Entangled states for compound state property systems

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We put forward a new axiomatization for standard quantum mechanics, starting with the basic notion of 'state property system' [1, 2] in which superposition and entanglement are introduced in an operational way. One of the hard problems in the foundations of quantum mechanics is that all type of product constructions on the level of the quantum logic structure give rise to a situation where the joint quantum entity only has product states of the subentities [3, 4, 5, 6]. On the level of the Hilbert space, the joint system of two quantum systems is described by means of the tensor product of the Hilbert spaces of the subsystems, and in this case there is an abundance of non-product states, giving rise to the well known phenomenon of quantum entanglement. We introduce the concept of entangled states for compound state property systems operationally, without the a priori assumption of a linear structure on the state space. We define 'pure' states of a state property system as primary objects by their 'state determination experiments' and the set of states by set-theoretic functions over the set of pure states. The set of pure states of a system composed of two subsystems is defined by taking the Cartesian product of the sets of pure states of the subsystems. The set of states of the compound system is defined by taking subsets over the pure states, introducing non-product states in a natural way. After formalizing state transition due to measurement, we derive an operational definition of entanglement. We illustrate our approach on a macroscopic model of a compound system in an entangled state [7] and show how Bell-inequalities [8, 9] are violated for a suitably chosen set of *deterministic* experiments.

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