

Projective Quantales: A General View

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Abstract.

This work is intended as a step towards the development of the non-commutative topology using the approach of the theory of (involutive) quantales developed by C.J. Mulvey, J.W. Pelletier and J. Rosický and others. Quantales are certain partially ordered algebraic structures which generalize frames (pointless topologies) as well as various lattices of multiplicative ideals from ring theory and functional analysis (C^* -algebras, von Neumann algebras). The relations on a set, under the operation of relational composition, also form an involutive quantale.

This note deals with a very general form of projectivity in the category of unital quantales.

Specifically, the setting here is the category Mon of monoids in which we consider subcategories K containing the category $UQuant$ of unital quantales reflectively, subject to a very simple natural condition. The projectivity in question is then taken relative to the onto unital quantale homomorphisms $h : L \rightarrow M$ for which the right adjoint $h_* : M \rightarrow L$ ($h(a) \leq b$ iff $a \leq h_*(b)$) belongs to K , referred to as K -flat projectivity introduced by Banaschewski for frames.

We establish both external and internal characterizations of K -flat projectivity. As a by-product of our investigation we solve the problem whether there is a weakly $*$ -stable complete distributive lattice which is not $*$ -stable affirmatively.