Mathematical Institute
Slovak Academy of Sciences, Bratislava

Preprint 10/2001

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Remarks on sequential envelopes II

September 24, 2001

PREPRINT SERIES

REMARKS ON SEQUENTIAL ENVELOPES II

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Abstract. We generalize the notion of a sequential envelope introduced by J. Novák. Our motivation comes from the foundations of probability. We discuss relationships between the envelope and fuzzy structures and mention possible applications, for example, to generalized fields of events having fuzzy nature.

1. Introduction

Let X be a set and let $\mathbb{A}$ be a field of subsets of X. Recall that $\mathbb{A}$ contains $\emptyset$ as the least element (representing the null event), X as the greatest element (representing the sure event), and it is closed with respect to all usual (finite) set operations. If $\mathbb{A}$ is closed with respect to countable set operations, then $\mathbb{A}$ is a σ-field. Otherwise $\mathbb{A}$ can be extended to the smallest σ-field $\sigma(\mathbb{A})$ containing $\mathbb{A}$, called the generated σ-field. Since in probability we are interested in limit theorems, the process of extending $\mathbb{A}$ to $\sigma(\mathbb{A})$ is essential.

Motivated by this example, J. Novák developed the theory of sequential envelopes (cf. NOVÁK [1962], [1965], [1968]). He observed that, in a certain sense, $\sigma(\mathbb{A})$ is the maximal field of subsets of X to which all σ-additive probability measures on $\mathbb{A}$ can be extended. Since σ-additivity is equivalent to sequential continuity (cf. NOVÁK [1958]), from the categorical viewpoint, the embedding of $\mathbb{A}$ into $\sigma(\mathbb{A})$ has a similar character as the embedding of a completely regular topological space X into its Čech-Stone compactification βX, or into its Hewitt realcompactification υX. Indeed, we construct an epireflector, assigning a "more perfect" object to each "less perfect" one. Observe that the additivity of a probability measure fails to be a topological notion. Hence Novák generalized his construction to quite arbitrary classes of sequentially continuous functions. The theory of sequential envelopes has been surveyed, e.g., in FRIČ [1988]. In the present paper we generalize sequential envelopes one step further. Besides using categorical methods, we present a fuzzy viewpoint.

As in FRIČ [200y], we use the language of mathematical structures (cf. ADÁMEK [1982], [1983]), equivalent to concrete categories, but more natural for our purpose. Categorical notions can be found, e.g., in HERRLICH and STRECKER

1991 *Mathematics Subject Classification.* (Mathematics Subject Classification 2000: Primary 54A20, 54D35, 54B30; Secondary 06D72, 54C20, 54H99, 60A05).

Key words and phrases. Sequential convergence, sequential continuity, extension of maps, E-sequential envelope, initial structure, product, convergence category, fuzzy structures, sober object, absolute object, epireflection, Novák envelope, ring of functions, completion of algebraic structures, foundations of probability, fuzzy events.

Supported by VEGA grant 2/6087/01

Typeset by $\mathcal{A}_\mathcal{M}\mathcal{S}$-TEX

[1976]. Interesting survey RODABAUGH [1991] is a stimulating source of fuzzy mathematics. Much more complex and advanced is the handbook HOEHLE and RODABAUGH [2000].

For the reader's convenience, in this section we recall some fundamental notions.

Recall that if X is a set, then by a sequential convergence on X we understand a set of pairs $(\langle x_n \rangle, x) \in X^N \times X$, specifying which sequence converges to which limit, and we assume the usual axioms of convergence: each sequence converges to at most one limit, each constant sequence converges to its value, each subsequence of a convergent sequence converges to the same limit, and if $\langle x_n \rangle$ is a sequence and x is a point such that for each subsequence $\langle x'_n \rangle$ of $\langle x_n \rangle$ there is a subsequence $\langle x''_n \rangle$ of $\langle x'_n \rangle$ which converges to x, then $\langle x_n \rangle$ converges x. A map is sequentially continuous if it preserves the convergence of sequences.

Let S be a complete concrete category whose objects are "structured sets" $A = (|A|, s)$ and whose morphisms are "structure preserving" point maps (cf. ADÁMEK [1982], [1983]). The underlying point map of a morphism f is denoted by $|f|$. A triple $(|f|, |A|, (|B|, r))$, where $(|B|, r)$ is an S-object, $|A|$ is a set, and $|f|$ is a point map of $|A|$ into $|B|$, is said to be an **arrow** in S. Let X be an index set and let $\{(|f_x|, |A|, (|A_x|, s_x));\ x \in X\}$ be an indexed family of arrows, called a **source**. Let $(|A|, s)$ be an S-object such that

- (i_1) For each $x \in X$, f_x is an S-morphism of $(|A|, s)$ into $(|A_x|, s_x)$;
- (i_2) For each S-object $(|B|, r)$ and for each point map $|h| : |B| \longrightarrow |A|$, if each $|f_x| \circ |h|$, $x \in X$, yields an S-morphism of $(|B|, r)$ into $(|A_x|, s_x)$, then $|h|$ yields an S-morphism h of $(|B|, r)$ into $(|A|, s)$.

Then $(|A|, s)$ is said to be **projectively generated** by $\{(|f_x|, |A|, (|A_x|, s_x));\ x \in X\}$ and s is said to be the **initial structure** w.r.t. $\{(|f_x|, |A|, (|A_x|, s_x));\ x \in X\}$.

Assume that $C = (|C|, t)$ is an S-object such that for each power $|C|^X$ and the corresponding source of projections $\{(|pr_x|, |C|^X, (|C_x|, t_x));\ x \in X\}$, $(|C_x|, t_x) = (|C|, t)$ for all $x \in X$, the resulting projectively generated object $(|C|^X, p)$ is the corresponding power C^X of C. Further, assume that C carries a sequential convergence (i.e. on the underlying set $|C|$ of C a convergence of sequences is defined and the usual axioms of convergence are assumed). Let S_C be the subcategory of S defined as follows. The **objects** of S_C are S-objects $\mathcal{X} = (|\mathcal{X}|, s)$ such that there exists a power $C^X = (|C|^X, p)$ of C for which:

- (i) $|\mathcal{X}|$ is a subset of $|C|^X$;
- (ii) $\mathcal{X}$ is reduced (equivalently, points of X are separated by $|\mathcal{X}|$, i.e. for $x, y \in X, x \neq y$, there exists $a \in |\mathcal{X}| \subseteq |C|^X$ such that $a(x) \neq a(y)$);
- (iii) s is the initial structure w.r.t. the single arrow $(|i_{\mathcal{X}}|, |\mathcal{X}|, (|C|^X, p))$, where $|i_{\mathcal{X}}|$ is the inclusion map of $|\mathcal{X}|$ into $|C|^X$.

The pointwise convergence in C^X is the categorical product convergence and each object of S_C carries the inherited pointwise convergence. The **morphisms** of S_C will be the sequentially continuous S-morphisms. If no confusion can arise, then $\mathcal{X}$ will denote both the object $(|\mathcal{X}|, s)$ and its underlying set $|\mathcal{X}|$. Further, $\mathcal{X} \hookrightarrow C^X$ means that the conditions (i), (ii), and (iii) are fulfilled.

Definition 1.1. Let S, C, and S_C be as above. Then S_C is said to be a **convergence category**.

If S is the structure of Boolean algebras and C is the two-element Boolean algebra carrying the discrete convergence, then S_C is the category whose objects are fields of sets (we identify each subset of X with its characteristic function) and whose morphisms are sequentially continuous Boolean homomorphisms. If S is the structure of MV-algebras (see CIGNOLI, D'OTTAVIANO and MUNDICI [2000]) and C is the closed unit interval carrying the usual Łukasiewicz operations, then S_C is the category of bold algebras.

The role of elementary events in the classical Kolmogorovian probability has been thoroughly discussed in ŁOŚ [1960], see also FRIČ [1997a]. Roughly, events have certain attributes and each event is represented by a two-valued map on the set of all attributes. In the "fuzzy" approach we replace the "yes - no" range by a more complex object C, possibly carrying additional algebraic, order, and convergence structures. Hence each generalized event is represented by a suitable C-valued map. The value at an attribute states "how much" of the attribute the event in question possesses. Different sets of attributes result in different models. For example, in the usual (compact) Stone representation of a Boolean algebra (by a field of sets) each additive probability measure is countably additive, while in some other representation some additive measures fail to be countably additive (cf.FRIČ [1997a], [1997b]). From the categorical viewpoint, the object C (belonging to a suitable category) "cogenerates" a category in which the generalized events live (cf. FRIČ [2000], [200x], [200y]).

2. The epireflection

Let S_C be a convergence category. Let $\mathcal{X} = (|\mathcal{X}|, s)$ be an S_C-object. Having in mind the construction of a sequential envelope, we would like to construct an S_C-object $\sigma(\mathcal{X}) = (|\sigma(\mathcal{X}|, s_\sigma)$ such that (cf. NOVÁK [1965]):

(i) $\mathcal{X}$ is a sequentially dense subobject of $\sigma(\mathcal{X})$;
(ii) Each S_C-morphism of $\mathcal{X}$ into C can be uniquely extended to an S_C-morphism of $\sigma(\mathcal{X})$ into C;
(iii) $\sigma(\mathcal{X})$ is maximal with respect to conditions (i) and (ii).

¿From the categorical viewpoint, we would like to construct a suitable epireflector σ, having the domain and the codomain as large as possible. The domain of σ will the subcategory SO_C of sober S_C-objects (sobriety will guarantee the extension of morphisms) and the codomain of σ will be the subcategory ASO_C of absolute sober S_C-objects (absolute objects will satisfy condition (iii) of maximality). The construction will require an additional assumption on S_C.

Definition 2.1. Let $\mathcal{X} \hookrightarrow C^X$. If for each morphism h of $\mathcal{X}$ into C there exists exactly one $x \in X$ such that $h = pr_x \circ i_{\mathcal{X}} = pr_x \restriction \mathcal{X}$ (i.e. h is the restriction of the x-th projection $pr_x : C^X \longrightarrow C$ to $\mathcal{X}$), then $\mathcal{X}$ is said to be **sober**.

Let SO_C be the category the objects of which are sober S_C-objects and the morphisms are S_C-morphisms. Let $\mathcal{X} = (|\mathcal{X}|, s) \hookrightarrow C^X$. Then $\sigma(\mathcal{X}) \hookrightarrow C^X$ will mean that there exists an S_C-object $\sigma(\mathcal{X}) = (|\sigma(\mathcal{X})|, s_\sigma)$ such that $|\sigma(\mathcal{X})|$ is the smallest sequentially closed subset in C^X containing $|\mathcal{X}|$ and s_σ is the initial structure w.r.t. the inclusion map of $|\sigma(\mathcal{X})|$ into $|C|^X$ (as a single arrow in S). Due to the fact that the convergence in C is quite arbitrary w.r.t. the original category S, the next condition (naturally fulfilled in applications) will be imposed on S_C:

(e) For each $\mathcal{X} \hookrightarrow C^X$, if $\mathcal{X}$ is sober, then $\sigma(\mathcal{X}) \hookrightarrow C^X$.

Lemma 2.2. *Let $\mathcal{X} = (|\mathcal{X}|, s) \hookrightarrow C^X$, $\sigma(\mathcal{X}) = (|\sigma(\mathcal{X})|, s_\sigma) \hookrightarrow C^X$.*

(i) *The inclusion map $|e_\mathcal{X}|$ of $|\mathcal{X}|$ into $|\sigma(\mathcal{X})|$ yields in S_C an epimorphism $e_\mathcal{X}$ of $\mathcal{X}$ into $\sigma(\mathcal{X})$;*

(ii) *If $\mathcal{X}$ is sober, then $\sigma(\mathcal{X})$ is sober.*

Proof. (i) Since the composition of $|e_\mathcal{X}|$ and the inclusion $|i_{\sigma(\mathcal{X})}|$ of $\sigma(\mathcal{X})$ into $|C|^X$, as the inclusion $|i_\mathcal{X}|$ of $|\mathcal{X}|$ into $|C|^X$, yields the morphism $i_\mathcal{X}$ and s_σ is the initial structure w.r.t. $|i_{\sigma(\mathcal{X})}|$, $e_\mathcal{X}$ is a morphism of $\mathcal{X}$ into $\sigma(\mathcal{X})$ in S. Clearly it is sequentially continuous and hence an S_C-morphism. Moreover, $|\sigma(\mathcal{X})|$ is the smallest sequentially closed subset in $\sigma(\mathcal{X})$ containing $|\mathcal{X}|$ and hence $e_\mathcal{X}$ is an epimorphism (remember, if two sequentially continuous maps agree on a sequentially dense subset and the convergence has unique sequential limits, then the two maps agree on the whole domain, cf. NOVÁK [1965]).

(ii) Let h be a morphism of $\sigma(\mathcal{X})$ into C. The composition $h \circ e_\mathcal{X}$ is a morphism of $\mathcal{X}$ into C. If $\mathcal{X}$ is sober, then there exists a unique $x \in X$ such that $h \circ e_\mathcal{X} = pr_x \circ i_\mathcal{X}$. Since $pr_x \circ i_{\sigma(\mathcal{X})}$ is a morphism and it agrees on $|\mathcal{X}|$ with h, it follows from (i) that $h = pr_x \circ i_{\sigma(\mathcal{X})}$. Thus $\sigma(\mathcal{X})$ is sober. This completes the proof.

Definition 2.3. Let $\mathcal{X} \hookrightarrow C^X$. If $|\mathcal{X}|$ is sequentially closed in C^X (i.e. $|\mathcal{X}| = |\sigma(\mathcal{X})|$), then $\mathcal{X}$ is said to be **absolute**.

Let ASO_C be the subcategory of S_C consisting of sober absolute objects.

Theorem 2.4. *Let S_C be a convergence category fulfilling condition (e). Then the assignment sending a sober S_C-object $\mathcal{X} \hookrightarrow C^X$ to $\sigma(\mathcal{X}) \hookrightarrow C^X$ yields a covariant functor σ of SO_C into ASO_C.*

Proof. Let h be a sequentially continuous S-morphism of a sober object $\mathcal{X} \hookrightarrow C^X$ into a sober object $\mathcal{Y} \hookrightarrow C^Y$. Then $\sigma(\mathcal{X}) \hookrightarrow C^X$ and $\sigma(\mathcal{Y}) \hookrightarrow C^Y$. For each $y \in Y$, $pr_y \circ h$ is a morphism of $\mathcal{X}$ into C. Since $\mathcal{X}$ is sober, for each $y \in Y$ there exists a unique $x(y) \in X$ such that $pr_y \circ h = pr_{x(y)} \circ i_\mathcal{X}$. For each $x \in X$, $pr_x \circ i_{\sigma(\mathcal{X})}$ is the unique morphism (in S_C) of $\sigma(\mathcal{X})$ into C which extends the morphism $pr_x \circ i_\mathcal{X}$ of $\mathcal{X}$ into C. Due to the fundamental property of products (for each object, if there is a family of morphism of the object into each of the factors of a product, then there exists a unique morphism of the object into the product through which the family factorizes), there exists a unique morphism g of $\sigma(\mathcal{X})$ into C^Y such that $pr_y \circ g = pr_{x(y)} \circ i_{\sigma(\mathcal{X})}$. Further, g is an extension of h. It follows from the sequential continuity of g that $|g|$ maps $|\sigma(\mathcal{X})|$ into $|\sigma(\mathcal{Y})|$. Then $i_{\sigma(\mathcal{Y})} \circ g$ is the unique morphism of $\sigma(\mathcal{X})$ into $\sigma(\mathcal{Y})$ which extends h. Put $\sigma(h) = i_{\sigma(\mathcal{Y})} \circ g$. Clearly, σ is a covariant functor of SO_C into ASO_C.

Corollary 2.5. *Let S_C be a convergence category fulfilling condition (e). Then σ is an epireflector of SO_C into ASO_C.*

Proof. It follows from Lemma 2.2 that $e_\mathcal{X} : \mathcal{X} \longrightarrow \sigma(\mathcal{X})$ is an epimorphism in S_C and that $\sigma(\mathcal{X})$ is sober. It is clear from the construction of $\sigma(h)$ in the proof of Theorem 2.4 that σ is an epireflector.

Definition 2.6. Let S_C be a convergence category satisfying condition (e). Let $\mathcal{X}$ be a sober object. Then $\sigma(\mathcal{X})$ is said to be the **Novák envelope** of $\mathcal{X}$.

3. Applications

To illustrate the results of the previous section, we present some examples and mention possible applications.

3.1. The E-sequential envelope revisited. Let E be a subset of the real line R containing at least two points. Let L be a convergence space (i.e.a set carrying a sequential convergence and the induced closure: for each subset A of L, clA is the set of all limits of sequences ranging in A). If the convergence in L is initial w.r.t. the set $\mathcal{C}(L, E)$ of all sequentially continuous functions on L into E, where E is considered as a subspace of R (i.e. a sequence $\langle x_n \rangle$ converges to x in L iff for each $f \in \mathcal{C}(L, E)$ the sequence $\langle f(x_n) \rangle$ converges to $f(x)$ in L), then L is said to be E-**sequentially regular**. Further, an E-sequentially regular space $\sigma_E(L)$ is said to be an E-**sequential envelope** of L if the following conditions are satisfied:

(i) L is sequentially dense in $\sigma_E(L)$ (i.e. if M is a sequentially closed subset of $\sigma_E(L)$ containing L, then $M = \sigma_E(L)$);

(ii) Each $f \in \mathcal{C}(L, E)$ can be continuously extended to $\overline{f} \in \mathcal{C}(\sigma_E(L), E)$;

(iii) $\sigma_E(L)$ is maximal w.r.t. conditions (i) and (ii).

Highly nontrivial results concerning E-sequential envelopes can be found, e.g., in FRIČ [1988], FRIČ and HUŠEK [1983], FRIČ and KOUTNÍK [1979], MIŠÍK [1984].

The E-sequential envelope can be described within the theory of Novák envelope. Indeed, let S be the category SET of sets and maps, let C be the set E carrying the inherited sequential convergence in the real line. Then objects of S_C are E-sequentially regular convergence spaces and each E-sequentially regular convergence space can be embedded in E^X, where X is any subset of $\mathcal{C}(L, E)$ which separates points of L. If $X = \mathcal{C}(L, E)$, then the embedding results in a sober object. Each $x \in X$ represents a sequentially continuous function on L into E. It is easy to see that condition (e) holds true. For sober S_C-objects, the E-sequential envelope coincides with the Novák envelope. Otherwise we embed an E-sequentially regular convergence space L into $E^{\mathcal{C}(L,E)}$ and the smallest sequentially closed subset of $E^{\mathcal{C}(L,E)}$, as a Novák envelope of the embedding, is isomorphic to the E-sequential envelope of L.

3.2. Completion of algebraic structures. Let L be an abelian group (more generally, ring, MV-algebra, ...) carrying a compatible sequential convergence: algebraic operations are sequentially continuous (i.e. if $\langle x_n \rangle$ converges to x and $\langle y_n \rangle$ converges to y, then $\langle -x_n \rangle$ converges to $-x$ and $\langle x_n + y_n \rangle$ converges to $x + y$). A sequence $\langle x_n \rangle$ is said to be **Cauchy** whenever for each pair of subsequences $\langle x'_n \rangle$ and $\langle x''_n \rangle$ their difference $\langle x'_n - x''_n \rangle$ converges to 0. If each Cauchy sequence converges, then L is said to be **complete**. If L is a sequentially dense subgroup of a complete abelian group L', then L' is said to be a **completion** of L. The completion theory for convergence algebras differs from the completion theory for topological algebras in many respects (cf. FRIČ and KOUTNÍK [1989], [1992]). For convergence rings a special type of completion has been constructed in FRIČ [1999] (see also FRIČ and JAKUBÍK [2001]). Again, such completions can be described within the theory of Novák envelope.

Let S be the category $RING$ of rings and ring homomorphisms. Let C be the ring R of real numbers carrying the usual sequential convergence. Then objects of S_C are exactly convergence rings of real functions w.r.t. the pointwise sequential convergence and each point of the domain represents a sequentially continuous ring homomorphism into R. Let $\mathcal{F} \subseteq R^X$ be an S_C-object. Let $\mathcal{F}$ be sober. Then X can be identified with the set $hom(\mathcal{F}, R)$ of all sequentially continuous ring homomorphisms of $\mathcal{F}$ into R. It follows from the construction of the Novák envelope $\sigma(\mathcal{F})$ of $\mathcal{F}$ that $\mathcal{F}$ is R-generated in the sense of Definition 2.1 in FRIČ [1999] and that $\sigma(\mathcal{F})$ is the R-completion of $\mathcal{F}$ in the sense of Definition 2.11 in FRIČ [1999]. If $\mathcal{F}$ is not sober, then $\mathcal{F}$ can be embedded into $R^{hom(\mathcal{F},R)}$, the embedding is a sober object isomorphic to $\mathcal{F}$, and the Novák envelope of the embedding is the R-completion of $\mathcal{F}$.

Analogously, if C is the ring $Z(2)$, then S_C-objects are fields of sets, sober objects are s-perfect fields of sets (cf. FRIČ [1997b]), and the generated σ-field of each s-perfect field coincides with its Novák envelope.

Similarly, if S is the category of MV-algebras and C is the closed unit interval I carrying the usual Łukasiewicz MV-algebra operations and the usual convergence of sequences in I, then the Novák envelope of a sober object (i.e. a sober bold algebra) is the generated Łukasiewicz tribe (cf. FRIČ [200x], [200y]). Observe that bold algebras generalize fields of sets and if a bold algebra fails to be sober, then the Novák envelope of the "sobrification" (embedding) yields its desired completion.

It might be interesting to study permanence properties (e.g. productivity) related to the Novák envelope and its generalizations (cf. KENT and RICHARDSON [1979], SCHRODER [1995]).

3.3. Foundations of probability. In the classical Kolmogorovian model of probability we start with an original probability space (i.e., a triple $(\Omega, \mathbb{S}, P)$, where Ω is a set, $\mathbb{S}$ is a σ-field of subsets, and P is a normed σ-additive measure) and we deal with random variables (i.e., measurable functions) and distributions of families of random variables. Each random variable induces (via the preimage of the underlying measurable function) a homomorphism of measurable sets of R into $\mathbb{S}$, called an observable. In various generalizations we deal with events forming an algebraic structure (more general than a field of subsets) and observables as morphisms preserving the structure. Since we are interested in limit theorems, we have to pass from finitary algebraic structures to their σ-completions and this is where the theory of sequential envelope and Novák envelope can be applied. The interested reader is referred to, e.g., SIKORSKI [1960], DVUREČENSKIJ and PULMANNOVÁ [2000], MUNDICI and RIEČAN [200z], HOEHLE and RODABAUGH [2000], RIEČAN and NEUBRUNN [1997], FRIČ [1997a], [1997b], [2000], [200x], [200y].

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