Towards a bicategorical interpetation of Quantum Mechanics

Emmanuel Galatoulas
Department of Mathematics, University of Athens

galas@tee.gr

Applying conceptual and formal tools from category theory in the understanding of Quantum Mechanics (QM) is not considered anymore a curiosity amongst theoretical physicists. Nevertheless, it is not at all straightforward how should someone interpet QM systems and their interactions in a categorical framework.

A consistent and meaningful categorical interpetation has certainly to account for the most important features of QM, namely entanglement, non-locality and indeterminism, enabling us to understand also what is that makes measurement a "puzzle" in QM or why should probabilistic aspects play such a fundamental role. What seems to be a key ingredient in such an interpetation is precisely an appropriate categorical grasping both of a quantum "structure" and its evolution or variation. Essentially, what we are looking for is a categorical formalisation of the notion of a *quantum varying structure*.

In the categorical context, notions of a varying structure usually employ the well known *topos-theoretical* machinery. This represents already a first source of complication. Topos theoretical structures seem rather incompatible with what has come to be considered the standard "logic" of QM. The logical features of quantum variation appear to implicate quite a different kind of "space of variation" or "generalised space" than topoi.

Actually interpeting both quantum variation as well as abstract quantum mechanical "objects", strongly suggest that these objects would have to live not in an ordinary category but rather in one of a higher order, namely that of *the categories "enriched" over a background category.* In fact there are pretty good reasons that this background category would have itself to be not an ordinary (monoidal) category, but rather a bicategory, providing a much deeper and proper understanding of the internal variation within the "objects" themselves (in terms of "*processes between processes*"), as well as of their variation with respect to the enriched environment in which they are "embedded".

A reasonable candidate for such a categorical interpetation of QM exists under the quite suggestive name of a *quantaloid*, a bicategory enriched over the category of complete lattices. What we would like to stress here though is the possibility that different background bicategories may represent and provide relative degrees of "approximation" to quantum structure and variation. For instance, the partial order underlying a quantaloid, although an advantage when it comes to calculations, represents at the same time, due to the fact that it is "induced" on the categories enriched over the quantaloid, a conceptual oversimplification which is rather restrictive on the structural features of the quantum mechanical objects involved. Intuitively, we could say that a quantaloid determines a certain type of quantum structure upon which it does impose a corresponding restriction of "accessible structural detail" resulting into a "coarser" view of that structure. Consequently, if we were to get a subtler or more "detailed" account of the complexity of a quantum mechanical system, background bicategories endowed with analogously subtler and more complex types of order should have to be employed. The implications of such a "*relativisation*" to the very process of the variation or "development" of a quantum mechanical object can also be discussed.