Congruences on orthomodular implication algebras

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1 Abstract

The classical two-valued propositional logic has its algebraic counterpart in a Boolean algebra. If one considers the logical connective implication of the classical logic only then the clone generated by this connective is not the clone of all Boolean functions. The algebraic counterpart of the mentioned case is the so-called implication algebra introduced and treated by Abbott. Similarly, an algebraic counterpart of the fragment of intuitionistic logic containing only the intuitionistic implication and the constant 1 (which serves as a true value) was introduced by Henkin and treated by Diego under the name Hilbert algebra.

In some considerations concerning quantum mechanics another type of logic turned out to be suitable. Algebraic counterparts of these logics are either orthomodular lattices or the so-called ortomodular algebras or certain generalizations of Boolean rings. These logics are related to the Hilbert space logic of quantum mechanics. This motivated to study their implication reducts. The notion of an orthologic was introduced by J.C.Abbott in 70'ties by weakening the axioms and rules of inference of the classical propositional calculus. The resulting Lindenbaum-Tarski algebra generalized the notion of an implication algebra.

It turned out that there are two types of orthoimplication algebras, namely orthomodular implication algebras with or without the so called compatibility condinition, briefly OMIA's with or without (CC).

The aim of our talk is to describe congruences on OMIA's. We already know that both OMIA's with or without (CC) are join semilattices having sections (=principal filters) orthomodular lattices. It is also well known that in OML's, congruences are completely determined by their congruence kernels, i.e. classes of the form $[1]_\theta$. These are in a 1-1 corespondence with p-filters. We shall show that in compatible case, congruence kernels of OMIA's are just subsets being p-filters on sectins. We shall present also a solution for a non-compatible case, where the situation is more complicated.

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