Reasoning about Compound Quantum Systems: How Dynamic-Epistemic Operators Capture Entanglement

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I will present recent joint work with A. Baltag on a dynamic logic of entanglement. This work improves on the one in [2], and extends the setting in [1] to the case of compound systems, by introducing *dynamic-epistemic logical operators* that express *temporal and spatial physical features of a compound system* $\Sigma = \bigotimes_{i \in I} \Sigma_i$; in particular, dynamic operators capture the *potential actions* that may affect the system, while the "epistemic" operators capture the notion of *subsystem* of Σ. In Hilbert spaces, both operators can be interpreted as "Kripke" modalities associated to binary relations on Σ; the relations corresponding to the dynamic modalities are induced by linear maps (projectors or unitary maps) describing the potential changes of the system; the relations corresponding to the "epistemic" modalities can be thought of as "indistinguishability" relations between global states of Σ: they express the fact that these states are "locally indistinguishable" from the point of view of some given subsystem Σ_i; in other words, *for all that this subsystem "knows", the two global states of* Σ *are the same.* This is what we call the "epistemic" aspect (in a very abstract sense) of our spatial operators, which should not be confused with the common usage of "knowledge" (in terms of actual observers) in the discussions on the interpretations of QM.

In our logic, the dynamic-epistemic modalities are taken as *basic*, and they are used to *define* the notions of a state change and of a subsystem. Using these operators, one can specify interesting physical properties, such as "separation" and "entanglement", and can characterize various special states, e.g. the Bell states, and the so-called "generalized (k-qubit) Bell states". Moreover, this logic setting can be used to capture various quantum-logical gates and to reason about many protocols used in Quantum Computation (e.g. teleportation, super-dense coding, entanglement swapping, quantum secret sharing etc.)

Our approach builds further on the dynamic-logical setting in [1], itself based on the older results (due to Piron, Soler, Mayet etc.) on the Hilbert-complete axiomatizations of algebraic quantum logic. By moving to a dynamic-logical setting, in which physical actions such as measurements and unitary evolutions are logically represented, one gains the necessary expressive power to overcome the old difficulties to prove completeness for quantum logic. Up till now, there are no known extensions of these completeness results to the compound case. It is our hope that our dynamic-epistemic setting opens a new way to approach the subject, and that moreover (in the view of the above applications) it may constitute a useful bridge between traditional Quantum Logic and the recent advances in Quantum Computation and Quantum Information.

[1] A. Baltag and S. Smets, "Complete Axiomatizations for Quantum Actions", International Journal of Theoretical Physics, to appear. Available at http://www.vub.ac.be/CLWF/SS/IQSA.pdf
[2] A. Baltag and S. Smets, "LQP: The Dynamic Logic of Quantum Information", to appear in Math. Struct. in Comp. Science, vol 16, pp. 1-35, 2006