

**AN EXAMPLE OF AN ATOMIC MV ALGEBRA  
WITH NON-ATOMIC SUBALGEBRA  
OF SHARP OR CENTRAL ELEMENTS**

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A set  $E$  equipped with a partial, commutative and associative operation  $\oplus$ , containing elements 0 and 1, in which the existence of a unique inverse element is guaranteed, and  $a \oplus 1$  is admitted only if  $a = 0$  is well known as *effect algebra* [1]. It was introduced by Foulis and Bennet (1994) and simultaneously by Kopka and Chovanec as a D-poset [1]. It can be equipped with partial order  $\leq$  and as such it can form a lattice, called a *lattice effect algebra* [1]. This structure generalizes both orthomodular lattices and MV algebras and is applied as a carrier of probability of unsharp or fuzzy events.

In connection with existence of states on lattice effect algebra properties of the subalgebra  $S(E)$  of sharp elements and the subalgebra  $C(E)$  of central elements are studied. In particular, Z. Riečanová in [2] formulated a sufficient condition under which  $S(E)$  and  $C(E)$  in an atomic Archimedean lattice effect algebra are atomic. Z. Riečanová in [3] described the structure of an Archimedean atomic lattice effect algebras in which  $C(E) = S(E)$ . In [4], as well as at several presentations, Z. Riečanová formulated a problem whether there exists an atomic lattice effect algebra with nonatomic subalgebra  $S(E)$  of sharp elements. We have found an affirmative answer to this question. In particular, we have found an example of atomic (non-Archimedean) MV algebra with nonatomic subalgebra of  $S(E) = C(E)$  of central or sharp elements.

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