Orthocomplemented lattices with a symmetric difference

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Modelling an abstract version of the set-theoretic operation of symmetric difference in the theory of orthocomplemented lattices, we introduce a class of orthocomplemented difference lattices.

Definition 1 Let $L = (X, \wedge, \vee, ^\perp, \mathbf{0}, \mathbf{1}, \triangle)$, where $(X, \wedge, \vee, ^\perp, \mathbf{0}, \mathbf{1})$ is an orthocomplemented lattice and $\triangle : X^2 \to X$ is a binary operation. Then L is said to be an *orthocomplemented difference lattice* (abbr., an ODL) if the following three properties are fulfilled for any $x, y, z \in X$:
(D_1) $x \triangle (y \triangle z) = (x \triangle y) \triangle z$,
(D_2) $x \triangle \mathbf{1} = x^\perp$, $\mathbf{1} \triangle x = x^\perp$,
(D_3) $x \triangle y \leq x \vee y$.

In the talk we shall be discussing algebraic properties of ODLs. Let us formulate three typical results.

Theorem 1 Let $L = (X, \wedge, \vee, ^\perp, \mathbf{0}, \mathbf{1}, \triangle)$ be an ODL. Then $(X, \wedge, \vee, ^\perp, \mathbf{0}, \mathbf{1})$ is an OML. Moreover, if L is finite, then $|L| = 2^n$.

Theorem 2 An ODL may not allow for a set representation.

Theorem 3 The class of all ODL's which allow for a set representation forms a variety of algebras.

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