Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$412 = 1 \cdot 335 + 77$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$412 = 1 \cdot \underline{335} + \underline{77}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned} 412 &= 1 \cdot \underline{335} + \underline{77} \\ 335 &= 4 \cdot 77 + 27 \end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$412 = 1 \cdot \underline{335} + \underline{77}$$

$$335 = 4 \cdot \underline{77} + \underline{27}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned} 412 &= 1 \cdot \underline{335} + \underline{77} \\ 335 &= 4 \cdot \underline{77} + \underline{27} \\ 77 &= 2 \cdot 27 + 23 \end{aligned}$$

Rozšírený Euklidov algoritmus

$(412, 335) = ?$

$$\begin{aligned} 412 &= 1 \cdot \underline{335} + \underline{77} \\ 335 &= 4 \cdot \underline{77} + \underline{27} \\ 77 &= 2 \cdot \underline{27} + \underline{23} \end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot 23 + 4
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4}
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot 4 + 3
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3}
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot 3 + \underline{1}
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \underline{1}
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = ?$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \underline{1} \\
3 &= 3 \cdot 1 + 0
\end{aligned}$$

Rozšírený Euklidov algoritmus

$(412, 335) =?$

$$
\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \underline{1} \quad \leftarrow \\
3 &= 3 \cdot 1 + 0
\end{aligned}
$$

Máme nsd
posledný nenulový zvyšok

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \boxed{1} \\
3 &= 3 \cdot 1 + 0
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \boxed{1} \rightarrow 1 \quad \text{-1} \\
3 &= 3 \cdot 1 + 0
\end{aligned}
$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$\begin{aligned}
412 &= 1 \cdot \underline{335} + \underline{77} \\
335 &= 4 \cdot \underline{77} + \underline{27} \\
77 &= 2 \cdot \underline{27} + \underline{23} \\
27 &= 1 \cdot \underline{23} + \underline{4} \\
23 &= 5 \cdot \underline{4} + \underline{3} \\
4 &= 1 \cdot \underline{3} + \boxed{1} \quad \rightarrow \quad 1 \quad \text{-1} \\
3 &= 3 \cdot 1 + 0
\end{aligned}$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcl}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} \\
3 & = & 3 & \cdot & 1 & + & 0
\end{array}
$$

$\rightarrow$

-1	1 - (-1)· 5
1	-1

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcl}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} \\
3 & = & 3 & \cdot & 1 & + & 0
\end{array}
$$

	-1	6
$\longrightarrow$	1	-1

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcrcl}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & & & \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & & & \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & & & \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & \text{-1 - 6} \cdot 1 & \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & \text{-1} & 6 & \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & \text{-1} & \\
3 & = & 3 & \cdot & 1 & + & 0 & & & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcrr}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & & \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & & \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & & \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & -7 \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & -1 & 6 \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & -1 \\
3 & = & 3 & \cdot & 1 & + & 0 & & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcll}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & & \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & & \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & -7 & 6 - (-7)\cdot 2 \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & -7 \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & -1 & 6 \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & -1 \\
3 & = & 3 & \cdot & 1 & + & 0 & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcrr}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & & \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & & \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & -7 & 20 \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & -7 \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & -1 & 6 \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & -1 \\
3 & = & 3 & \cdot & 1 & + & 0 & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcl}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} \\
3 & = & 3 & \cdot & 1 & + & 0
\end{array}
$$

$\longrightarrow$

20	-7 - 20 · 4
-7	20
6	-7
-1	6
1	-1

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcrr}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & & \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & 20 & -87 \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & -7 & 20 \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & -7 \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & -1 & 6 \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & -1 \\
3 & = & 3 & \cdot & 1 & + & 0 & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$(412, 335) = 1$

412	$=$	1	$\cdot$	$\underline{335}$	$+$	$\underline{77}$		-87	20 - (-87)· 1
335	$=$	4	$\cdot$	$\underline{77}$	$+$	$\underline{27}$		20	-87
77	$=$	2	$\cdot$	$\underline{27}$	$+$	$\underline{23}$		-7	20
27	$=$	1	$\cdot$	$\underline{23}$	$+$	$\underline{4}$		6	-7
23	$=$	5	$\cdot$	$\underline{4}$	$+$	$\underline{3}$		-1	6
4	$=$	1	$\cdot$	$\underline{3}$	$+$	$\boxed{1}$	$\rightarrow$	1	-1
3	$=$	3	$\cdot$	1	$+$	0			

Rozšírený Euklidov algoritmus

$$(412, 335) = 1$$

$$
\begin{array}{rcrcrcrcrr}
412 & = & 1 & \cdot & \underline{335} & + & \underline{77} & & -87 & 107 \\
335 & = & 4 & \cdot & \underline{77} & + & \underline{27} & & 20 & -87 \\
77 & = & 2 & \cdot & \underline{27} & + & \underline{23} & & -7 & 20 \\
27 & = & 1 & \cdot & \underline{23} & + & \underline{4} & & 6 & -7 \\
23 & = & 5 & \cdot & \underline{4} & + & \underline{3} & & -1 & 6 \\
4 & = & 1 & \cdot & \underline{3} & + & \boxed{1} & \rightarrow & 1 & -1 \\
3 & = & 3 & \cdot & 1 & + & 0 & &
\end{array}
$$

Rozšírený Euklidov algoritmus

$(412, 335) = 1$

412	$=$	1	$\cdot$	$\underline{335}$	$+$	$\underline{77}$		$\boxed{-87}$	$\boxed{107}$
335	$=$	4	$\cdot$	$\underline{77}$	$+$	$\underline{27}$		20	-87
77	$=$	2	$\cdot$	$\underline{27}$	$+$	$\underline{23}$		-7	20
27	$=$	1	$\cdot$	$\underline{23}$	$+$	$\underline{4}$		6	-7
23	$=$	5	$\cdot$	$\underline{4}$	$+$	$\underline{3}$		-1	6
4	$=$	1	$\cdot$	$\underline{3}$	$+$	$\boxed{1}$	$\rightarrow$	1	-1
3	$=$	3	$\cdot$	1	$+$	0			

Výsledok: $412 \cdot (-87) + 335 \cdot 107 = 1 = (412, 335)$