A generalization of local fuzzy and quantum structures

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A *bounded residuated lattice-ordered monoid* (*$R\ell$-monoid*) is an algebra $M = (M; \odot, \vee, \wedge, \to, \rightsquigarrow, 0, 1)$ of type $\langle 2, 2, 2, 2, 2, 0, 0, \rangle$ satisfying the following conditions:

(i) $(M;\ \odot, 1)$ is a monoid (need not be commutative).

(ii) $(M;\ \vee, \wedge, 0, 1)$ is a bounded lattice.

(iii) $x \odot y \leq z$ iff $x \leq y \to z$ iff $y \leq x \rightsquigarrow z$ for any $x,\ y \in M$.

(iv) $(x \to y) \odot x = x \wedge y = y \odot (y \rightsquigarrow x)$.

Bounded $R\ell$-monoids form a variety of algebras of the indicated type and they can be also recognized as bounded integral generalized BL-algebras.

The class of bounded residuated lattice ordered monoids ($R\ell$-monoids) contains as proper subclasses the class of pseudo BL-algebras (and consequently those of pseudo MV-algebras, BL-algebras and MV-algebras) and of Heyting algebras.

Local MV-algebras, that means MV-algebras having a unique maximal ideal, were studied by Belluce, DiNola and Lettieri. More generally, local BL-algebras were investigated by Turunen and Sessa and local bounded commutative $R\ell$-monoids by Rachůnek and Šalounová. Furthermore, the notions of local algebras were generalized and studied also for pseudo MV-algebras by Leustean and for pseudo BL-algebras by Georgescu and Leustean.

We define and study local bounded $R\ell$-monoids which need not be commutative. Many of results are obtained for the variety of good normal $R\ell$-monoids that contains both the variety of good pseudo BL-algebras and that of Heyting algebras. Moreover, we introduce perfect $R\ell$-monoids and characterize them by means of the filter of elements of infinite order.