The General Structure of Group Representations and Imprimitivity Systems

— abstract IQSA 2006 —

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The history of physics indicates that symmetry considerations play an important role in our understanding of life, the universe and everything. It is therefore important to incorporate the representation of various symmetry groups in our descriptions of physical reality. Given the recent and subrecent proliferation of mathematical structures used in our descriptions, it becomes important to recognize the general structural properties required of such representations. Accordingly, I share the view of Foulis & Wilce (2000) that it is urgent to develop a more general representation theory in this broader framework. The language *par excellence* to do so is category theory, since it is, *inter alia*, concerned with the elucidation of common constructions in different mathematical frameworks. The two archetypical descriptions of physical systems — classical systems and quantum systems — live essentially in $\underline{\mathrm{Set}}$-like categories and $\underline{\mathrm{Vect}}_{\mathbb{F}}$-like categories, respectively. The corresponding constructions lead to a developed theory of group actions on sets and related constructs on the one hand, and (unitary) group representations on Hilbert spaces and related structures on the other. I analyze some of the common structural ideas in these two types of theories from a general (categorical) perspective, emphasizing the unity of the underlying ideas, using appropriate categories of functor categories. In slightly more detail, a group G acting on some category $\underline{\mathrm{C}}$ corresponds with an elementary functor category $\mathbf{Fun}(\underline{\mathrm{G}}, \underline{\mathrm{C}})$, and the subgroups of the group G lead to a corresponding network of forgetful functors and their adjoints (whenever they exist) between the corresponding functor categories. I consider the relation with the notion of imprimitivity systems and the theory of observables. Time permitting, I investigate some of the general structural

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properties required of abstract categories that allow for a more developed theory of groups acting on them.

References

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