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Quantum tomography and the metaplectic representation

Under the name of Quantum Tomography it is usually meant the problem of "measuring a quantum state"; in other words, the problem of reconstructing the (unknown) state of a quantum system from a (suitable) set of experimental data. Optical homodyne tomography is a well-established method for measuring the quantum state of the radiation field, based on the measurement of the field quadratures, with varying phase. In a previous work we have shown that the method of homodyne tomography follows (in a precise mathematical way) from the fact that there is a unitary irreducible square-integrable representation of the Heisemberg group acting on the Hilbert space of quantum states of the radiation field. The aim of this talk is discussing another scheme of quantum tomography, based on the properties of square-integrable representations of the group SL(2,R). SL(2,R) is a Lie group that has an infinite sequence of unitary irreducible square-integrable representations: the so called "discrete series of SL(2,R)". These representations have a standard realization on Hilbert spaces of (square-integrable) holomorphic functions on the Poincare' upper half plane (equivalently, on the unit disc of the complex plane). Such Hilbert spaces cannot be interpreted, in a natural way, as spaces of states of any quantum system. We show that there are (equivalent) realizations of the representation of the discrete series of SL(2,R) acting on Hilbert spaces which can be physically interpreted as spaces of states of quantum systems. This can be achieved by using the metaplectic (or Segal-Shale-Weil, or oscillator) representation. This construction is well known, and seems to have an ubiquitous role in Mathematics. It is a unitary projective reducible representation of SL(2,R). Its tensor square is a true unitary representation of SL(2,R). It can be realized on the Bargmann space of quantum states of the two modes radiation field, and it decomposes into a sum of all the representations of the discrete series of SL(2,R). This shows that there is a realization of the discrete series with a physical interpretation. Using this model, it is possible to derive a scheme of quantum tomography. Its physical interpretation appears to be linked to the properties of "squeezed states", well known in quantum optics