On quantum statistical experiments and sufficiency.

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Abstract: We present an attempt to develop the theory of quantum statistical experiments, parallel to the classical theory [2]. The talk is based on joint work with Dénes Petz [1] and Madalin Guta.

By a quantum statistical experiment, we mean a couple $(\mathcal{M}, \mathcal{S})$, where $\mathcal{M}$ is a von Neumann algebra and $\mathcal{S} = \{\varphi_\theta, \theta \in \Theta\}$ is a set of normal states on $\mathcal{M}$, Θ is a parameter set. This can be interpreted as follows: the algebra $\mathcal{M}$ represents a quantum system in an unknown state φ and $\mathcal{S}$ represents our knowledge of φ. We perform an experiment to obtain information on the value of θ.

Let $\mathcal{N}$ be another von Neumann algebra, then a linear mapping $\alpha : \mathcal{N} \to \mathcal{M}$, which is 2-positive, unital and normal, is called a coarse-graining. The experiment $(\mathcal{N}, \mathcal{S} \circ \alpha)$, $\mathcal{S} \circ \alpha = \{\varphi_\theta \circ \alpha, \theta \in \Theta\}$ is called a randomization of $(\mathcal{M}, \mathcal{S})$. An example of this situation is a simple measurement, that is a POV measure E on a finite set X. Here, the coarse-graining $\alpha : \mathcal{C}(X) \to \mathcal{M}$ is given by

$$f \mapsto \sum_{x \in X} f(x) E(x)$$

and $\mathcal{S} \circ \alpha$ is the set of induced probability measures on X. Clearly, in general, some information on θ is lost by randomization. It can happen, however, that also $(\mathcal{M}, \mathcal{S})$ is a randomization of $(\mathcal{N}, \mathcal{S} \circ \alpha)$, then we say that α is a sufficient coarse-graining and the two experiments are statistically equivalent.

We present several equivalent characterizations of sufficiency of coarse - grainings. Moreover, inspired by the classical theory of experiments, we suggest a representation of equivalence classes of experiments. This allows to define a convex structure and a topology on the set of equivalence clases of experiments having the same parameter set. Some examples and applications are treated.

References

[1] A. Jenčová, D. Petz, Sufficiency in quantum statistical inference, *Commun. Math. Phys.*, in press,

[2] H. Strasser, *Mathematical theory of statistics. Statistical experiments and asymptotic decision theory*, Walter de Gruyter, Berlin, 1985.