Semigroups of Binary Systems as Generalizations of Dynamical Systems

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Given binary systems $(\mathbf{X}, *)$ and $(\mathbf{X}, \circ)$, a product (composition) is defined by

$$(\mathbf{X}, *)\triangle(\mathbf{X}, \circ) = (\mathbf{X}, \triangle),$$

if for all $x, y \in \mathbf{X}$,

$$x \triangle y = (x * y) \circ (y * x).$$

If $f : \mathbf{X} \to \mathbf{X}$ is a function, then $x * y = f(x)$ defines a binary system $(\mathbf{X}, *, f)$ such that

$$(\mathbf{X}, *, f)\triangle(\mathbf{X}, \circ, g) = (\mathbf{X}, \triangle, f \circ g).$$

The operation $\triangle$ is associative and $\Phi(f) = (\mathbf{X}, *, f)$ defines an injective homomorphism of the semigroup $(\mathbf{X}^{\mathbf{X}}, \circ)$ to the semigroup $(\mathrm{Bin}(\mathbf{X}), \triangle)$, where $\mathrm{Bin}(\mathbf{X})$ consists of all binary systems $(\mathbf{X}, *)$ defined on $\mathbf{X}$. One may now consider special subsemigroups of $(\mathrm{Bin}(\mathbf{X}), \triangle)$ and analyze them analogously to the analysis of iterated function systems for example, including notions of ergodicity. Among systems which yield interesting questions and sometimes interesting answers as well are groups, D-posets, BCK-algebras, and a host of other systems, some of which are new but natural in the context. If $\mathbf{X}$, $\mathbf{X}^2$, $\mathbf{X}^3, \ldots$ permit notions of integration, then expressions $\int\int_D (x * y)\, dA$ and the like show up naturally as objects of study also.