Topological study of contextuality in quantum mechanics

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Abstract

Kochen-Specker (KS) theorem rules out the non-contextual assignment of values to physical magnitudes [1]. We give a topological version of this theorem and study its consequences when the discourse is enriched with modal operators.

We consider the family $\mathcal{W}$ of all Boolean subalgebras of the lattice of closed subspaces of the Hilbert space of the (pure) states of a quantum system ordered by inclusion and a topological space E over it whose canonical base is given by principal decreasing sets of subalgebras. It may be proved that the map $p : E \to \mathcal{W}$ such that $(W, f) \mapsto W$ is a sheaf over $\mathcal{W}$, the *spectral sheaf* [2]. From the physical perspective, we may say that the spectral sheaf takes into account the whole set of possible ways of assigning truth values to the propositions associated with the projectors of the spectral decomposition of operators representing physical magnitudes. The continuity of a local section of p guarantees that the truth value of a proposition is maintained when considering the inclusion of subalgebras. In these terms, KS theorem is equivalent to the fact that the spectral sheaf p has no global sections.

Finally, we include modalities. This enlargement of the structure allows to refer consistently to actual and possible properties of the system [3]. Following topological considerations as in the previous case, it may be proved that, even in the enriched structure, contextuality remains a central feature of quantum systems.

References

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