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LOGICS of J-PROJECTIONS of TYPE (B).

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Let H be a complex Hilbert space. Fix a selfadjoint symmetry operator J ($J = J^* = J^{-1}$, $J \neq \pm I$). The operator $A^{\#} := JA^*J$ is called to be *J-adjoint* to A. Note, A is J-self-adjoint (J-positive, J-negative) iff JA - self-adjoint (positive, negative, respectively). Let $\mathcal{P} := \{p \in B(H) : p^2 = p = p^{\#}\}$. Any $p \in \mathcal{P}$ is said to be *J-projection*. Let $\mathcal{P}^+$ ($\mathcal{P}^-$) be the set of all J-positive (J-negative, respectively) J-projections. Assume that there are orthogonal projections P, Q such that $P + Q = I$, $Q^{\#} = P$. Put $\mathcal{J} := P - Q$. A von Neumann algebra $\mathcal{A}$ in H is said to be a *von Neumann J-algebra of type* (B) if $A \in \mathcal{A}$ implies $A^{\#} \in \mathcal{A}$ and if P, Q are central projections in $\mathcal{A}$. The logic $\mathcal{P}^{\mathcal{A}} := \mathcal{A} \cap \mathcal{P}$ is called to be *the logic of type* (B) and any J-projection from $\mathcal{P}^{\mathcal{A}}$ is said to be a *J-projection of type* (B). A J-projection $R \in \mathcal{P}^+$ is said to be a *generator* to a J-projection $\mathcal{R}$ if $\mathcal{R} = R + \mathcal{J}R\mathcal{J}$ and subspaces RH, $\mathcal{J}RH$ are mutually orthogonal. Let F^+ be the cover projection of P^+RP^+. Here $P^+ := (1/2)(I + J)$ and $P^- := I - P^+$.

The aim of the report J-projections from $\mathcal{A}$ are studied for the first time.

Let $\mathcal{A}$ be a von Neumann J-algebra of type (B). Then $\mathcal{P}^{\mathcal{A}} \cap \mathcal{P}^+ = \{0\} = \mathcal{P}^{\mathcal{A}} \cap \mathcal{P}^-$. By analogy with $\mathcal{P}$ *if* $\dim H = \infty$ *then the logic* $\mathcal{P}^{\mathcal{A}}$ *is not a σ-logic. The function $\sigma(P) := \mathcal{J}P\mathcal{J}$ is an outomorphism of $\mathcal{P}$, for wich $\mathcal{J}\mathcal{P}^+\mathcal{J} = \mathcal{P}^-$. In particular, $\mathcal{J}P^+\mathcal{J} = P^-$. Let $\mathcal{L}_P^{\mathcal{A}}$ be the set of all projections from $P\mathcal{A}P$. Then $\mathcal{P}^{\mathcal{A}} = \{q + Jq^*J : q \in \mathcal{L}_P^{\mathcal{A}}\}$. Let q, $p \in \mathcal{P}$ and let q be a J-projection of type (B). Then $p \leq q$ iff $\mathcal{J}p\mathcal{J} \leq q$.*

Theorem 1. *Let R be a J-projection. The following conditions are equivalent:* 1) *R has type (B);* 2) *there exists a (unique!) generator to R;* 3) *$R = \mathcal{J}R\mathcal{J}$;* 4) *$QRP = PRQ = 0$.*

Remark 1. 1) *From $R \in \mathcal{P}^+$ ($\in \mathcal{P}^-$) and $PRQ = 0$ ($QRP = 0$) it is $R = 0$ hold.* 2) *Let $\dim H \geq 4$. Then there is $R \in \mathcal{P}$ such that $PRQ = 0$, $QRP \neq 0$.* 3) *If $R \in \mathcal{P}$ and $PRQ = 0$, then PRP and QRQ are projections.*

Theorem 2. *Let $R \in \mathcal{P}^+$ and let $P^-RP^+ = U|P^-RP^+|$ be the polar decomposition for P^-RP^+. Then R is a generator iff subspaces $\mathcal{J}F^+H$ and UH are mutually orthogonal.*

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