_{H',j}(k) \cdot |M_{n,i,j}(k)| =$$
$$(k-1) \cdot F_{G-e}(k) \cdot |M_n(k)| +$$
$$\sum_{i=1}^{p}\sum_{j=1}^{p}(-1)^{2p+1+i+j} \cdot (k-1) \cdot F_{H-e,i}(k) \cdot F_{H',j}(k) \cdot |M_{n,i,j}(k)| =$$
$$(k-1) \cdot |M_{G-e,C}(k)|.$$

If e is not a loop of G, then e is not a loop of $[H, i]$ for every $i = 1, \ldots, p$, whence by Lemma 1(c) and (1), we have

$$|M_{G,C}(k)| = F_G(k) \cdot |M_n(k)| +$$
$$\sum_{i=1}^{p}\sum_{j=1}^{p}(-1)^{2p+1+i+j} \cdot F_{H,i}(k) \cdot F_{H',j}(k) \cdot |M_{n,i,j}(k)| =$$
$$(F_{G/e}(k) - F_{G-e}(k)) \cdot |M_n(k)| +$$
$$\sum_{i=1}^{p}\sum_{j=1}^{p}(-1)^{2p+1+i+j} \cdot (F_{H/e,i}(k) - F_{H-e,i}(k)) \cdot F_{H',j}(k) \cdot |M_{n,i,j}(k)| =$$
$$|M_{G/e,C}(k)| - |M_{G-e,C}(k)|.$$

Since $|E(G/e)|, |E(G-e)| < |E(G)|$, we have $|M_{G,C}(k)| = 0$ by the induction hypothesis. $\square$

Lemma 2. *$|M_n(k)|$ is a nonzero polynomial for every $n \geq 2$.*

Proof. By the definition of determinant, $|M_n(k)| = \sum_\alpha \operatorname{sgn}(\alpha)(\prod_{i=1}^{p} F_{i,\alpha(i)}(k))$ where $p = p(n)$ and the sum runs through the set of all possible permutations α of numbers $1, \ldots, p$ and $\operatorname{sgn}(\alpha)$ is 1 (-1) if α is even (odd). Let r_i denote the number of sets in the partition $P_{n,i}$, $i \in \{1, \ldots, p\}$. Then the graph $[i, j]$, $i, j \in \{1, \ldots, p\}$, has n edges, $r_i + r_j$ vertices and the number of its components, denoted by $c_{i,j}$, satisfies $c_{i,j} \leq \min\{r_i, r_j\}$. Clearly, $c_{i,i} = r_i$ for $i = 1, \ldots, p$, whence by Lemma 1(e), the polynomial $\prod_{i=1}^{p} F_{i,i}(k)$ has degree $np - \sum_{i=1}^{p} 2r_i + \sum_{i=1}^{p} r_i$. Let α be a permutation of numbers $1, \ldots, p$ such that $s \neq \alpha(s)$ where $s \in \{1, \ldots, p\}$. By Lemma 1(d)–(e), the polynomial $\prod_{i=1}^{p} F_{i,\alpha(i)}(k)$ is either 0 or has degree $np - \sum_{i=1}^{p} 2r_i + \sum_{i=1}^{p} c_{i,\alpha(i)}$. If $r_s \neq r_{\alpha(s)}$, then there exists $t \in \{1, \ldots, p\}$ such that $r_{\alpha(t)} < r_t$, whence $c_{t,\alpha(t)} \leq \min\{r_t, r_{\alpha(t)}\} < r_t$. If $r_s = r_{\alpha(s)}$, then $c_{s,\alpha(s)} < r_s$ (it is easy to check that the graph $[i, j]$ has less than r_i components if $i \neq j$ and $r_i = r_j$). Since $c_{i,\alpha(i)} \leq \min\{r_i, r_{\alpha(i)}\} \leq r_i$ for $i = 1, \ldots, p$, we have $\sum_{i=1}^{p} c_{i,\alpha(i)} < \sum_{i=1}^{p} r_i$, whence $np - \sum_{i=1}^{p} 2r_i + \sum_{i=1}^{p} c_{i,\alpha(i)} < np - \sum_{i=1}^{p} 2r_i + \sum_{i=1}^{p} r_i = \sum_{i=1}^{p}(n - r_i)$. Thus $|M_n(k)|$ is the sum of polynomials so that exactly one of them has the maximal possible degree $\sum_{i=1}^{p}(n - r_i)$. Hence $|M_n(k)| \neq 0$. $\square$

We have also proved that $|M_n(k)|$ has degree $\sum_{i=1}^{p(n)}(n - r_i)$. Using Lemma 2, Theorem 1 and (1) we can compute $F_G(k)$ if we know $F_{H,i}(k)$, $F_{H',j}(k)$ and $F_{i,j}(k)$ for every $i, j \in \{1, \ldots, p(n)\}$.

3. Small cuts

In this section we compute $M_n(k)$ for $n = 2, \dots, 5$.

Let L_n be the graph consisting of one vertex and n loops. By Lemma 1(b), $F_{L_n}(k) = (k-1)^n$ for every $n \ge 1$. Let I_n be the graph having two vertices joined with n parallel edges. By Lemma 1(c), $F_{I_n}(k) = F_{I_n/e}(k) - F_{I_n - e}(k) = F_{L_{n-1}}(k) - F_{I_{n-1}}(k)$ for every $n \ge 2$ and $e \in E(I_n)$. Thus using induction on n we can check that for every $n \ge 2$,

$$F_{I_n}(k) = \sum_{i=1}^{n-1} (-1)^{i+1} (k-1)^{n-i}. \tag{2}$$

If $n = 2$ ($n = 3$) then $\mathcal{P}_n$ has exactly one partition and by (2), $|M_n(k)|$ is equal to $k-1$ ($(k-1)^2 - (k-1) = (k-1)(k-2) = k^2 - 3k + 2$). We get the following statement proved also by Sekine and Zhang [ZH]. (Note that $[H,1]$ and $[H',1]$ arise from H and H' after identifying the vertices $u_1, \dots, u_n$ and $u'_1, \dots, u'_n$ to vertices x_1 and x'_1, respectively.)

Corollary 1. *Let G be a graph with an edge cut C of cardinality $n = 2$ or 3. Then $F_G(k) = F_{H,1}(k) \cdot F_{H',1}(k)/|M_n(k)|$, where $|M_2(k)| = k-1$ and $|M_3(k)| = k^2 - 3k + 2$.*

Proof. Follows immediately from Theorem 1 and (1). □

It is known that two homeomorphic graphs have the same flow polynomial (see, e. g., [BO], [T3]). If G_1 and G_2 are two graphs, then denote by $G_1 + G_2$ the graph arising from two vertex-disjoint copies of G_1 and G_2. Let $G_1 \cdot G_2$ arise from $G_1 + G_2$ after identifying a vertex from the copy of G_1 with a vertex from the copy of G_2. $G_1 \cdot G_2$ is not uniquely determined, but we know that $F_{G_1+G_2}(k) = F_{G_1 \cdot G_2}(k) = F_{G_1}(k) \cdot F_{G_2}(k)$ (see, e. g., [BO], [T3]).

$\mathcal{P}_4$ contains partitions $P_{4,1} = \{\{1,2,3,4\}\}$, $P_{4,2} = \{\{1,2\},\{3,4\}\}$, $P_{4,3} = \{\{2,3\},\{4,1\}\}$ and $P_{4,4} = \{\{1,3\},\{2,4\}\}$. Thus in the case $n = 4$ the graph $[1,1]$ is I_4, for $i = 2,3,4$, the graphs $[1,i]$ and $[i,1]$ are $I_2 \cdot I_2$ and the graph $[i,i]$ is $I_2 + I_2$ and, for $i,j = 2,3,4$, $i \ne j$, the graph $[i,j]$ is a circuit of length 4, which is homeomorphic with I_2. Therefore by the aforementioned formulas and (2), we have

$$M_4(k) = (k-1) \cdot \begin{pmatrix} k^2-3k+3 & k-1 & k-1 & k-1 \\ k-1 & k-1 & 1 & 1 \\ k-1 & 1 & k-1 & 1 \\ k-1 & 1 & 1 & k-1 \end{pmatrix}.$$

$\mathcal{P}_5$ contains the following partitions (considering the sums in brackets mod 5):

$$\begin{aligned} P_{5,1} &= \{1,2,3,4,5\}, \\ P_{5,1+i} &= \{\{i,(i+1)\},\{(i+2),(i+3),(i+4)\}\}, \quad (i = 1,\dots,5), \\ P_{5,6+i} &= \{\{i,(i+2)\},\{(i+1),(i+3),(i+4)\}\}, \quad (i = 1,\dots,5). \end{aligned}$$

Thus in the case $n = 5$ the graph $[1,1]$ is I_5, for $i = 2,\dots,5$, the graphs $[1,i]$ and $[i,1]$ are $I_3 \cdot I_2$ and the graph $[i,i]$ is $I_3 + I_2$ and for $i,j = 2,\dots,5$, $i \ne j$, the graph

$$
M_5(k) = \begin{pmatrix}
h(k) & g(k) & g(k) & g(k) & g(k) & g(k) & g(k) & g(k) & g(k) & g(k) & g(k) \\
g(k) & g(k) & f(k) & 0 & 0 & f(k) & f(k) & f(k) & 0 & f(k) & f(k) \\
g(k) & f(k) & g(k) & f(k) & 0 & 0 & f(k) & f(k) & f(k) & 0 & f(k) \\
g(k) & 0 & f(k) & g(k) & f(k) & 0 & f(k) & f(k) & f(k) & f(k) & 0 \\
g(k) & 0 & 0 & f(k) & g(k) & f(k) & 0 & f(k) & f(k) & f(k) & f(k) \\
g(k) & f(k) & 0 & 0 & f(k) & g(k) & f(k) & 0 & f(k) & f(k) & f(k) \\
g(k) & f(k) & f(k) & f(k) & 0 & f(k) & g(k) & 0 & f(k) & f(k) & 0 \\
g(k) & f(k) & f(k) & f(k) & f(k) & 0 & 0 & g(k) & 0 & f(k) & f(k) \\
g(k) & 0 & f(k) & f(k) & f(k) & f(k) & f(k) & 0 & g(k) & 0 & f(k) \\
g(k) & f(k) & 0 & f(k) & f(k) & f(k) & f(k) & f(k) & 0 & g(k) & 0 \\
g(k) & f(k) & f(k) & 0 & f(k) & f(k) & 0 & f(k) & f(k) & 0 & g(k)
\end{pmatrix}
$$

$$
\begin{aligned}
f(k) &= (k-1)(k-2) \\
g(k) &= (k-1)(k^2-3k+2) \\
h(k) &= (k-1)(k^3-4k^2+6k-4)
\end{aligned}
$$

Table 1.

$[i, j]$ is either homeomorphic with I_3, or has a bridge. Matrix $M_5(k)$ is indicated in Table 1.

We give a recursive formula for computing $p(n)$. For $i = 2, \ldots, n-2$ (resp. $i = n$), $\mathcal{P}_n$ contains exactly $\binom{n-1}{i-1} p(n-i)$ (resp. 1) partitions such that the element n is contained in an i-element subset. Hence for every $n \geq 2$, we have

$$
p(n) = 1 + \sum_{i=2}^{n-2} \binom{n-1}{i-1} p(n-i). \tag{3}
$$

4. Planar graphs

We identify a planar graph with its embedding in the plane. An edge cut C of a planar graph G is called *planar* if there exists a closed Jordan curve in the plane which intersects each edge of C exactly once and does not intersect any other edge or vertex of G.

Let $P = \{Q_1, \ldots, Q_r\}$ be a partition of the set $\{1, \ldots, n\}$. Take a circuit C_n having vertices $v_1, \ldots, v_n$ and add vertices $u_1, \ldots, u_n$ and edges $v_1u_1, \ldots, v_nu_n$. For $j = 1, \ldots, r$, identify the set of vertices $\{u_i; i \in Q_j\}$ to a new vertex x_j. Suppose that the resulting graph has an embedding in the plane so that no vertex x_j, $j = 1, \ldots, r$, is inside the circuit C_n and no two edges intersect. Then P is called *planar*. Let $\overline{\mathcal{P}}_n$ be the set of nontrivial planar partitions of $\{1, \ldots, n\}$ and $\overline{p}(n) = |\overline{\mathcal{P}}_n|$. For the case of simplicity we can choose the notation so that $\overline{\mathcal{P}}_n = \{P_{n,1}, \ldots, P_{n,\overline{p}(n)}\}$. (For example, $\overline{\mathcal{P}}_4 = \{P_{4,1}, P_{4,2}, P_{4,3}\}$ and $\overline{\mathcal{P}}_5 = \{P_{5,1}, \ldots, P_{5,6}\}$).

Let $\overline{M}_n(k)$ and $\overline{M}_{G,C}(k)$ be the matrices arising from $M_n(k)$ and $M_{G,C}(k)$ after deleting the rows and columns corresponding to nonplanar partitions of $\mathcal{P}_n$, respectively. Let $\overline{M}_{n,i,j}(k)$ denote the matrix arising from $\overline{M}_n(k)$ after deleting the ith row and the jth column, $i, j \in \{1, \ldots, \overline{p}(n)\}$. We have

$$|\overline{M}_{G,C}(k)| = F_G(k) \cdot |\overline{M}_n(k)| + \sum_{i=1}^{\overline{p}(n)} \sum_{j=1}^{\overline{p}(n)} (-1)^{2\overline{p}(n)+1+i+j} \cdot F_{H,i}(k) \cdot F_{H',j}(k) \cdot |\overline{M}_{n,i,j}(k)|. \tag{4}$$

Theorem 2. *Let G be a planar graph and $C = \{e_1, \ldots, e_n\}$ be a planar edge cut of G. Then $|\overline{M}_{G,C}(k)| = 0$.*

Proof. If $|E(G)| = n$, then there exist planar partitions P and P' of the set $\{1, \ldots, n\}$ so that $G = [P, P']$. The rest of the proof runs in a similar way as in Theorem 1 after replacing $p(n)$, $M_n(k)$, $M_{n,i,j}(k)$ and $M_{G,C}(k)$ by $\overline{p}(n)$, $\overline{M}_n(k)$, $\overline{M}_{n,i,j}(k)$ and $\overline{M}_{G,C}(k)$, respectively. $\square$

Lemma 3. *$|\overline{M}_n(k)|$ is a nonzero polynomial for every $n \geq 2$.*

Proof. The proof runs in a similar way as in Lemma 2 after replacing $p(n)$ and $M_n(k)$ by $\overline{p}(n)$ and $\overline{M}_n(k)$, respectively. $\square$

The degree of $|\overline{M}_n(k)|$ is $\sum_{i=1}^{\overline{p}(n)}(n - r_i)$.

Note that $\overline{M}_2(k) = M_2(k)$, $\overline{M}_3(k) = M_3(k)$, $\overline{M}_4(k)$ arises from $M_4(k)$ after deleting the last row and the last column and $\overline{M}_5(k)$ arises from $M_5(k)$ after deleting the last five rows and the last five columns. By (3), $p(6) = 1 + \binom{5}{1}4 + \binom{5}{2} + \binom{5}{3} = 41$. We can check that $\overline{p}(6) = 15$. Thus from (4) and Theorem 2 we can get much simpler formulas then in the general case.

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