Loop Lemma in MV-algebra Pastings

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A method of a construction of quantum logics (orthomodular posets and orthomodular lattices) making use of the pasting of Boolean algebras was originally suggested by Greechie in 1971 [4]. These quantum logics are called *Greechie logics*. In Greechie logics, Boolean algebras generate blocks with the intersection of each pair of blocks containing at most one atom. Such system of Boolean algebras is called *almost disjoint*. One of useful tools in order to construct interesting orthomodular posets and orthomodular lattices is Greechie's Loop Lemma. Loop Lemma gives the necessary and sufficient conditions for a Boolean algebra pasting to be an orthomodular lattice.

Later the method of the pasting of Boolean algebras was generalized by Dichtl [3]. Dichtl has succeeded in obtaining characterizations of orthomodular posets and orthomodular lattices under assumptions more general than those of the Loop Lemma.

Riečanová [5] proved that every difference lattice is a set-theoretical union of maximal sub-D-lattices of pairwise compatible elements, i. e. maximal sub-MV-algebras, called *blocks*.

In [1] was suggested a method of a construction of difference posets [2] by the MV-algebra pasting. In the present paper we give some re-formulations of the basic notions introduced in [1] and we give a considerable generalization of the Loop Lemma.

References

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