Planck′s constant in the light of quantum logic

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The goal of quantum logic is the "bottom-top" reconstruction of Hilbert lattices and of quantum mechanics in Hilbert space. Starting from a weak quantum ontology of sharp or unsharp properties, a long sequence of arguments leads to a quantum language, a quantum logic, and to to a complete, irreducible, atomic orthomodular lattice, which satisfies the covering law. Finally, by means of the theorems of Piron and of Solèr one obtains the three classical Hilbert spaces and in particular the complex numbers Hilbert space of quantum mechanics.

However, this abstract and very general framework of quantum mechanics does not yet contain Planck′s constant $\hbar$, which is often considered to indicate the border line between the quantum world and the classical world. We discuss the reason for this deficiency and argue that at the present stage of the reconstruction the theory does not grasp the classical reality at all. Hence, there is no interface between classical and quantum physics that could provide the constant $\hbar$. This characteristic number of quantum mechanics can be obtained only, if the abstract and "empty" theory is applied to real entities and if the conceptual framework is extended by incorporating concepts that are usually considered as classical notions.

We proceed in two steps: Introducing the concepts of localizability and homogeneity of space we can define objects by means of symmetry groups and systems of imprimitivity. For elementary systems, the irreducible representations of the Galileo group turn out to be projective and contain a real parameter z, that is not determined by the theory. The physical meaning of z becomes obvious if we consider the dynamics of elementary particles in more detail. From commutation relations and the equations of motion it follows that $z = m/\hbar$, where m is the inertial mass of the particle and $\hbar$ a universal constant that must be determined by experiments. Although there is no obvious relation to classical physics, $\hbar$ has a meaning within the context of quantum mechanics and can be determined experimentally, for instance by the smallest possible uncertainty of jointly measured unsharp complementary observables. - Possible relations to classical physics are briefly discussed.