On the convex-set approach

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Different approaches to the mathematical foundations of physical theories can be generally characterized by different choices of which basic ingredients in the description of a physical system are adopted as primitive undefined notions, and which ones are viewed as derived.

Here attention will be focused on the approach that adopts the notion of states as a primitive one. The convexity of the set of states mirrors the basic fact that the states of a physical system can be mixed up. This approach has proved to offer a flexible and meaningful format for physical statistical theories, even beyond the strictly quantum frame.

The typical branching between the classical and the quantum behaviour can be traced back, in this context, to the fact that in the classical case the convex decomposition of a non pure state into pure states is unique, hence the set of states becomes a simplex, while in the quantum case such a decomposition is never unique and the set of states is no longer a simplex.

By adopting a natural definition of observable the notion of convexity model emerges, a notion that will prove to be sufficiently structured to allow a rich array of results but general enough to encompass the relevant physical models, in particular the classical and the quantum ones. The relationship between convexity models and effect algebras will be sketched.

A convexity model which adopts a simplex structure for the states, as in the classical case, but calls into play observables that carry an intrinsic indeterministic nature, will be considered. This model received considerable attention in the literature and is often referred to as the *operational* framework.

It is a remarkable fact that typically quantum behaviours can be hosted in this framework. We will outline the occurrence of probability contexts that do not fit into the classical probabilistic pattern characterized by a deterministic background, and we will comment on the possible occurrence of non classical correlations among observables. The possibility of embedding the quantum model in the operational framework will then be outlined.

The present contribution is intended as an homage to S. Bugajski with whom the present author had the privilege of a long collaboration.