# MEASURES ON SUBSPACE STRUCTURES OF INNER PRODUCT SPACES

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ABSTRACT. For an inner product space $S$ we consider the complete lattice of orthogonally closed subspaces (denoted by $F(S)$) and the orthomodular poset of splitting subspaces (denoted by $E(S)$). We recall that $E(S)$ coincides with $F(S)$ if, and only if, $S$ is a Hilbert space [1]. It is shown that only 'free' charges can exist on $F(S)$ when $S$ is incomplete [2]. Strongly dense inner product spaces are introduced, and it is shown that for such inner product spaces the state space of $F(S)$ is affinely homeomorphic to the face consisting of the free states on $F(\overline{S})$ (i.e. the singular states on $B(\overline{S})$, where $B(\overline{S})$ is the algebra of bounded operators on $\overline{S}$) [4]. The range of bounded charges on $F(S)$ is investigated and it is shown that this is always convex. This result is used to exhibit a counterexample of a regular charge on $F(H)$ (where $H$ is a Hilbert space) which is not completely-additive [3].

Regular bounded charges on $E(S)$ are characterized as follows: "Every regular bounded charge on $E(S)$ is the restriction of a normal functional on $B(\overline{S})$". However, it is shown that the set of regular bounded charges need not be sequentially complete when $S$ is not complete [5]. This is in contrast to the situation when we take $S$ to be a Hilbert space. The Vitali-Hahn-Saks theorem for $B(H)$ is also discussed. It is shown that this theorem cannot be carried over directly to the non-commutative case. In fact, the signed-measure version of the Vitali-Hahn-Saks theorem fails for $B(H)$ when $H$ is infinite dimensional [6]. On the other-hand, if we restrict to positive measures, the Vitali-Hahn-Saks theorem holds for the projection lattice of a Hilbert space, but not for the projection logic of an incomplete inner product space.

## REFERENCES

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