1. $\int (3x-11)^9 \mathrm{d}x = \left| \begin{matrix} t = 3x-11 \\ \mathrm{d}t = 3\,\mathrm{d}x \end{matrix} \right| = \frac{1}{3}\int t^9 \mathrm{d}t = \frac{1}{3}\frac{t^{10}}{10} + C$

$$\boxed{\frac{1}{30}(3x-11)^{10} + C}$$

2. $\int x(a+bx)^n \mathrm{d}x = \left| \begin{matrix} t = a+bx & \mathrm{d}x = \frac{\mathrm{d}t}{b} \\ \mathrm{d}t = b \cdot \mathrm{d}x & x = \frac{t-a}{b} \end{matrix} \right| = \int \frac{t-a}{b} t^n \frac{\mathrm{d}t}{b} = \frac{1}{b^2}\int t^{n+1}\mathrm{d}t - \frac{a}{b^2}\int t^n \mathrm{d}t =$

$= \frac{1}{b^2} \cdot \frac{1}{n+2} t^{n+2} - \frac{a}{b^2} \cdot \frac{1}{n+1} \cdot t^{n+1} + C$

$$\boxed{\frac{1}{b^2}\left(\frac{(a+bx)^{n+2}}{n+2} - a \cdot \frac{(a+bx)^{n+1}}{n+1}\right) + C}$$

3. $\int \frac{x}{(a+bx)^n}\mathrm{d}x = \left| \begin{matrix} t = a+bx & x = \frac{t-a}{b} \\ \mathrm{d}t = b\,\mathrm{d}x \end{matrix} \right| = \int \frac{t-a}{b}\frac{1}{t^n}\frac{\mathrm{d}t}{b} = \frac{1}{b^2}\int \frac{t-a}{t^n}\,dt = \frac{1}{b^2}\left(\int t^{1-n}\,\mathrm{d}t - a\int t^{-n}\,\mathrm{d}t\right) =$

$= \frac{1}{b^2}\left(\frac{t^{2-n}}{2-n} - a\frac{t^{1-n}}{1-n}\right) + C$

$$\boxed{\frac{1}{b^2}\left(\frac{(a+bx)^{2-n}}{2-n} - a\frac{(a+bx)^{1-n}}{1-n}\right) + C}$$

4. $\int \frac{x^2}{(a+bx)^n}\mathrm{d}x = \left| \begin{matrix} t = a+bx & x = \frac{t-a}{b} \\ \mathrm{d}t = b\,\mathrm{d}x \end{matrix} \right| = \int (\frac{t-a}{b})^2 \frac{1}{t^n}\frac{\mathrm{d}t}{b} = \frac{1}{b^3}\int \frac{(t-a)^2}{t^n}\mathrm{d}t = \frac{1}{b^3}\int \frac{t^2-2at+a^2}{t^n} =$

$= \frac{1}{b^3}\left(\int t^{2-n}\mathrm{d}t - 2a\int t^{1-n}\mathrm{d}t + a^2\int t^{-n}\mathrm{d}t\right) = \frac{1}{b^3}\left(\frac{t^{3-n}}{3-n} - 2a\frac{t^{2-n}}{2-n} + a^2\frac{t^{1-n}}{1-n}\right) + C$

$$\boxed{\frac{1}{b^3}\left(\frac{(a+bx)^{3-n}}{3-n} - 2a\frac{(a+bx)^{2-n}}{2-n} + a^2\frac{(a+bx)^{1-n}}{1-n}\right) + C}$$

5. $\int \frac{\mathrm{d}x}{9x^2+4} = \int \frac{\mathrm{d}x}{4(\frac{9}{4}x^2+1)} = \frac{1}{4}\int \frac{\mathrm{d}x}{1+(\frac{3}{2}x)^2} = \left| \begin{matrix} t = \frac{3}{2}x \\ \mathrm{d}t = \frac{3}{2}\mathrm{d}x \end{matrix} \right| = \frac{2}{3}\frac{1}{4}\int \frac{\mathrm{d}t}{1+t^2} = \frac{1}{6}\mathrm{arctg}\,t + C$

$$\boxed{\frac{1}{6}\mathrm{arctg}\,\frac{3}{2}x + C}$$

6. $\int \frac{\mathrm{d}x}{x^2+5x+11} = \int \frac{\mathrm{d}x}{(x+\frac{5}{2})^2+11-(\frac{5}{2})^2} = \left| \begin{matrix} t = x+\frac{5}{2} \\ \mathrm{d}t = \mathrm{d}x \end{matrix} \right| = \int \frac{\mathrm{d}t}{t^2+\frac{19}{4}} = \frac{4}{19}\int \frac{\mathrm{d}t}{(\frac{2}{\sqrt{19}}t)^2+1} = \left| \begin{matrix} s = \frac{2}{\sqrt{19}}t \\ \mathrm{d}s = \frac{2}{\sqrt{19}}\mathrm{d}t \end{matrix} \right| =$

$= \frac{4}{19}\frac{\sqrt{19}}{2}\int \frac{\mathrm{d}s}{1+s^2} = \frac{2}{\sqrt{19}}\mathrm{arctg}\,s + C = \frac{2}{\sqrt{19}}\mathrm{arctg}\,\frac{2}{\sqrt{19}}t + C$

$$\boxed{\frac{2}{\sqrt{19}}\mathrm{arctg}\,\frac{x+5}{\sqrt{19}} + C}$$

7. $\int \frac{\mathrm{d}x}{\sqrt{x^2+4x+5}} = \int \frac{\mathrm{d}x}{\sqrt{(x+\frac{4}{2})^2+5-(\frac{4}{2})^2}} = \int \frac{\mathrm{d}x}{\sqrt{(x+2)^2+1}} = \left| \begin{matrix} t = x+2 \\ \mathrm{d}t = \mathrm{d}x \end{matrix} \right| = \int \frac{\mathrm{d}t}{\sqrt{1+t^2}} = \ln(t+\sqrt{t^2+1}) + C$

$$\boxed{\ln(x+2+\sqrt{x^2+4x+5}) + C}$$

8. $\int \frac{\mathrm{d}x}{\sqrt{1-3x^2}} = \int \frac{\mathrm{d}x}{\sqrt{1-(\sqrt{3}x)^2}} = \left| \begin{matrix} t = \sqrt{3}x \\ \mathrm{d}t = \sqrt{3}\,\mathrm{d}x \end{matrix} \right| = \frac{1}{\sqrt{3}}\int \frac{\mathrm{d}t}{\sqrt{1-t^2}} = \frac{1}{\sqrt{3}}\arcsin t + C$

$$\boxed{\frac{1}{\sqrt{3}}\arcsin\sqrt{3}x + C}$$

9. $\int \frac{\mathrm{d}x}{(x-\sqrt{x^2-1})^2} = \int \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\frac{\mathrm{d}x}{(x-\sqrt{x^2-1})^2} = \int \frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}\mathrm{d}x = \int (x+\sqrt{x^2-1})^2\mathrm{d}x =$

$= \int (2x^2-1+2x\sqrt{x^2-1})\mathrm{d}x = \frac{2}{3}x^3 - x + \int 2x\sqrt{x^2-1}\mathrm{d}x = \left| \begin{matrix} t = x^2 \\ \mathrm{d}t = 2x\,\mathrm{d}x \end{matrix} \right| = \frac{2}{3}x^3 - x + \int \sqrt{t-1}\mathrm{d}t =$

$= \frac{2}{3}x^3 - x + \frac{2}{3}(t-1)^{\frac{3}{2}} + C$

$$\boxed{\frac{2}{3}x^3 - x + \frac{2}{3}(x^2-1)^{\frac{3}{2}} + C}$$

10. $\int \frac{e^x}{4+e^x}\mathrm{d}x = \left| \begin{matrix} t = e^x \\ \mathrm{d}t = e^x\mathrm{d}x \end{matrix} \right| = \int \frac{\mathrm{d}t}{4+t} = \ln|4+t| + C$

$$\boxed{\ln(4+e^x) + C}$$

11. $\int \frac{\mathrm{d}x}{1+3^x} = \left| \begin{array}{c} t = 1+3^x \\ \mathrm{d}t = 3^x \ln 3 \, \mathrm{d}x \end{array} \right| = \frac{1}{\ln 3} \int \frac{\mathrm{d}t}{t(t-1)} = \frac{1}{\ln 3} \int \frac{t-(t-1)}{t(t-1)} \mathrm{d}t = \frac{1}{\ln 3} \left(\int \frac{\mathrm{d}t}{t-1} - \int \frac{\mathrm{d}t}{t} \right) =$

$= \frac{1}{\ln 3} \ln \left| \frac{t-1}{t} \right| + C$

$$\boxed{\frac{1}{\ln 3} \ln \frac{3^x}{1+3^x} + C = -\frac{1}{\ln 3} \ln(1+3^{-x}) + C}$$

12. $\int \frac{\mathrm{d}x}{\sqrt{2^x+1}} = \left| \begin{array}{cc} t = \sqrt{2^x+1} & \frac{\mathrm{d}x}{\sqrt{2^x+1}} = \frac{2}{\ln 2} \frac{\mathrm{d}t}{2^x} \\ \mathrm{d}t = \frac{1}{2} \frac{2^x \ln 2}{\sqrt{2^x+1}} \mathrm{d}x & \frac{\mathrm{d}x}{\sqrt{2^x+1}} = \frac{2}{\ln 2} \frac{\mathrm{d}t}{t^2-1} \end{array} \right| = \frac{2}{\ln 2} \int \frac{\mathrm{d}t}{t^2-1} = \frac{1}{\ln 2} \int \frac{(t+1)-(t-1)}{t^2-1} =$

$= \frac{1}{\ln 2} \left(\int \frac{\mathrm{d}t}{t-1} - \int \frac{\mathrm{d}t}{t+1} \right) = \frac{1}{\ln 2} \ln \left| \frac{t-1}{t+1} \right| + C$

$$\boxed{\frac{1}{\ln 2} \ln \left| \frac{\sqrt{2^x+1}-1}{\sqrt{2^x+1}+1} \right| + C}$$

13. $\int \frac{\ln^4 x}{x} \mathrm{d}x = \left| \begin{array}{c} t = \ln x \\ \mathrm{d}t = \frac{1}{x} \mathrm{d}x \end{array} \right| = \int t^4 \, \mathrm{d}t = \frac{1}{5} t^5 + C$

$$\boxed{\frac{1}{5} \ln^5 x + C}$$

14. $\int \frac{\cos x}{\sin^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \sin x \\ \mathrm{d}t = \cos x \, \mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{t^2} = -\frac{1}{t} + C$

$$\boxed{-\frac{1}{\sin x} + C}$$

15. $\int \frac{\sin x}{\sqrt{\cos^5 x}} \mathrm{d}x = \left| \begin{array}{c} t = \cos x \\ \mathrm{d}t = -\sin x \, \mathrm{d}x \end{array} \right| = -\int \frac{\mathrm{d}t}{t^{\frac{5}{2}}} = -\int t^{-\frac{5}{2}} \mathrm{d}t = \frac{2}{3} t^{-\frac{3}{2}} + C =$

$$\boxed{\frac{2}{3} \frac{1}{\sqrt{\cos^3 x}} + C}$$

16. $\int \frac{\sqrt[3]{\mathrm{tg}^2 x}}{\cos^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \mathrm{tg}\, x \\ \mathrm{d}t = \frac{\mathrm{d}x}{\cos^2 x} \end{array} \right| = \int t^{\frac{2}{3}} \mathrm{d}t = \frac{t^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5} t^{\frac{5}{3}} + C$

$$\boxed{\frac{3}{5} \mathrm{tg}^{\frac{5}{3}} x + C}$$

17. $\int \frac{\sin 2x}{\sin^2 x + 3} \mathrm{d}x = \left| \begin{array}{c} t = \sin^2 x \\ \mathrm{d}t = 2 \sin x \cos x \, \mathrm{d}x = \sin 2x \, \mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{t^2+3} = \frac{1}{3} \int \frac{\mathrm{d}t}{\frac{t^2}{3}+1} = \frac{1}{3} \int \frac{\mathrm{d}t}{(\frac{t}{\sqrt{3}})^2+1} =$

$= \left| \begin{array}{cc} s = \frac{t}{\sqrt{3}} & \\ \mathrm{d}s = \frac{1}{\sqrt{3}} \mathrm{d}t & \mathrm{d}t = \sqrt{3} \mathrm{d}s \end{array} \right| = \frac{\sqrt{3}}{3} \int \frac{\mathrm{d}s}{s^2+1} = \frac{\sqrt{3}}{3} \mathrm{arctg}\, s + C = \frac{\sqrt{3}}{3} \mathrm{arctg}\, \frac{t}{\sqrt{3}} + C$

$$\boxed{\frac{\sqrt{3}}{3} \mathrm{arctg}\, \frac{\sin^2 x}{\sqrt{3}} + C}$$

18. $\int \cos^2 x \, \mathrm{d}x = \int \frac{1+\cos 2x}{2} \mathrm{d}x =$

$$\boxed{\frac{1}{2} x + \frac{1}{4} \sin 2x + C}$$

19. $\int \sin^2 x \, \mathrm{d}x = \int \frac{1-\cos 2x}{2} \mathrm{d}x$

$$\boxed{\frac{1}{2} x - \frac{1}{4} \sin 2x + C}$$

20. $\int \frac{\mathrm{d}x}{\cos x} = \int \frac{\cos x}{\cos^2 x} \mathrm{d}x = \int \frac{\cos x}{1-\sin^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \sin x \\ \mathrm{d}t = \cos x \, \mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{1-t^2} = \int \frac{\frac{(1+t)+(1-t)}{2}}{(1-t)(1+t)} \mathrm{d}t =$

$= \frac{1}{2} \int \frac{\mathrm{d}t}{1-t} + \frac{1}{2} \int \frac{\mathrm{d}t}{1+t} = -\frac{1}{2} \ln|1-t| + \frac{1}{2} \ln|1+t| + C = \frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| + C$

$$\boxed{\frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C}$$

21. $\int \frac{\mathrm{d}x}{\sin x} = \int \frac{\sin x}{\sin^2 x} \mathrm{d}x = \frac{\sin x}{1-\cos^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \cos x \\ \mathrm{d}t = -\sin x \, \mathrm{d}x \end{array} \right| = -\int \frac{\mathrm{d}t}{1-t^2} = \int \frac{\mathrm{d}t}{t^2-1} = \int \frac{\frac{(t+1)-(t-1)}{2}}{(t-1)(t+1)} \mathrm{d}t =$

$= \frac{1}{2} \int \frac{\mathrm{d}t}{t-1} - \frac{1}{2} \int \frac{\mathrm{d}t}{t+1} = \frac{1}{2} \ln|t-1| - \frac{1}{2} \ln|t+1| + C = \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + C$

$$\boxed{\frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C}$$

22. $\int \cos 3x \sin 4x \,\mathrm{d}x = \int \frac{\sin 7x + \sin x}{2} \mathrm{d}x$

$$\boxed{-\frac{\cos 7x}{14} - \frac{\cos x}{2} + C}$$

23. $\int \sqrt{\frac{\arccos x}{1-x^2}} \mathrm{d}x = \left| \begin{array}{c} t = \arccos x \\ \mathrm{d}t = -\frac{1}{\sqrt{1-x^2}} \mathrm{d}x \end{array} \right| = -\int \sqrt{t}\,\mathrm{d}t = -\frac{2}{3} t^{\frac{3}{2}} + C$

$$\boxed{-\frac{2}{3}\sqrt{\arccos^3 x} + C}$$

24. $\int \frac{x^2}{\sin x^3} \mathrm{d}x = \left| \begin{array}{c} t = x^3 \\ \mathrm{d}t = 3x^2\,dx \end{array} \right| = \frac{1}{3} \int \frac{\mathrm{d}t}{\sin t} = \frac{1}{6} \ln \left| \frac{\cos t - 1}{\cos t + 1} \right| + C$

$$\boxed{\frac{1}{6} \ln \left| \frac{\cos x^3 - 1}{\cos x^3 + 1} \right| + C}$$

25. $\int \frac{x - \operatorname{arctg} x}{1+x^2} \mathrm{d}x = \int \frac{x}{1+x^2} \mathrm{d}x - \int \frac{\operatorname{arctg} x}{1+x^2} \mathrm{d}x = \left| \begin{array}{cc} t = x^2 & s = \operatorname{arctg} x \\ \mathrm{d}t = 2x\,\mathrm{d}x & \mathrm{d}s = \frac{1}{1+x^2}\mathrm{d}x \end{array} \right| = \frac{1}{2} \int \frac{\mathrm{d}t}{1+t} - \int s\,\mathrm{d}s =$

$= \frac{1}{2} \ln |1+t| - \frac{1}{2} s^2 + C$

$$\boxed{\frac{1}{2} \ln(1+x^2) - \frac{1}{2} \operatorname{arctg}^2 x + C}$$

26. $\int \frac{\mathrm{d}x}{\cosh x} = \int \frac{2}{e^x + e^{-x}} \mathrm{d}x = \left| \begin{array}{c} t = e^x \\ \mathrm{d}t = e^x \mathrm{d}x \end{array} \right| = \int \frac{2\mathrm{d}t}{e^x} \frac{1}{e^x + e^{-x}} = \int \frac{2\mathrm{d}t}{1+t^2} = 2 \operatorname{arctg} t + C$

$$\boxed{2 \operatorname{arctg} e^x + C}$$

27. $\int \frac{\mathrm{d}x}{\cosh x} = \int \frac{\cosh x}{\cosh^2 x} \mathrm{d}x = \int \frac{\cosh x}{1 + \sinh^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \sinh x \\ \mathrm{d}t = \cosh x \mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{1+t^2} = \operatorname{arctg} t + C$

$$\boxed{\operatorname{arctg} \sinh x + C}$$

28. $\int \frac{1}{\cosh^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \operatorname{tgh} x \\ \mathrm{d}t = \frac{\mathrm{d}x}{\cosh^2 x} \end{array} \right| = \int 1\,\mathrm{d}t = t + C$

$$\boxed{\operatorname{tgh} x + C}$$

29. $\int \frac{\mathrm{d}x}{\operatorname{tg} x \ln^2 \sin x} = \left| \begin{array}{c} t = \ln \sin x \\ \mathrm{d}t = \frac{\cos x}{\sin x} \mathrm{d}x = \frac{\mathrm{d}x}{\operatorname{tg} x} \end{array} \right| = \int \frac{\mathrm{d}t}{t^2} = -\frac{1}{t} + C$

$$\boxed{-\frac{1}{\ln \sin x} + C}$$

30. $\int x e^{2x} \mathrm{d}x = \left| \begin{array}{cc} x & e^{2x} \\ 1 & \frac{1}{2} e^{2x} \end{array} \right| = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} \mathrm{d}x$

$$\boxed{\frac{e^{2x}}{4}(2x-1) + C}$$

31. $\int x \ln x \mathrm{d}x = \left| \begin{array}{cc} \ln x & x \\ \frac{1}{x} & \frac{x^2}{2} \end{array} \right| = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x\,\mathrm{d}x$

$$\boxed{\frac{x^2}{4}(2 \ln x - 1) + C}$$

32. $\int x a^x \mathrm{d}x = \left| \begin{array}{cc} x & a^x \\ 1 & \frac{1}{\ln a} a^x \end{array} \right| = \frac{1}{\ln a} x a^x - \frac{1}{\ln a} \int a^x \mathrm{d}x$

$$\boxed{\frac{1}{\ln a} x a^x - \frac{1}{\ln^2 a} a^x + C}$$

33. $\int \ln x\,dx = \left| \begin{array}{cc} \ln x & 1 \\ \frac{1}{x} & x \end{array} \right| = x \ln x - \int 1 \mathrm{d}x$

$$\boxed{x(\ln x - 1) + C}$$

34. $\int \arcsin x \,\mathrm{d}x = \left| \begin{array}{cc} \arcsin x & 1 \\ \frac{1}{\sqrt{1-x^2}} & x \end{array} \right| = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \mathrm{d}x$

$$\boxed{x \arcsin x + \sqrt{1-x^2} + C}$$

35. $I_n = \int \frac{\mathrm{d}x}{(x^2+1)^n} = \begin{vmatrix} \frac{1}{(x^2+1)^n} & 1 \\ \frac{-n(2x)}{(x^2+1)^{n+1}} & x \end{vmatrix} = \frac{x}{(x^2+1)^n} - \int \frac{-2nx^2}{(x^2+1)^{n+1}}\mathrm{d}x = \frac{x}{(x^2+1)^n} + 2n\int \frac{x^2}{(x^2+1)^{n+1}}\mathrm{d}x =$

$= \frac{x}{(x^2+1)^n} + 2n\int \frac{x^2+1-1}{(x^2+1)^{n+1}} = \frac{x}{(x^2+1)^n} + 2n\int \frac{\mathrm{d}x}{(x^2+1)^n} - 2n\int \frac{\mathrm{d}x}{(x^2+1)^{n+1}} = \frac{x}{(x^2+1)^n} + 2nI_n - 2nI_{n+1}$

$\boxed{I_1 = \operatorname{arctg} x + C}$ $\boxed{I_{n+1} = \frac{2n-1}{2n}I_n + \frac{1}{2n}\frac{x}{(x^2+1)^n} + C, n \in N}$

36. $\int e^{\arcsin x}\mathrm{d}x = \begin{vmatrix} e^{\arcsin x} & 1 \\ \frac{e^{\arcsin x}}{\sqrt{1-x^2}} & x \end{vmatrix} = xe^{\arcsin x} - \int \frac{xe^{\arcsin x}}{\sqrt{1-x^2}}\mathrm{d}x = \begin{vmatrix} e^{\arcsin x} & \frac{x}{\sqrt{1-x^2}} \\ \frac{e^{\arcsin x}}{\sqrt{1-x^2}} & -\sqrt{1-x^2} \end{vmatrix} =$

$= xe^{\arcsin x} + \sqrt{1-x^2}\arcsin x - \int e^{\arcsin x}\mathrm{d}x$

$\boxed{\frac{1}{2}xe^{\arcsin x} + \frac{1}{2}\sqrt{1-x^2}\arcsin x + C}$

37. $\int \sinh^2 x\,\mathrm{d}x = \begin{vmatrix} \sinh x & \sinh x \\ \cosh x & \cosh x \end{vmatrix} = \sinh x\cosh x - \int \cosh^2 x\,\mathrm{d}x$, t.j. $C+S = \sinh x\cosh x$ a $C-S = x$.

$\boxed{\frac{1}{2}(\sinh x\cosh x - x) + C}$

38. $\int \frac{xe^x}{(1+x)^2}\mathrm{d}x = \begin{vmatrix} xe^x & \frac{1}{(1+x)^2} \\ e^x + xe^x & -\frac{1}{1+x} \end{vmatrix} = -\frac{x}{x+1}e^x + \int e^x\mathrm{d}x$

$\boxed{\frac{e^x}{x+1} + C}$

39. $\int \frac{x\arcsin x}{\sqrt{1-x^2}}\mathrm{d}x = \begin{vmatrix} \arcsin x & \frac{x}{\sqrt{1-x^2}} \\ \frac{1}{\sqrt{1-x^2}} & -\sqrt{1-x^2} \end{vmatrix} = -\sqrt{1-x^2}\arcsin x + \int 1\mathrm{d}x$

$\boxed{x - \sqrt{1-x^2}\arcsin x + C}$

40. $\int e^{2x}\cos x\,\mathrm{d}x = \begin{vmatrix} e^{2x} & \cos x \\ 2e^{2x} & \sin x \end{vmatrix} = e^{2x}\sin x - 2\int e^{2x}\sin x\,\mathrm{d}x = \begin{vmatrix} e^{2x} & \sin x \\ 2e^{2x} & -\cos x \end{vmatrix} =$

$e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$

$\boxed{\frac{e^{2x}}{5}(\sin x + 2\cos x) + C}$

41. $\int \sin x\ln(\operatorname{tg} x)\mathrm{d}x = \begin{vmatrix} \ln(\operatorname{tg} x) & \sin x \\ \frac{1}{\cos^2 x}\cdot\frac{1}{\operatorname{tg} x} & -\cos x \end{vmatrix} = -\cos x\ln(\operatorname{tg} x) + \int \frac{1}{\sin x}\mathrm{d}x$

$\boxed{-\cos x\ln(\operatorname{tg} x) + \frac{1}{2}\ln\left|\frac{1-\cos x}{1+\cos x}\right| + C}$

42. $\int x\operatorname{tg}^2 x\,dx = \int x\frac{\sin^2 x}{\cos^2 x}\mathrm{d}x = \int x\frac{1-\cos^2 x}{\cos^2 x} = \begin{vmatrix} x & \frac{1}{\cos^2 x} - 1 \\ 1 & \operatorname{tg} x - x \end{vmatrix} = x\operatorname{tg} x - x^2 - \int \operatorname{tg} x + \int x\,\mathrm{d}x$

$\boxed{x\operatorname{tg} x - \frac{1}{2}x^2 + \ln|\cos x| + C}$

43. $\int \frac{\operatorname{arctg} e^x}{e^x} = \begin{vmatrix} \operatorname{arctg} e^x & e^{-x} \\ \frac{e^x}{1+e^{2x}} & -e^{-x} \end{vmatrix} = -e^{-x}\operatorname{arctg} e^x + \int \frac{1}{1+e^{2x}}\mathrm{d}x$

$\int \frac{1}{1+e^{2x}}\mathrm{d}x = \begin{vmatrix} t = e^{2x} & \\ \mathrm{d}t = 2e^{2x}\mathrm{d}x & \mathrm{d}x = \frac{1}{2}e^{-2x}\mathrm{d}t \end{vmatrix} = \int \frac{1}{1+t}\frac{\mathrm{d}t}{t} = \int \frac{t+1-t}{(1+t)t}\mathrm{d}t = \int \frac{\mathrm{d}t}{t} - \int \frac{\mathrm{d}t}{t+1} = \ln\left|\frac{t}{t+1}\right| + C =$

$= \ln\left|\frac{e^{2x}}{e^{2x}+1}\right| + C$

$\boxed{-\frac{\operatorname{arctg} x}{e^x} + \ln\left|\frac{e^{2x}}{e^{2x}+1}\right| + C}$

44. $\int \ln(x+\sqrt{1+x^2})\,\mathrm{d}x = \begin{vmatrix} \ln(x+\sqrt{1+x^2}) & 1 \\ \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} & x \end{vmatrix} = x\ln(x+\sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}}\mathrm{d}x$

$\boxed{x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + C}$

45. $\int \frac{e^{\operatorname{arctg} x}}{\sqrt{(1+x^2)^3}}\mathrm{d}x = \begin{vmatrix} \frac{1}{\sqrt{1+x^2}} & \frac{e^{\operatorname{arctg} x}}{1+x^2} \\ -\frac{1}{2}(1+x^2)^{-\frac{3}{2}}2x = -\frac{x}{\sqrt{(1+x^2)^3}} & e^{\operatorname{arctg} x} \end{vmatrix} = \frac{e^{\operatorname{arctg} x}}{\sqrt{1+x^2}} + \int \frac{x\, e^{\operatorname{arctg} x}}{\sqrt{(1+x^2)^3}} =$

$= \begin{vmatrix} \frac{x}{\sqrt{1+x^2}} & \frac{e^{\operatorname{arctg} x}}{1+x^2} \\ \frac{\sqrt{1+x^2}-x\frac{x}{\sqrt{1+x^2}}}{1+x^2} = \frac{1}{\sqrt{(1+x^2)^3}} & e^{\operatorname{arctg} x} \end{vmatrix} = \frac{e^{\operatorname{arctg} x}}{\sqrt{1+x^2}} + \frac{x\, e^{\operatorname{arctg} x}}{\sqrt{1+x^2}} - \int \frac{e^{\operatorname{arctg} x}}{\sqrt{(1+x^2)^3}}\mathrm{d}x$

$$\boxed{\frac{1}{2}e^{\operatorname{arctg} x}\frac{x+1}{\sqrt{1+x^2}} + C}$$

46. $\int \operatorname{arcsin}\sqrt{\frac{x}{x+1}}\mathrm{d}x = \begin{vmatrix} \operatorname{arcsin}\sqrt{\frac{x}{x+1}} & 1 \\ \frac{1}{\sqrt{1-(\sqrt{\frac{x}{x+1}})^2}}\frac{1}{2}\sqrt{\frac{x+1}{x}}\frac{1}{(1+x)^2} & x \end{vmatrix} = x\operatorname{arcsin}\sqrt{\frac{x}{x+1}} - \frac{1}{2}\int \frac{\sqrt{x}}{(1+x)}\mathrm{d}x$

$\int \frac{\sqrt{x}}{1+x}\mathrm{d}x = \begin{vmatrix} t = \sqrt{x} \\ \mathrm{d}t = \frac{1}{2}\frac{\mathrm{d}x}{\sqrt{x}} \end{vmatrix} = 2\int \frac{t^2}{1+t^2}\mathrm{d}t = 2\int \frac{t^2+1-1}{1+t^2} = 2\int 1\,\mathrm{d}t - 2\int \frac{\mathrm{d}t}{1+t^2} = 2t - 2\operatorname{arctg} t + C =$

$= 2\sqrt{x} - 2\operatorname{arctg}\sqrt{x} + C$

$$\boxed{x\operatorname{arcsin}\sqrt{\frac{x}{x+1}} + \operatorname{arctg}\sqrt{x} - \sqrt{x} + C}$$