Chains in D–posets

František KÔPKA
Department of Engineering Fundamentals
Faculty of Electrical Engineering University of Žilina
Workplace Liptovský Mikuláš
Slovakia
E-mail: kopka@lm.utc.sk

D-posets ([5]), or effect algebras ([3]), are the joint generalizations of the classical algebraic structures, which are the algebraic models of quantum mechanics and the fuzzy sets theory systems. In these systems the important role is played by the Boolean D-posets (MV-algebras) ([2, 1]), which are the non–commutative generalizations of the Boolean algebras. The Boolean D-posets simultaneously represent the compatible sets in the D–posets ([4]). The basis for operations of observables on D-posets is provided by, so called, representation theorem ([5]), which is based on the properties of the ranges of chains.

In this contribution the properties of D-posets in relation to the chains in them are studied. For example, the following assertions are valid.

1. If $\mathcal{M}$ is a maximal chain in an finite Boolean D–poset $\mathcal{P}$, then for every element $a \in \mathcal{P}$ there exists a decreasing sequence $(c_k)_{k=1}^n \subseteq \mathcal{M}$, such that $a = c_1 - (c_2 - (c_3 - \cdots - (c_{n-1} - c_n)))$.

2. The D-poset created by the differences of the elements of a chain with the greatest and the smallest elements of given finite D-poset, is a Boolean D-poset.

These properties of D–posets can be used with much benefit in the probability theory on D-posets, mainly in the assignment of the distribution of probability.

References

[1] CHANG, C. C.: Algebraic analysis of many–valued logics. Transaction of the American Mathematical Society, 88 (1958), 467– 490.

[2] CHOVANEC, F., KÔPKA, F.: Boolean D-posets, Tatra Mountains Mathematical Publications 10, (1997), 183–197.

[3] FOULIS, D. J., BENNET, M. K.: Effect algebras and unsharp quantum logics, Found. Phys. 19 (1994), 1325–1346.

[4] KÔPKA, F.: Compatibility in D–posets. Inter. J. Theor. Phys., 34 (1995), No. 8, 1525–1531.

[5] KÔPKA, F., CHOVANEC, F.: D–posets, Math. Slovaca 44 (1994), 21–34.