WIGNER TYPE THEOREMS FOR PROJECTIONS

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The original Wigner's theorem states that if H is a complex Hilbert space and S a bijective transformation of the set of all one-dimensional linear subspaces of H which preserves angles between any pair of such subspaces then S is induced by either a unitary or an antiunitary operator on H. If $\dim H \geq 3$, U. Uhlhorn improved this result by requiring that S only preserves the orthogonality between the one-dimensional subspaces.

The aim of this talk is to generalize the Uhlhorn's version of the Wigner's theorem to posets of projections.

Let L be a DAC-lattice (a lattice which axiomatizes lattices of closed subspaces). Define

$$P(L) = \{(a, b) \in L^2 \mid a \vee b = 1,\ a \wedge b = 0,\ (a, b)M,\ (a, b)M^*\}$$

where $(a, b)M$ and $(a, b)M^*$ mean, respectively, (a, b) is a modular pair and (a, b) is a dual modular pair. P(L) is an atomic orthomodular poset, called the projection poset of L. If E is a locally convex space and L its lattice of all closed subspaces then the poset of weakly continuous linear projections (= idempotents) defined on E is isomorphic to the projection poset $P(L)$ by the mapping $p \to (Im\, p, Ker\, p)$.

An orthogonality relation is defined on $P(L)$ by $(a, b) \perp (c, d) \Leftrightarrow a \leq d$ and $b \geq c$

Every bijection of the atoms of a projection poset $P(L)$ which preserves $\perp$ in both directions extends to an automorphism of $P(L)$ and is induced by an automorphism or an antiautomorphism of the lattice L. In the first case, the bijection is said to be even and, otherwise, it is an odd bijection.

By using a generalization of the First Fundamental Theorem of projective geometry we obtain, in particular, Wigner type theorems for

- even bijections preseving $\perp$ and $P(L)$ the projection poset of a real locally convex space or an infinite dimensional complex normed space;
- odd bijections preserving $\perp$ and $P(L)$ the projection poset of a Hermitian space.

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