Transportation of probability measures

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Abstract

Classical random variable f transports the original probability p on events to the distribution p_f on the real Borel sets. In general, a measurable map transports each probability measure on a given σ-field of subsets of its domain to a probability measure on a given σ-field of subsets on its range; point probability measures (we identify each of them with the corresponding point) are transported to point probability measures. A statistical map (cf. [1], [3]) is a map of the probability measures on one σ-field of subsets to the probability measures on another σ-field of subsets such that a certain measurability condition is satisfied. It generalizes the classical measurable map and the novelty is that a point probability measure can be transported to a nondegenerated probability measure. We discuss diagrams describing the transportation of probability measures between three (and more) sets of probability measures. The results contribute to the understanding the nature of statistical maps and the difference between measurable maps and statistical maps. The category of D-posets (equivalently effect algebras) of fuzzy sets (cf. [2], [4]) plays a fundamental role.

References

[1] BUGAJSKI, S.: Statistical maps I. Basic properties. Math. Slovaca **51** (2001), 321–342.

[2] FRIČ, R.: Remarks on statistical maps. Tatra Mt. Math. Publ. **30** (2005), 21–34.

[3] GUDDER, S.: Fuzzy probability theory. Demonstratio Math. **31** (1998), 235–254.

[4] PAPČO, M.: On measurable spaces and measurable maps. Tatra Mountains Mathematical Publ. **28** (2004), 125–140.

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