Pastings and related constructions of quantum structures

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From the early times of quantum structures, pasting belonged to techniques which enriched the theory by numerous counterexamples, as well as positive results [4, 5]. The original intention was to paste Boolean algebras together in order to obtain orthomodular posets or lattices. The possibilities of this tool were clarified in [3, 8]. It was generalized to pastings of orthomodular posets in [10]. Later on, more general quantum structures were introduced and the pasting was generalized to orthoalgebras [6, 9]. Here the necessary and sufficient conditions are even simpler.

The recent boom of more general quantum structures – effect algebras – has been followed by numerous attempts to describe them also as pastings of simpler structures [1, 2, 7, 11], mostly MV-algebras. These constructions allowed to obtain only a specific class of *homogeneous* effect algebras, including lattice effect algebras. A different pasting allows to generate also some non-homogenoeous effect algebras, but it is known that not all effect algebras can be obtained this way.

As a by-product of the main line of research, several other constructions of quantum logics using pastings have been derived [9, 10].

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