(aaa)

$$\int \frac{\mathrm{d}x}{\sqrt[3]{(4-3x)^2}} = \left| \begin{array}{c} t = 4-3x \\ \mathrm{d}t = -3\,\mathrm{d}x \end{array} \right| = -\frac{1}{3}\int \frac{\mathrm{d}t}{t^{\frac{2}{3}}} = -\frac{1}{3}\int t^{-\frac{2}{3}}\mathrm{d}t = -t^{\frac{1}{3}} + C$$

$$\boxed{-(4-3x)^{\frac{1}{3}} + C}$$

(aab)

$$\int e^{-x}\sin^2 x\,\mathrm{d}x = \left| \begin{array}{cc} \sin^2 x & e^{-x} \\ 2\sin x\cos x & -e^{-x} \end{array} \right| = -e^{-x}\sin^2 x + \int e^{-x}\sin 2x\,\mathrm{d}x$$

$$\int e^{-x}\sin 2x\,\mathrm{d}x = \left| \begin{array}{cc} \sin 2x & e^{-x} \\ 2\cos 2x & -e^{-x} \end{array} \right| = -e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\,\mathrm{d}x$$

$$\int e^{-x}\cos 2x\,\mathrm{d}x = \left| \begin{array}{cc} \cos 2x & e^{-x} \\ -2\sin 2x & -e^{-x} \end{array} \right| = -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,\mathrm{d}x$$

$$\begin{array}{ll} s - 2c = -e^{-x}\sin 2x & 5S = -e^{-x}\left(\sin 2x + 2\cos 2x\right) \\ 2s + c = -e^{-x}\cos 2x & S = -\frac{1}{5}e^{-x}\left(\sin 2x + 2\cos 2x\right) + C \end{array}$$

$$\boxed{-\frac{1}{5}e^{-x}\left(5\sin^2 x + 2\cos 2x + \sin 2x\right) + C}$$

(aac)

$$\int e^{ax}\cos bx\,\mathrm{d}x = \left| \begin{array}{cc} e^{ax} & \cos bx \\ ae^{ax} & \frac{1}{b}\sin bx \end{array} \right| = \frac{1}{b}e^{ax}\sin bx - \frac{a}{b}\int e^{ax}\sin bx\,\mathrm{d}x$$

$$\int e^{ax}\sin bx\,\mathrm{d}x = \left| \begin{array}{cc} e^{ax} & \sin bx \\ ae^{ax} & -\frac{1}{b}\cos bx \end{array} \right| = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\,\mathrm{d}x$$

$$\begin{array}{ll} c + \frac{a}{b}s = \frac{1}{b}e^{ax}\sin bx & c = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + C \\ -\frac{a}{b}c + s = -\frac{1}{b}e^{ax}\cos bx & s = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + C \end{array}$$

$$\boxed{\frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + C}$$

(aad)

$$\int(3x^2+2x+1)\sin\frac{x}{3}\mathrm{d}x = \left| \begin{array}{cc} 3x^2+2x+1 & \sin\frac{x}{3} \\ 6x+2 & -3\cos\frac{x}{3} \end{array} \right| = -3(3x^2+2x+1)\cos\frac{x}{3} + 6\int(3x+1)\cos\frac{x}{3}\mathrm{d}x =$$

$$= \left| \begin{array}{cc} 3x+1 & \cos\frac{x}{3} \\ 3 & 3\sin\frac{x}{3} \end{array} \right| = -(9x^2+6x+3)\cos\frac{x}{3} + 6(3x+1)\sin\frac{x}{3} - 54\int\sin\frac{x}{3}\mathrm{d}x =$$

$$= -(9x^2+6x+3)\cos\frac{x}{3} + (18x+6)\sin\frac{x}{3} + 162\cos\frac{x}{3} + C$$

$$\boxed{-(9x^2+6x-159)\cos\frac{x}{3} + (18x+6)\sin\frac{x}{3} + C}$$

(aae)

$$\int \sin x\sqrt{(3+2\cos x)^5}\,\mathrm{d}x = \left| \begin{array}{c} t = 3+2\cos x \\ \mathrm{d}t = -2\sin x\,\mathrm{d}x \end{array} \right| = -\frac{1}{2}\int t^{\frac{5}{2}}\mathrm{d}t = -\frac{1}{2}\frac{2}{7}t^{\frac{7}{2}} + C = -\frac{1}{7}t^{\frac{7}{2}} + C$$

$$\boxed{-\frac{1}{7}(3+2\cos x)^{\frac{7}{2}} + C}$$

(aaf)

$$\int(3x^2+1)\ln(x-4)\,\mathrm{d}x = \left| \begin{array}{cc} \ln(x-4) & 3x^2+1 \\ \frac{1}{x-4} & x^3+x \end{array} \right| = x(x^2+1)\ln(x-4) - \int\frac{x^3+x}{x-4}\mathrm{d}x$$

$$\left(x^3 + x = x^2(x-4) + 4x(x-4) + 17(x-4) + 68\right)$$

$$= x(x^2+1)\ln(x-4) - \int\left(x^2+4x+17+\frac{68}{x-4}\right)\mathrm{d}x = x(x^2+1)\ln(x-4) - \frac{1}{3}x^3 + 2x^2 + 17x + 68\ln|x-4| + C$$

$$\boxed{x(x^2+1)\ln(x-4) - \frac{1}{3}x^3 + 2x^2 + 17x + 68\ln|x-4| + C}$$

(aag)

$\int \left(\frac{\ln x}{x}\right)^2 \mathrm{d}x = \begin{vmatrix} \ln^2 x & \frac{1}{x^2} \\ 2\frac{\ln x}{x} & -\frac{1}{x} \end{vmatrix} = -\frac{\ln^2 x}{x} + 2\int \frac{\ln x}{x^2}\mathrm{d}x$

$\int \frac{\ln x}{x^2}\mathrm{d}x = \begin{vmatrix} \ln x & \frac{1}{x^2} \\ \frac{1}{x} & -\frac{1}{x} \end{vmatrix} = -\frac{\ln x}{x} + \int \frac{\mathrm{d}x}{x^2} = -\frac{\ln x}{x} - \frac{1}{x} + C$

$$\boxed{-\frac{1}{x}(\ln^2 x - 2\ln x - 2) + C}$$

(aah)

$\int x^2 \operatorname{arctg} 3x \,\mathrm{d}x = \begin{vmatrix} \operatorname{arctg} 3x & x^2 \\ \frac{3}{1+9x^2} & \frac{x^3}{3} \end{vmatrix} = \frac{1}{3}x^3 \operatorname{arctg} 3x - \int \frac{x^3}{1+9x^2}\mathrm{d}x = |\, x^3 : (9x^2+1) = \frac{1}{9}x \quad \text{zv. } -\frac{1}{9}x \,| =$

$= \frac{1}{3}x^3 \operatorname{arctg} 3x - \int \frac{1}{9}x \,\mathrm{d}x + \frac{1}{9}\int \frac{x}{1+9x^2}\mathrm{d}x = \frac{1}{3}x^3 \operatorname{arctg} 3x - \frac{1}{18}x^2 + \frac{1}{162}\ln(1+9x^2) + C$

$$\boxed{\frac{1}{3}x^3 \operatorname{arctg} 3x - \frac{1}{18}x^2 + \frac{1}{162}\ln(1+9x^2) + C}$$

(aai)

$\int \arcsin^2 x \,\mathrm{d}x = \begin{vmatrix} \arcsin^2 x & 1 \\ 2\frac{\arcsin x}{\sqrt{1-x^2}} & x \end{vmatrix} = x\arcsin^2 x - 2\int \arcsin x \frac{x}{\sqrt{1-x^2}}\mathrm{d}x = \begin{vmatrix} \arcsin x & \frac{x}{\sqrt{1-x^2}} \\ \frac{1}{\sqrt{1-x^2}} & -\sqrt{1-x^2} \end{vmatrix} =$

$= x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2\int 1\,\mathrm{d}x = x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C$

$$\boxed{x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C}$$

(aaj)

$\int \sin x \sinh x \,\mathrm{d}x = \begin{vmatrix} \sin x & \sinh x \\ \cos x & \cosh x \end{vmatrix} = \sin x \cosh x - \int \cos x \cosh x \,\mathrm{d}x$

$\int \cos x \cosh x \,\mathrm{d}x = \begin{vmatrix} \cos x & \cosh x \\ -\sin x & \sinh x \end{vmatrix} = \cos x \sinh x + \int \sin x \sinh x \,\mathrm{d}x$

$$\begin{aligned} ss + cc &= \sin x \cosh x & 2ss &= \sin x\cosh x - \cos x \sinh x \\ -ss + cc &= \cos x \sinh x & ss &= \tfrac{1}{2}(\sin x \cosh x - \cos x \sinh x) \end{aligned}$$

$$\boxed{\frac{1}{2}(\sin x\cosh x - \cos x \sinh x) + C}$$

(aak)

$\int (4x^3+2x)\operatorname{arctg} x \,\mathrm{d}x = \begin{vmatrix} \operatorname{arctg} x & 4x^3+2x \\ \frac{1}{1+x^2} & x^4+x^2 \end{vmatrix} = (x^4+x^2)\operatorname{arctg} x - \int \frac{x^4+x^2}{1+x^2}\mathrm{d}x =$

$= (x^4+x^2)\operatorname{arctg} x - \int x^2 \mathrm{d}x = (x^4+x^2)\operatorname{arctg} x - \frac{1}{3}x^3 + C$

$$\boxed{(x^4+x^2)\operatorname{arctg} x - \frac{1}{3}x^3 + C}$$

(aal)

$\int \frac{\mathrm{d}x}{(2x^2+2)\sqrt{\operatorname{arccotg}^3 x}} = \begin{vmatrix} t = \operatorname{arccotg} x \\ \mathrm{d}t = -\frac{1}{1+x^2}\mathrm{d}x \end{vmatrix} = -\frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t^3}} = -\frac{1}{2}\int t^{-\frac{3}{2}}\mathrm{d}t = -\frac{1}{2}(-2)t^{-\frac{1}{2}} + C = t^{-\frac{1}{2}} + C$

$$\boxed{\frac{1}{\sqrt{\operatorname{arccotg} x}} + C}$$

(aam)

$\int (2x-1)\arccos x \,\mathrm{d}x = \begin{vmatrix} \arccos x & 2x-1 \\ -\frac{1}{\sqrt{1-x^2}} & x^2-x \end{vmatrix} = (x^2-x)\arccos x + \int \frac{x^2-x}{\sqrt{1-x^2}}\mathrm{d}x$

$= (x^2-x)\arccos x + \int x\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$

$\int x\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x = \begin{vmatrix} t = \sqrt{\frac{1-x}{1+x}} & x = \frac{1-t^2}{1+t^2} \\ \mathrm{d}x = \frac{(-2t)(1+t^2)-(1-t^2)(2t)}{(1+t^2)^2}\mathrm{d}t & \mathrm{d}x = \frac{-4t}{(1+t^2)^2}\mathrm{d}t \end{vmatrix} = \int \frac{1-t^2}{1+t^2}\cdot t \cdot \frac{-4t\,\mathrm{d}t}{(1+t^2)^2} = 4\int \frac{t^4-t2}{(1+t^2)^3}\mathrm{d}t =$

$= 4\int \frac{t^2(1+t^2)-2(1+t^2)+2}{(1+t^2)^3}\mathrm{d}t = 4\int \frac{t^2-2}{(1+t^2)^2} + 8\int \frac{\mathrm{d}t}{(1+t^2)^3} = 4\int \frac{t^2+1-3}{(1+t^2)^2}\mathrm{d}t + 8\int \frac{\mathrm{d}t}{(1+t^2)^3} =$

$= 4\int \frac{\mathrm{d}t}{1+t^2} - 12\int \frac{\mathrm{d}t}{(1+t^2)^2} + 8\int \frac{\mathrm{d}t}{(1+t^2)^3} =: I$

$$\operatorname{arctg} t = \int \frac{1}{1+t^2} = \begin{vmatrix} \frac{1}{1+t^2} & 1 \\ -\frac{2t}{1+t^2} & t \end{vmatrix} = \frac{t}{1+t^2} + 2\int \frac{t^2}{1+t^2}\mathrm{d}t = \frac{t}{1+t^2} + 2\int \frac{t^2+1-1}{(t^2+1)^2}\mathrm{d}t =$$
$$= \frac{t}{1+t^2} + 2\int \frac{1}{1+t^2}\mathrm{d}t - 2\int \frac{\mathrm{d}t}{(1+t^2)^2} = \frac{t}{1+t^2} + 2\operatorname{arctg} t - 2\int \frac{\mathrm{d}t}{(1+t^2)^2}$$
$$\int \frac{\mathrm{d}t}{(1+t^2)^2} = \frac{1}{2}\frac{t}{1+t^2} + \frac{1}{2}\operatorname{arctg} t + C$$
Podobne $\int \frac{\mathrm{d}t}{(1+t^2)^3} = \frac{3}{4}\int \frac{\mathrm{d}t}{(1+t^2)^2} + \frac{1}{4}\frac{t}{(1+t^2)^2} + C = \frac{3}{8}\operatorname{arctg} t + \frac{3}{8}\frac{t}{1+t^2} + \frac{1}{4}\frac{t}{(1+t^2)^2} + C$
$$I = 4\operatorname{arctg} t - 6\operatorname{arctg} t - 6\frac{t}{1+t^2} + 3\operatorname{arctg} t + 3\frac{t}{1+t^2} + 2\frac{t}{(1+t^2)^2} + C = \operatorname{arctg} t - 3\frac{t}{1+t^2} + 2\frac{t}{(1+t^2)^2} + C =$$
$$= \operatorname{arctg}\sqrt{\frac{x-1}{x+1}} - 3\frac{\sqrt{\frac{x-1}{x+1}}}{1+\frac{1-x}{1+x}} + 2\frac{\sqrt{\frac{1-x}{1+x}}}{(1+\frac{1-x}{1+x})^2} + C = \operatorname{arctg}\sqrt{\frac{x-1}{x+1}} - \frac{3(1+x)}{2}\sqrt{\frac{1-x}{1+x}} + \frac{(1+x)^2}{2}\sqrt{\frac{x-1}{x+1}} + C$$
$$\boxed{\operatorname{arctg}\sqrt{\frac{x-1}{x+1}} + \frac{1}{2}(x^2 - x - 2)\sqrt{\frac{x-1}{x+1}} + C}$$

(aan)

$$\int (x^2-3x+1)\cosh 2x\,\mathrm{d}x = \begin{vmatrix} x^2-3x+1 & \cosh 2x \\ 2x-3 & \frac{1}{2}\sinh 2x \end{vmatrix} = \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{2}\int (2x-3)\sinh 2x\,\mathrm{d}x =$$
$$= \begin{vmatrix} 2x-3 & \sinh 2x \\ 2 & \frac{1}{2}\cosh 2x \end{vmatrix} = \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{4}(2x-3)\cosh 2x + \frac{1}{2}\int \cosh 2x\,\mathrm{d}x =$$
$$= \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{4}(2x-3)\cosh 2x + \frac{1}{4}\sinh 2x + C$$
$$\boxed{\frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{4}(2x-3)\cosh 2x + \frac{1}{4}\sinh 2x + C}$$

(aao)

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\mathrm{d}x = \begin{vmatrix} t = \sqrt{x} & 2\sqrt{x}\,\mathrm{d}t = \mathrm{d}x \\ \mathrm{d}t = \frac{1}{2}\frac{1}{\sqrt{x}}\mathrm{d}x & 2t\,\mathrm{d}t = \mathrm{d}x \end{vmatrix} = \int \frac{t\cdot 2t\,\mathrm{d}t}{1+t} = 2\int \frac{t^2}{1+t}\mathrm{d}t = 2\int \frac{t^2-1+1}{1+t}\mathrm{d}t =$$
$$2\left(\int (t-1)\,\mathrm{d}t + \int \frac{1}{1+t}\mathrm{d}t\right) = t^2 - 2t + 2\ln|t+1| + C$$
$$\boxed{x - 2\sqrt{x} + 2\ln|\sqrt{x}+1| + \mathrm{C}}$$

(aap)

$$\int \frac{\mathrm{d}x}{(2-x)\sqrt{1-x}} = \begin{vmatrix} t = \sqrt{1-x} & -2\sqrt{1-x}\,\mathrm{d}t = \mathrm{d}x \\ \mathrm{d}t = -\frac{1}{2}\frac{1}{\sqrt{1-x}}\mathrm{d}x & -2t\,\mathrm{d}t = \mathrm{d}x \end{vmatrix} = \int \frac{-2t\,\mathrm{d}t}{(1+t^2)t} = -2\int \frac{\mathrm{d}t}{1+t^2} = -2\operatorname{arctg} t + C$$
$$\boxed{-2\operatorname{arctg}\sqrt{x-1} + C}$$

(aaq)

$$\int \frac{\sqrt{x}}{x+2}\mathrm{d}x = \begin{vmatrix} t = \sqrt{x} & 2\sqrt{x}\mathrm{d}t = \mathrm{d}x \\ \mathrm{d}t = \frac{1}{2\sqrt{x}}\,\mathrm{d}x & 2t\,\mathrm{d}t = \mathrm{d}x \end{vmatrix} = \int \frac{t\cdot 2t\,\mathrm{d}t}{2+t^2} = \int \frac{2(t^2+2)-4}{t^2+2}\mathrm{d}t = \int 2\,\mathrm{d}t - 4\int \frac{\mathrm{d}t}{2+t^2} =$$
$$= 2t - 4\int \frac{1}{2}\frac{\mathrm{d}t}{1+\frac{t^2}{2}} = 2t - 2\int \frac{\mathrm{d}t}{1+\left(\frac{t}{\sqrt{2}}\right)^2} = \begin{vmatrix} s = \frac{t}{\sqrt{2}} & \\ \mathrm{d}s = \frac{1}{\sqrt{2}}\mathrm{d}t & \sqrt{2}\,\mathrm{d}s = \mathrm{d}t \end{vmatrix} = 2t - 2\sqrt{2}\int \frac{\mathrm{d}s}{1+s^2} =$$
$$= 2t - 2\sqrt{2}\operatorname{arctg} s + C = 2t - 2\sqrt{2}\operatorname{arctg}\frac{t}{\sqrt{2}} + C$$
$$\boxed{2\sqrt{x} - 2\sqrt{2}\operatorname{arctg}\sqrt{\frac{x}{2}} + C}$$

(aar)

$$\int \frac{\mathrm{d}x}{1+\sqrt[3]{x}} = \begin{vmatrix} t^3 = x & t = \sqrt[3]{x} \\ 3t^2\mathrm{d}t = \mathrm{d}x & \end{vmatrix} = \int \frac{3t^2\mathrm{d}t}{1+t} = 3\int \frac{t^2-1+1}{1+t}\mathrm{d}t = 3\int (t-1)\,\mathrm{d}t + 3\int \frac{\mathrm{d}t}{1+t} =$$
$$= \frac{3}{2}t^2 - 3t + 3\ln|1+t| + C$$
$$\boxed{\frac{3}{2}\sqrt[3]{x^2} - 3\sqrt[3]{x} + 3\ln|1+\sqrt[3]{x}| + C}$$

(aas)

$$\int \frac{\sqrt{x}}{1-\sqrt[3]{x}}\mathrm{d}x = \begin{vmatrix} x = t^6 & t = \sqrt[6]{x} \\ \mathrm{d}x = 6t^5\mathrm{d}t & \end{vmatrix} = \int \frac{t^3}{1-t^2}6t^5\mathrm{d}t = -6\int \frac{t^8}{t^2-1}\mathrm{d}t =$$

$$\left| t^8 = t^6(t^2-1)+t^4(t^2-1)+t^2(t^2-1)+(t^2-1)+1 \right| = \int \left(t^6+t^4+t^2+1+\frac{1}{t^2-1} \right) \mathrm{d}t =$$
$$= \frac{t^7}{7}+\frac{t^5}{5}+\frac{t^3}{3}+t+\frac{1}{2}\int \frac{t+1-(t-1)}{t^2-1}\mathrm{d}t = \ldots + \int \frac{\mathrm{d}t}{t-1} - \int \frac{\mathrm{d}t}{t+1} = \ldots + \ln\left|\frac{t-1}{t+1}\right| + C$$
$$\boxed{\frac{1}{7}x^{\frac{7}{6}}+\frac{1}{5}x^{\frac{5}{6}}+\frac{1}{3}x^{\frac{1}{2}}+x^{\frac{1}{6}}+\ln\left|\frac{x^{\frac{1}{6}}-1}{x^{\frac{1}{6}}+1}\right|+C}$$

(aat)

$$\int \frac{\mathrm{d}x}{x\sqrt{x-4}} = \left| \begin{matrix} t^2 = x-4 & t=\sqrt{x-4} \\ 2t\,\mathrm{d}t = \mathrm{d}x & \end{matrix} \right| = \int \frac{2t\,\mathrm{d}t}{(4+t^2)t} = 2\int \frac{\mathrm{d}t}{4+t^2} = \frac{1}{2}\int \frac{\mathrm{d}t}{1+\left(\frac{t}{2}\right)^2} =$$
$$= \left| \begin{matrix} s = \frac{t}{2} & \\ \mathrm{d}s = \frac{1}{2}\mathrm{d}t & 2\mathrm{d}s = \mathrm{d}t \end{matrix} \right| = \int \frac{\mathrm{d}s}{1+s^2} = \operatorname{arctg} s + C = \operatorname{arctg}\frac{t}{2}+C$$
$$\boxed{\operatorname{arctg}\frac{\sqrt{x-4}}{2}+C}$$

(aau)

$$\int \sqrt{\frac{1+x}{1-x}}\,\mathrm{d}x = \left| \begin{matrix} t=\sqrt{\frac{1+x}{1-x}} & x=\frac{t^2-1}{t^2+1} & 1-x=\frac{2}{1+t^2} \\ \mathrm{d}t=\frac{1}{2}\sqrt{\frac{1-x}{1+x}}\frac{2\,\mathrm{d}x}{(1-x)^2} & \mathrm{d}x=\frac{4t\,\mathrm{d}t}{(1+t^2)^2} & \end{matrix} \right| = \int t\frac{4t\,\mathrm{d}t}{(1+t^2)^2} = 4\int \frac{t^2+1-1}{(1+t^2)^2} =$$
$$= 4\int \frac{\mathrm{d}t}{1+t^2} - 4\int \frac{\mathrm{d}t}{(1+t^2)^2} = 4\operatorname{arctg} t - 2\operatorname{arctg} t - \frac{2t}{1+t^2} + C$$
$$\boxed{2\operatorname{arctg}\sqrt{\frac{1+x}{1-x}} - \sqrt{1-x^2}+C}$$

(aav)

$$\int \sqrt{\frac{1+x}{1-x}}\frac{1}{(1-x)(1+x)^2}\mathrm{d}x = \left| \begin{matrix} t=\sqrt{\frac{1+x}{1-x}} & x=\frac{t^2-1}{t^2+1} & 1-x=\frac{2}{1+t^2} \\ \mathrm{d}t=\frac{1}{2}\sqrt{\frac{1-x}{1+x}}\frac{2\,\mathrm{d}x}{(1-x)^2} & \mathrm{d}x=\frac{4t\,\mathrm{d}t}{(1+t^2)^2} & 1+x=\frac{2t^2}{1+t^2} \end{matrix} \right| =$$
$$\int t\frac{1+t^2}{2}\frac{(1+t^2)^2}{4t^4}\frac{4t\,\mathrm{d}t}{1+t^2} = \frac{1}{2}\int \frac{(1+t^2)^2}{t^2}\mathrm{d}t = \frac{1}{2}\int \left(\frac{1}{t^2}+2+t^2\right)\mathrm{d}t = -\frac{1}{2t}+t+\frac{1}{6}t^3+C$$
$$\boxed{-\frac{1}{2}\sqrt{\frac{1-x}{1+x}}+\sqrt{\frac{1+x}{1-x}}+\frac{1}{6}\left(\frac{1+x}{1-x}\right)^{\frac{3}{2}}+C}$$

(aaw)

$$\int \frac{\mathrm{d}x}{\sqrt{(x-2)^3(x-3)}} = \left| \begin{matrix} t=\sqrt{\frac{x-2}{x-3}} & x=\frac{3t^2-2}{t^2-1} & x-2=\frac{t^2}{t^2-1} \\ \mathrm{d}t=\frac{1}{2}\sqrt{\frac{x-3}{x-2}}\frac{-1\,\mathrm{d}x}{(x-3)^2} & \mathrm{d}x=-\frac{2t\,\mathrm{d}t}{(t^2-1)^2} & x-3=\frac{1}{t^2-1} \end{matrix} \right| = \int \frac{-\frac{2t}{(t^2-1)^2}}{\sqrt{\frac{t^6}{(t^2-1)^3}\frac{1}{t^2-1}}}\mathrm{d}t =$$
$$= -2\int \frac{t}{t^3}\mathrm{d}t = -2\int \frac{\mathrm{d}t}{t^2} = \frac{2}{t}+C$$
$$\boxed{2\sqrt{\frac{x-3}{x-2}}+C}$$

(aax)

$$\int \frac{\mathrm{d}x}{\sqrt{3-2x-5x^2}} = \int \frac{\mathrm{d}x}{\sqrt{(3-5x)(x+1)}} = \left| \begin{matrix} t=\sqrt{\frac{3-5x}{x+1}} & x=\frac{3-t^2}{5+t^2} & x+1=\frac{8}{5+t^2} \\ \mathrm{d}t=\frac{1}{2}\sqrt{\frac{x+1}{3-5x}}\frac{-8\,\mathrm{d}x}{(x+1)^2} & \mathrm{d}x=\frac{-16t\,\mathrm{d}t}{(5+t^2)^2} & 3-5x=\frac{8t^2}{5+t^2} \end{matrix} \right| =$$
$$= \int \frac{-\frac{16t}{(5+t^2)^2}}{\frac{8t}{5+t^2}}\mathrm{d}t = -2\int \frac{\mathrm{d}t}{5+t^2} = -\frac{2}{5}\int \frac{\mathrm{d}t}{1+\left(\frac{t}{\sqrt{5}}\right)^2} = \left| \begin{matrix} s=\frac{t}{\sqrt{5}} & \\ \mathrm{d}s=\frac{\mathrm{d}t}{\sqrt{5}} & \sqrt{5}\,\mathrm{d}s=\mathrm{d}t \end{matrix} \right| = -\frac{2}{\sqrt{5}}\int \frac{\mathrm{d}s}{1+s^2} =$$
$$= -\frac{2}{\sqrt{5}}\operatorname{arctg} s + C = -\frac{2}{\sqrt{5}}\operatorname{arctg}\frac{t}{\sqrt{5}}+C$$
$$\boxed{-\frac{2}{\sqrt{5}}\operatorname{arctg}\sqrt{\frac{3-5x}{5+5x}}+C}$$

(aay)

$$\int \frac{x-1}{\sqrt{x^2-2x+2}}\mathrm{d}x = \int \frac{x-1}{\sqrt{(x-1)^2+1}}\mathrm{d}x = \left| \begin{matrix} t=x-1 \\ \mathrm{d}t=\mathrm{d}x \end{matrix} \right| = \int \frac{t}{\sqrt{t^2+1}}\mathrm{d}t = \left| \begin{matrix} s=t^2+1 \\ \mathrm{d}s=2t\,\mathrm{d}t \end{matrix} \right| = \frac{1}{2}\int \frac{\mathrm{d}s}{\sqrt{s}} =$$
$$= \sqrt{s}+C = \sqrt{t^2+1}+C$$

$$\int \frac{x-1}{\sqrt{x^2-2x+2}}\mathrm{d}x = \left| \begin{array}{c} t = x^2-2x+2 \\ \mathrm{d}t = (2x-2)\,\mathrm{d}x \end{array}\right| = \frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t}} = \sqrt{t} + C$$

$$\boxed{\sqrt{x^2-2x+2}+C}$$

(aaz)

$$\int \frac{\mathrm{d}x}{(9+x^2)\sqrt{9+x^2}} = \left| \begin{array}{c} x = 3\operatorname{tg} t \\ \mathrm{d}x = \frac{3}{\cos^2 t}\mathrm{d}t = 3(1+\operatorname{tg}^2 t)\,\mathrm{d}t \end{array}\right| = \int \frac{3(1+\operatorname{tg}^2 t)}{9(1+\operatorname{tg}^2 t)\cdot 3\sqrt{1+\operatorname{tg}^2 t}} = \frac{1}{9}\int \cos t\,\mathrm{d}t =$$

$$= \frac{1}{9}\sin t + C$$

$$\boxed{\frac{1}{9}\frac{x}{\sqrt{9+x^2}} + C}$$

(aba)

$$\int \sqrt{3-2x-x^2}\,\mathrm{d}x = \int \sqrt{(x+3)(1-x)}\mathrm{d}x = \left| \begin{array}{ccc} t = \sqrt{\frac{1-x}{x+3}} & x = \frac{1-3t^2}{1+t^2} & x+3 = \frac{4}{1+t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{x+3}{1-x}}\frac{-4\,\mathrm{d}x}{(x+3)^2} & \mathrm{d}x = \frac{-8t\,\mathrm{d}t}{(1+t^2)^2} & 1-x = \frac{4t^2}{1+t^2} \end{array}\right| =$$

$$= \int \frac{4t}{(1+t^2)}\frac{-8t}{(1+t^2)^2}\mathrm{d}t = -32\int \frac{t^2\mathrm{d}t}{(1+t^2)^3} = -32\int \frac{t^2+1-1}{(1+t^2)^3}\mathrm{d}t = -32\int \frac{\mathrm{d}t}{(1+t^2)^2} + 32\int \frac{\mathrm{d}t}{(1+t^2)^3} =$$

$$= -32\left(\frac{1}{2}\operatorname{arctg} t + \frac{1}{2}\frac{t}{1+t^2}\right) + 32\left(\frac{3}{8}\operatorname{arctg} t + \frac{3}{8}\frac{t}{1+t^2} + \frac{1}{4}\frac{t}{(1+t^2)^2}\right) + C = 8\frac{t}{(1+t^2)^2} - 4\frac{t}{1+t^2} - 4\operatorname{arctg} t + C$$

$$\boxed{-4\operatorname{arctg}\sqrt{\frac{1-x}{x+3}} - \frac{x+1}{2}\sqrt{3-2x-x^2} + C}$$

(abb)

$$\int \frac{2x+1}{\sqrt{x^2+x}}\mathrm{d}x = \left| \begin{array}{c} t = x^2+x \\ \mathrm{d}t = (2x+1)\,\mathrm{d}x \end{array}\right| = \int \frac{\mathrm{d}t}{\sqrt{t}} = 2\sqrt{t} + C$$

$$\boxed{2\sqrt{x^2+x}+C}$$

(abc)

$$\int \frac{\sqrt{x^2+2x}}{x}\mathrm{d}x = \left| \begin{array}{ccc} t = \sqrt{\frac{x}{x+2}} & x = \frac{2t^2}{1-t^2} & x+2 = \frac{2}{1-t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{x+2}{x}}\frac{2}{(x+2)^2} & \mathrm{d}x = \frac{4t\,\mathrm{d}t}{(1-t^2)^2} & \end{array}\right| = \int \frac{\frac{2t}{1-t^2}\frac{4t\,\mathrm{d}t}{(1-t^2)^2}}{\frac{2t^2}{1-t^2}} = 4\int \frac{\mathrm{d}t}{(1-t^2)^2} =$$

$$= 4J_2 = \frac{2t}{1-t^2} + \ln\left|\frac{1+t}{1-t}\right| + C$$

$$J_{n-1} = \int \frac{\mathrm{d}t}{(1-t^2)^{n-1}} = \left| \begin{array}{cc} \frac{1}{(1-t^2)^{n-1}} & 1 \\ -(n-1)(1-t^2)^{-n}(-2t) & t \end{array}\right| = \frac{t}{(1-t^2)^{n-1}} + 2(n-1)\int \frac{-t^2\mathrm{d}t}{(1-t^2)^n} =$$

$$= \frac{t}{(1-t^2)^{n-1}} + 2(n-1)\int \frac{1-t^2-1}{(1-t^2)^n}\mathrm{d}t = \frac{t}{(1-t^2)^{n-1}} + 2(n-1)\int \frac{\mathrm{d}t}{(1-t^2)^{n-1}} - 2(n-1)\int \frac{\mathrm{d}t}{(1-t^2)^n} =$$

$$= \frac{t}{(1-t^2)^{n-1}} + (2n-2)J_{n-1} - (2n-2)J_n \Rightarrow J_n = \frac{1}{2n-2}\frac{t}{(1-t^2)^{n-1}} + \frac{2n-3}{2n-2}J_{n-1},\ n>1$$

$$J_1 = \int \frac{1}{1-t^2} = \frac{1}{2}\int \frac{1-t+1+t}{1-t^2}\mathrm{d}t = \frac{1}{2}\int \frac{\mathrm{d}t}{1+t} + \frac{1}{2}\int \frac{\mathrm{d}t}{1-t} = \frac{1}{2}\ln|1+t| - \frac{1}{2}\ln|1-t| + C = \frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C$$

$$J_2 = \frac{1}{2}\frac{t}{1-t^2} + \frac{1}{2}J_1 = \frac{1}{2}\frac{t}{1-t^2} + \frac{1}{4}\ln\left|\frac{1+t}{1-t}\right| + C$$

$$\boxed{\ln|x+1+\sqrt{x^2+2x}| + \sqrt{x^2+2x} + C}$$

(abd)

$$\int \frac{\mathrm{d}x}{\sqrt{25+9x^2}} = \left| \begin{array}{c} x = \frac{5}{3}\operatorname{tg} t \\ \mathrm{d}x = \frac{5}{3}\frac{\mathrm{d}t}{\cos^2 t} \end{array}\right| = \int \frac{5}{3}\frac{\frac{\mathrm{d}t}{\cos^2 t}}{\sqrt{25+9\frac{25}{9}\operatorname{tg}^2 t}} = \frac{1}{3}\int \frac{\mathrm{d}t}{\cos t} = \frac{1}{3}\int \frac{\cos t\,\mathrm{d}t}{1-\sin^2 t} = \left| \begin{array}{c} s = \sin t \\ \mathrm{d}s = \cos t\,\mathrm{d}t \end{array}\right| =$$

$$= \frac{1}{3}\int \frac{\mathrm{d}s}{1-s^2} = \frac{1}{6}\ln\left|\frac{1+s}{1-s}\right| + C = \frac{1}{6}\ln\left|\frac{1+\sin t}{1-\sin t}\right| + C = \left| \begin{array}{cc} \operatorname{tg} t = \frac{3x}{5} & \cos t = \frac{1}{\sqrt{1+\operatorname{tg}^2 t}} = \frac{5}{\sqrt{25+9x^2}} \\ \sin t = \frac{3x}{\sqrt{25+9x^2}} & \end{array}\right| =$$

$$= \frac{1}{6}\ln\left|\frac{1+\frac{3x}{\sqrt{25+9x^2}}}{1-\frac{3x}{\sqrt{25+9x^2}}}\right| + C = \frac{1}{6}\ln\left|\frac{\sqrt{25+9x^2}+3x}{\sqrt{25+9x^2}-3x}\right| + C$$

$$\boxed{\frac{1}{6}\ln\left|\frac{\sqrt{25+9x^2}+3x}{\sqrt{25+9x^2}-3x}\right| + C}$$

(abe)

$$\int \frac{3\,\mathrm{d}x}{\sqrt{9x^2-1}} = \left| \begin{array}{c} t = 3x \\ \mathrm{d}t = 3\,\mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{\sqrt{t^2-1}} = \left| \begin{array}{c} t = \frac{1}{\cos s} \\ \mathrm{d}t = \mathrm{tg}\, s\, \mathrm{d}s \end{array} \right| = \int \frac{\mathrm{tg}\, s\, \mathrm{d}s}{\sqrt{\frac{1}{\cos^2 s} - 1}} = \int \frac{\mathrm{tg}\, s}{\mathrm{tg}\, s} \mathrm{d}s = \int 1\, \mathrm{d}s = s + C =$$

$$= \arccos \frac{1}{t} + C = \arccos \frac{1}{3x} + C$$

$$\boxed{\arccos \frac{1}{3x} + C}$$

(abf)

$$\int \frac{\mathrm{d}x}{x^2\sqrt{9-x^2}} = \left| \begin{array}{ccc} t = \sqrt{\frac{3-x}{3+x}} & x = \frac{3-3t^2}{1+t^2} & 3 - x = \frac{6t^2}{1+t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{3+x}{3-x}}\frac{-6\,\mathrm{d}x}{(3+x)^2} & \mathrm{d}x = \frac{-12t\,\mathrm{d}t}{(1+t^2)^2} & 3 + x = \frac{6}{1+t^2} \end{array} \right| = \int \frac{\frac{-12t\,\mathrm{d}t}{(1+t^2)^2}}{9\left(\frac{1-t^2}{1+t^2}\right)^2 \frac{6t}{1+t^2}} =$$

$$\frac{2}{9}\int \frac{-1-t^2}{(1-t^2)^2}\mathrm{d}t == \frac{2}{9}\int \frac{1-t^2-2}{(1-t^2)^2}\mathrm{d}t = \frac{2}{9}\int \frac{\mathrm{d}t}{1-t^2} - \frac{4}{9}\int \frac{\mathrm{d}t}{(1-t^2)^2} = \frac{1}{9}\ln\left|\frac{1+t}{1-t}\right| - \frac{4}{9}\left(\frac{1}{2}\frac{t}{1-t^2} + \frac{1}{4}\ln\left|\frac{1+t}{1-t}\right|\right) + C =$$

$$-\frac{2}{9}\frac{t}{1-t^2} + C$$

$$\boxed{-\frac{1}{9}\frac{\sqrt{9-x^2}}{x} + C}$$

(abg)

$$\int \sin^3 x \cos x\,\mathrm{d}x = \left| \begin{array}{c} t = \sin x \\ \mathrm{d}t = \cos x\,\mathrm{d}x \end{array} \right| = \int t^3 \mathrm{d}t = \frac{1}{4}t^4 + C$$

$$\boxed{\frac{1}{4}\sin^4 x + C}$$

(abh)

$$\int \cos^5 2x \sin 2x\,\mathrm{d}x = \left| \begin{array}{c} t = \cos 2x \\ \mathrm{d}t = -2\sin 2x\,\mathrm{d}x \end{array} \right| = -\frac{1}{2}\int t^5 \mathrm{d}t = -\frac{1}{12}t^6 + C$$

$$\boxed{-\frac{1}{12}\cos^6 2x + C}$$

(abi)

$$\int \mathrm{tg}\, 4x\,\mathrm{d}x = \int \frac{\sin 4x}{\cos 4x}\mathrm{d}x = \left| \begin{array}{c} t = \cos 4x \\ \mathrm{d}t = -4\sin 4x\,\mathrm{d}x \end{array} \right| = -\frac{1}{4}\frac{\mathrm{d}t}{t} = -\frac{1}{4}\ln|t| + C$$

$$\boxed{-\frac{1}{4}\ln|\cos 4x| + C}$$

(abj)

$$\int \cos^2 2x\,\mathrm{d}x = \frac{1}{2}\int (\cos 4x + 1)\,\mathrm{d}x = \frac{1}{2}\int \cos 4x\,\mathrm{d}x + \frac{1}{2}\int 1\,\mathrm{d}x = \frac{1}{8}\sin 4x + \frac{1}{2}x + C$$

$$\boxed{\frac{1}{8}\sin 4x + \frac{1}{2}x + C}$$

(abk)

$$\int \cos^5 x\,\mathrm{d}x = \int \cos x(1 - \sin^2 x)^2 \mathrm{d}x = \left| \begin{array}{c} t = \sin x \\ \mathrm{d}t = \cos x\,\mathrm{d}x \end{array} \right| = \int (1-t^2)^2 \mathrm{d}t = \int (1 - 2t^2 + t^4)\mathrm{d}t =$$

$$= t - \frac{2}{3}t^3 + \frac{1}{5}t^5 + C$$

$$\boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C}$$

(abl)

$$\int \frac{\mathrm{d}x}{\sin x} = \int \frac{\sin x}{\sin^2 x}\mathrm{d}x = \int \frac{\sin x}{1-\cos^2 x}\mathrm{d}x = \left| \begin{array}{c} t = \cos x \\ \mathrm{d}t = -\sin x\,\mathrm{d}x \end{array} \right| = -\int \frac{\mathrm{d}t}{1-t^2} = -\frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C = \frac{1}{2}\ln\left|\frac{1-t}{1+t}\right| + C$$

$$\boxed{\frac{1}{2}\ln\left|\frac{1-\cos x}{1+\cos x}\right| + C}$$

(abm)

$$\int \frac{\sin^3 x}{\cos^4 x}\mathrm{d}x = \int \frac{\sin^2 x}{\cos^4 x}\sin x\,\mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^4 x}\sin x\,\mathrm{d}x = \left| \begin{array}{c} t = \cos x \\ \mathrm{d}t = -\sin x\,\mathrm{d}x \end{array} \right| = -\int \frac{1-t^2}{t^4}\mathrm{d}t =$$

$$= -\int t^{-4}\mathrm{d}t + \int t^{-2}\mathrm{d}t = \frac{1}{3}\frac{1}{t^3} - \frac{1}{t} + C$$

$$\boxed{\frac{1}{3\cos^3 x} - \frac{1}{\cos x} + C}$$

(abn)

$$\int \frac{\mathrm{d}x}{\sin x \cos^3 x} = \int \frac{\frac{\mathrm{d}x}{\cos^2 x}}{\frac{\sin x}{\cos x}\cos^2 x} = \left| \begin{array}{ll} t = \operatorname{tg} x & \\ \mathrm{d}t = \frac{\mathrm{d}x}{\cos^2 x} & \frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x \end{array} \right| = \int \frac{(1+t^2)\mathrm{d}t}{t} = \ln|t| + \frac{1}{2}t^2 + C$$

$$\boxed{\ln|\operatorname{tg} x| + \frac{1}{2}\operatorname{tg}^2 x + C}$$

(abo)

$$\int \operatorname{cotg}^3 x\,\mathrm{d}x = \int \frac{\cos^3 x}{\sin^3 x}\mathrm{d}x = \left| \begin{array}{ll} \cos^2 x & \frac{\cos x}{\sin^3 x} \\ -2\cos x \sin x & -\frac{1}{2}\frac{1}{\sin^2 x} \end{array} \right| = -\frac{1}{2}\operatorname{cotg}^2 x - \int \frac{\cos x}{\sin x}\mathrm{d}x = \left| \begin{array}{l} t = \sin x \\ \mathrm{d}t = \cos x\,\mathrm{d}x \end{array} \right| =$$

$$= -\frac{1}{2}\operatorname{cotg}^2 x - \int \frac{\mathrm{d}t}{t} = -\frac{1}{2}\operatorname{cotg}^2 x - \ln|t| + C$$

$$\boxed{-\frac{1}{2}\operatorname{cotg}^2 x - \ln|\sin x| + C}$$

(abp)

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}\mathrm{d}x = \left| \begin{array}{ll} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{array} \right| = \int \frac{\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \frac{2\,\mathrm{d}t}{1+t^2} = 2\int \frac{t^2+2t-1}{(1+2t-t^2)(1+t^2)}\mathrm{d}t =$$

$$= -2\int \frac{-t^2+2t+1-4t}{(1+2t-t^2)(1+t^2)}\mathrm{d}t = -2\int \frac{\mathrm{d}t}{1+t^2} + \int \frac{8t\,\mathrm{d}t}{(1+2t-t^2)(1+t^2)} = -2\operatorname{arctg} t + \int \frac{2t-2}{1+2t-t^2}\mathrm{d}t + \int \frac{2t+2}{1+t^2}\mathrm{d}t =$$

$$\left| \frac{8t}{(1+2t-t^2)(1+t^2)} = \frac{At+B}{1+2t-t^2} + \frac{Ct+D}{1+t^2} = \frac{t^3(A-C)+t^2(B+2C-D)+t(A+C-2D)+B+D}{(1+2t-t^2)(1+t^2)} \Rightarrow \begin{array}{c} A = C = D = 2 \\ B = -2 \end{array} \right|$$

$$= -2\operatorname{arctg} t - \int \frac{2-2t}{1+2t-t^2}\mathrm{d}t + \int \frac{2t\,\mathrm{d}t}{1+t^2} + 2\int \frac{\mathrm{d}t}{1+t^2} = -2\operatorname{arctg} t - \ln|1+2t-t^2| + \ln|1+t^2| + 2\operatorname{arctg} t + C =$$

$$= \ln\left|\frac{1+t^2}{1+2t-t^2}\right| + C = \ln\left|\frac{1+\operatorname{tg}^2\frac{x}{2}}{1+2\operatorname{tg} x - \operatorname{tg}^2 x}\right| + C = \ln\left|\frac{\frac{1}{\cos^2\frac{x}{2}}}{1+2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} - \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}\right| + C = \ln\left|\frac{1}{\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} - \sin^2\frac{x}{2}}\right| + C =$$

$$= \ln\left|\frac{1}{\cos x + \sin x}\right| + C = -\ln|\sin x + \cos x| + C$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}\mathrm{d}x = \left| \begin{array}{l} t = \sin x + \cos x \\ \mathrm{d}t = \cos x - \sin x \end{array} \right| = -\int \frac{\mathrm{d}t}{t} = -\ln|t| + C = -\ln|\sin x + \cos x| + C$$

$$\boxed{-\ln|\sin x + \cos x| + C}$$

(abq)

$$\int \frac{\mathrm{d}x}{5-3\cos x} = \left| \begin{array}{ll} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{array} \right| = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{5-3\frac{1-t^2}{1+t^2}} = \int \frac{2\,\mathrm{d}t}{2+8t^2} = \int \frac{\mathrm{d}t}{1+4t^2} = \left| \begin{array}{l} 2t = s \\ 2\mathrm{d}t = \mathrm{d}s \end{array} \right| =$$

$$\frac{1}{2}\int \frac{\mathrm{d}s}{1+s^2} = \frac{1}{2}\operatorname{arctg} s + C = \frac{1}{2}\operatorname{arctg} 2t + C$$

$$\boxed{\frac{1}{2}\operatorname{arctg}(2\operatorname{tg}\frac{x}{2}) + C}$$

(abr)

$$\int \frac{\cos x}{1+\cos x}\mathrm{d}x = \left| \begin{array}{ll} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{array} \right| = \int \frac{\frac{1-t^2}{1+t^2}}{1+\frac{1-t^2}{1+t^2}} \frac{2\,\mathrm{d}t}{1+t^2} = \int \frac{1-t^2}{1+t^2}\mathrm{d}t = -\int \frac{t^2+1-2}{1+t^2}\mathrm{d}t =$$

$$= -\int 1\,\mathrm{d}t + 2\int \frac{\mathrm{d}t}{1+t^2} = -t + 2\operatorname{arctg} t + C = x - \operatorname{tg}\frac{x}{2} + C = x - \frac{\sin x}{1+\cos x} + C$$

$$\boxed{x - \frac{\sin x}{1+\cos x} + C}$$

(abs)

$$\int \frac{\sin x}{1-\sin x}\mathrm{d}x = \left| \begin{array}{ll} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{array} \right| = \int \frac{\frac{2t}{1+t^2}}{1-\frac{2t}{1+t^2}} \frac{2\,\mathrm{d}t}{1+t^2} = \int \frac{4t\,\mathrm{d}t}{(1-2t+t^2)(1+t^2)} = \int \frac{4t\,\mathrm{d}t}{(1-t)^2(1+t^2)} =$$

$$\frac{4t}{(1-t)^2(1+t^2)} = \frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{Ct+D}{1+t^2} = \frac{t^3(-A+C)+t^2(A+B-2C+D)+t(-A+C-2D)+A+B+D}{(1-t)^2(1+t^2)} \Rightarrow \begin{array}{c} A = C = 0 \\ B = 2, D = -2 \end{array}$$

$$= 2\int \frac{\mathrm{d}t}{(1-t)^2} - 2\int \frac{\mathrm{d}t}{1+t^2} = \frac{2}{1-t} - 2\operatorname{arctg} t + C = \frac{2}{1-\operatorname{tg}\frac{x}{2}} - 2\operatorname{arctg}\operatorname{tg}\frac{x}{2} + C = \frac{2}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}} - x + C =$$

$$= -x + \frac{2\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} = -x + \frac{2\cos\frac{x}{2}(\cos\frac{x}{2}+\sin\frac{x}{2})}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} = -x + \frac{2\frac{\cos x+1}{2}+\sin x}{\cos x} = -x+1+\frac{1+\sin x}{\cos x}+C$$

$$\boxed{-x+\frac{1+\sin x}{\cos x}+C}$$

(abt)

$$\int \frac{\mathrm{d}x}{\sin x+\cos x} = \begin{vmatrix} t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{1+t^2} \\ \mathrm{d}x=\frac{2\,\mathrm{d}t}{1+t^2} & \cos x=\frac{1-t^2}{1+t^2}\end{vmatrix} = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{2\,\mathrm{d}t}{1+2t-t^2}$$

$$\frac{2}{1+2t-t^2} = \frac{-2}{t^2-2t-1} = \frac{A}{t-1+\sqrt{2}} + \frac{B}{t-1-\sqrt{2}} = \frac{(A+B)t-(A+B)+\sqrt{2}(-A+B)}{t^2-2t-1} \Rightarrow A=\frac{\sqrt{2}}{2}, B=-\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{2}\int\frac{\mathrm{d}t}{t-1+\sqrt{2}} - \frac{\sqrt{2}}{2}\int\frac{\mathrm{d}t}{t-1-\sqrt{2}} = \frac{\sqrt{2}}{2}\ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}-1+\sqrt{2}}{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}-1-\sqrt{2}}\right| + C =$$

$$= \frac{\sqrt{2}}{2}\ln\left|\frac{\sin\frac{x}{2}+(\sqrt{2}-1)\cos\frac{x}{2}}{\sin\frac{x}{2}-(\sqrt{2}+1)\cos\frac{x}{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{\left(\sin\frac{x}{2}+(\sqrt{2}-1)\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+(\sqrt{2}+1)\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}-(\sqrt{2}+1)\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+(\sqrt{2}+1)\cos\frac{x}{2}\right)}\right| + C =$$

$$= \frac{\sqrt{2}}{2}\ln\left|\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sqrt{2}\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}-(3+2\sqrt{2})\cos^2\frac{x}{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{1+\sqrt{2}\sin x}{\sqrt{2}+1+(2+\sqrt{2})\cos x}\right| + C$$

$$\boxed{\frac{\sqrt{2}}{2}\ln\left|\frac{1+\sqrt{2}\sin x}{\sqrt{2}+1+(2+\sqrt{2})\cos x}\right| + C}$$

(abu)

$$\int \frac{\mathrm{d}x}{\cos x+2\sin x+3} = \begin{vmatrix} t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{1+t^2} \\ \mathrm{d}x=\frac{2\,\mathrm{d}t}{1+t^2} & \cos x=\frac{1-t^2}{1+t^2}\end{vmatrix} = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{\frac{1-t^2}{1+t^2}+\frac{4t}{1+t^2}+3} = \int\frac{2\,\mathrm{d}t}{2t^2+4t+4} = \int\frac{\mathrm{d}t}{t^2+2t+2} =$$

$$= \int\frac{\mathrm{d}t}{(t+1)^2+1} = \begin{vmatrix} s=t+1 \\ \mathrm{d}s=\mathrm{d}t\end{vmatrix} = \int\frac{\mathrm{d}s}{1+s^2} = \operatorname{arctg} s + C = \operatorname{arctg}(t+1)+C = \operatorname{arctg}(1+\operatorname{tg}\frac{x}{2})+C$$

$$\boxed{\operatorname{arctg}(1+\operatorname{tg}\frac{x}{2})+C}$$

(abv)

$$\int\sin 3x\sin 5x\,\mathrm{d}x = \frac{1}{2}\int(\cos 2x-\cos 8x)\mathrm{d}x = \frac{1}{2}\frac{\sin 2x}{2} - \frac{1}{2}\frac{\sin 8x}{8} + C = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$$

$$\cos 8x = \cos(3x+5x) = \cos 3x\cos 5x - \sin 3x\sin 5x \qquad \sin 3x\cos 5x = \frac{1}{2}(\cos 2x-\cos 8x)$$

$$\cos 2x = \cos(-2x) = \cos(3x-5x) = \cos 3x\cos 5x + \sin 3x\sin 5x$$

$$\boxed{\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C}$$

(abw)

$$\int\sin\frac{x}{4}\cos\frac{3x}{4}\,\mathrm{d}x = \frac{1}{2}\int(\sin x-\sin\frac{x}{2})\mathrm{d}x = -\frac{1}{2}\cos x+\cos\frac{x}{2}+C$$

$$\boxed{-\frac{1}{2}\cos x+\cos\frac{x}{2}+C}$$

(abx)

$$\int\sin x\sin 2x\sin 3x\,\mathrm{d}x = \frac{1}{2}\int\sin x(\cos x-\cos 5x)\,\mathrm{d}x = \frac{1}{4}\int\sin 2x\,\mathrm{d}x - \frac{1}{4}\int(\sin 6x-\sin 4x)\mathrm{d}x =$$

$$= -\frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x + C$$

$$\boxed{-\frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x + C}$$

(aby)

$$\int\cosh^3 x\,\mathrm{d}x = \begin{vmatrix} \cosh^2 x & \cosh x \\ 2\cosh x\sinh x & \sinh x\end{vmatrix} = \cosh^2 x\sinh x - 2\int\cosh x\sinh^2 x\,\mathrm{d}x = \begin{vmatrix} t=\sinh x \\ \mathrm{d}t=\cosh x\,\mathrm{d}x\end{vmatrix} =$$

$$= \cosh^2 x\sinh x - 2\int t^2\mathrm{d}t = \cosh^2 x\sinh x - \frac{2}{3}t^3 + C = \frac{2}{3}(\cosh^2 x-\sinh^2 x)\sinh x + \frac{1}{3}\cosh^2 x\sinh x + C =$$

$$= \frac{2}{3}\sinh x - \frac{1}{3}\cosh^2 x\sinh x + C$$

$$\boxed{\frac{2}{3}\sinh x - \frac{1}{3}\cosh^2 x\sinh x + C}$$

(abz)

$$\int\operatorname{tgh} x\,\mathrm{d}x = \int\frac{\sinh x}{\cosh x}\mathrm{d}x = \begin{vmatrix} t=\cosh x \\ \mathrm{d}t=\sinh x\end{vmatrix} = \int\frac{\mathrm{d}t}{t} = \ln\cosh x + C = -x+\ln(1+e^{2x})+C$$

$$\boxed{\ln\cosh x + C}$$

(aca)

$\int \frac{x^2-5x+9}{x^2-5x+6}\mathrm{d}x = \int \frac{x^2-5x+6+3}{x^2-5x+6}\mathrm{d}x = \int 1\,\mathrm{d}x + 3\int \frac{\mathrm{d}x}{(x-2)(x-3)} = x + 3\int \frac{(x-2)-(x-3)}{(x-2)(x-3)}\mathrm{d}x =$

$= x + 3\int \frac{\mathrm{d}x}{x-3} - 3\int \frac{\mathrm{d}x}{x-2} = x + 3\ln|x-3| - 3\ln|x-2| + C = x + 3\ln\left|\frac{x-3}{x-2}\right| + C$

$$\boxed{x + 3\ln\left|\frac{x-3}{x-2}\right| + C}$$

(acb)

$\int \frac{5x^3+2}{x^3-5x^2+4x}\mathrm{d}x = \int \frac{5(x^3-5x^2+4x)+25x^2-20x+2}{(x^2-5x+4)x}\mathrm{d}x = \int 5\,\mathrm{d}x + \int \frac{25x^2-20x+2}{x(x-1)(x-4)}\mathrm{d}x =$

$\left| \frac{25x^2-20x+2}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4} = \frac{A(x^2-5x+4)+B(x^2-4x)+C(x^2-x)}{x(x-1)(x-4)} = \frac{x^2(A+B+C)+x(-5A-4B-C)+4A}{x(x-1)(x-4)} \quad A = \frac{1}{2},\quad B = -\frac{7}{3},\quad C = \frac{161}{6} \right|$

$= 5x + \frac{1}{2}\int\frac{\mathrm{d}x}{x} - \frac{7}{3}\int\frac{dx}{x-1} + \frac{161}{6}\int\frac{\mathrm{d}x}{x-4} = 5x + \frac{1}{2}\ln|x| - \frac{7}{3}\ln|x-1| + \frac{161}{6}\ln|x-4| + C$

$$\boxed{5x + \frac{1}{2}\ln|x| - \frac{7}{3}\ln|x-1| + \frac{161}{6}\ln|x-4| + C}$$

(acc)

$\int \frac{x^2\mathrm{d}x}{x^2-6x+10} = \int \frac{x^2-6x+10+6x-10}{x^2-6x+10}\mathrm{d}x = \int 1\,\mathrm{d}x + \int \frac{6x-10}{x^2-6x+10}\mathrm{d}x = x + \int \frac{3(2x-6)-8}{x^2-6x+10}\mathrm{d}x =$

$= x + 3\int \frac{2x-6}{x^2-6x+10}\mathrm{d}x - 8\int \frac{\mathrm{d}x}{x^2-6x+10} = x + 3\ln(x^2-6x+10) - 8\int \frac{\mathrm{d}x}{(x-3)^2+1} = \left|\begin{matrix} t = x-3 \\ \mathrm{d}t = \mathrm{d}x \end{matrix}\right| =$

$= \ldots - 8\int\frac{\mathrm{d}t}{1+t^2} = \ldots - 8\operatorname{arctg} t + C = \ldots - 8\operatorname{arctg}(x-3) + C$

$$\boxed{5x + 3\ln(x^2-6x+10) - 8\operatorname{arctg}(x+3) + C}$$

(acd)

$\int \frac{x^3+x+1}{x(x^2+1)}\mathrm{d}x = \int 1\,\mathrm{d}x + \int \frac{\mathrm{d}x}{x(x^2+1)} = x + \int\frac{\mathrm{d}x}{x} - \int \frac{x}{1+x^2}\mathrm{d}x = x + \ln|x| - \frac{1}{2}\ln|1+x^2| + C$

$\left| \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(1+x^2)} = \frac{A(1+x^2)+(Bx+C)x}{x(x^2+1)} = \frac{x^2(A+B)+x(C)+A}{x(x^2+1)} \Rightarrow A = 1, B = -1, C = 0 \right|$

$$\boxed{x + \ln|x| - \frac{1}{2}\ln(1+x^2) + C}$$

(ace)

$\int \frac{(x-1)^2}{x^2+3x+4}\mathrm{d}x = \int \frac{x^2-2x+1}{x^2+3x+4}\mathrm{d}x = \int \frac{x^2+3x+4-5x-3}{x^2+3x+4}\mathrm{d}x = \int 1\,\mathrm{d}x - \int \frac{5x+3}{x^2+3x+4} = x - \int \frac{\frac{5}{2}(2x+3)-\frac{9}{2}}{x^2+3x+4} =$

$= x - \frac{5}{2}\ln(x^2+3x+4) - \frac{9}{2}\int\frac{\mathrm{d}x}{x^2+3x+4} = \ldots - \frac{9}{2}\int\frac{\mathrm{d}x}{(x+\frac{3}{2})^2+\frac{7}{4}} = \ldots - \frac{9}{2}\frac{4}{7}\int\frac{\mathrm{d}x}{\left(\frac{2x+3}{\sqrt{7}}\right)^2+1} = \left|\begin{matrix} t = \frac{2x+3}{\sqrt{7}} \\ \mathrm{d}t = \frac{2}{\sqrt{7}}\mathrm{d}x \end{matrix}\right| =$

$= \ldots - \frac{18}{7}\frac{\sqrt{7}}{2}\int\frac{\mathrm{d}t}{1+t^2} = \ldots - \frac{9\sqrt{7}}{7}\operatorname{arctg} t + C = \ldots - \frac{9\sqrt{7}}{7}\operatorname{arctg}\left(\frac{2x+3}{\sqrt{7}}\right) + C$

$$\boxed{x - \frac{5}{2}\ln(x^2+3x+4) - \frac{9\sqrt{7}}{7}\operatorname{arctg}\left(\frac{2x+3}{\sqrt{7}}\right) + C}$$

(acf)

$\int \frac{x^4}{x^4-1}\mathrm{d}x = \int \frac{x^4-1+1}{x^4-1}\mathrm{d}x = \int 1\,\mathrm{d}x + \int \frac{\mathrm{d}x}{(x-1)(x+1)(x^2+1)} = x + \frac{1}{4}\int\frac{\mathrm{d}x}{x-1} - \frac{1}{4}\int\frac{\mathrm{d}x}{x+1} + \frac{1}{2}\int\frac{\mathrm{d}x}{1+t^2} =$

$\left| \frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1} = \frac{A(x+1)(x^2+1)+B(x-1)(x^2+1)+Cx(x^2-1)+D(x^2-1)}{(x-1)(x+1)(x^2+1)} = \frac{A(x^3+x^2+x+1)+B(x^3-x^2+x-1)+C(x^3-x)+D(x^2-1)}{(x-1)(x+1)(x^2+1)} = \frac{x^3(A+B+C)+x^2(A-B+D)+x(A+B-C)+A-B-D}{(x-1)(x+1)(x^2+1)} \Rightarrow A = \frac{1}{4},\quad B = -\frac{1}{4},\quad C = 0,\quad D = \frac{1}{2} \right|$

$= x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\operatorname{arctg} x + C = x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\operatorname{arctg} x + C$

$$\boxed{x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\operatorname{arctg} x + C}$$

(acg)

$\int \frac{2x-3}{(x^2-3x+2)^2}\,\mathrm{d}x = \left| \begin{array}{l} t = x^2-3x+2 \\ \mathrm{d}t = (2x-3)\,\mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{t^2} = -\frac{1}{t} + C$

$$\boxed{-\frac{1}{x^2-3x+2} + C}$$

(ach)

$\int \frac{x^3+x-1}{x(x^2+1)}\,\mathrm{d}x$

Riešenie je takmer rovnaké ako v úlohe $\int \frac{x^3+x+1}{x(x^2+1)}\,\mathrm{d}x$

$$\boxed{x - \ln|x| + \frac{1}{2}\ln(1+x^2) + C}$$

(aci)

$\int \frac{\mathrm{d}x}{x^2+x} = \int \frac{(x+1)-x}{x(x+1)}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{x} - \int \frac{\mathrm{d}x}{x+1} = \ln|x| - \ln|x+1| + C = \ln\left|\frac{x}{x+1}\right| + C$

$$\boxed{\ln\left|\frac{x}{x+1}\right| + C}$$

(acj)

$\int \frac{\mathrm{d}x}{x^2-1} = \int \frac{\frac{(x+1)-(x-1)}{2}}{(x+1)(x-1)}\,\mathrm{d}x = \frac{1}{2}\int \frac{\mathrm{d}x}{x-1} - \frac{1}{2}\int \frac{\mathrm{d}x}{x+1} = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$

$$\boxed{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C}$$

(ack)

$\int \frac{\mathrm{d}x}{x^3+x} = \int \frac{\mathrm{d}x}{x(x^2+1)} = \int \frac{x^2+1-x^2}{x(x^2+1)}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{x} - \int \frac{x}{1+x^2} = \ln|x| - \frac{1}{2}\ln(1+x^2) + C$

$$\boxed{\ln\frac{|x|}{\sqrt{1+x^2}} + C}$$

(acl)

$\int \frac{\mathrm{d}x}{(x-1)(x+2)(x+3)} = \frac{1}{12}\int \frac{\mathrm{d}x}{x-1} - \frac{1}{3}\int \frac{\mathrm{d}x}{x+2} + \frac{1}{4}\int \frac{\mathrm{d}x}{x+3} = \frac{1}{12}\ln|x-1| - \frac{1}{3}\ln|x+2| + \frac{1}{4}\ln|x+3| + C$

$$\left| \begin{array}{l} \frac{1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3} = \frac{A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)}{(x-1)(x+2)(x+3)} = \\ = \frac{A(x^2+5x+6)+B(x^2+2x-3)+C(x^2+x-2)}{(x-1)(x+2)(x+3)} = \frac{x^2(A+B+C)+x(5A+2B+C)+6A-3B-2C}{(x-1)(x+2)(x+3)} \Rightarrow \\ A = \frac{1}{12}, \quad B = -\frac{1}{3}, \quad C = \frac{1}{4} \end{array} \right|$$

$$\boxed{\frac{1}{12}\ln|x-1| - \frac{1}{3}\ln|x+2| + \frac{1}{4}\ln|x+3| + C}$$

(acm)

$\int \frac{\mathrm{d}x}{x(x+1)^2} = \int \frac{(x+1)-x}{x(x+1)^2}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{x(x+1)} - \int \frac{\mathrm{d}x}{(x+1)^2} = \int \frac{(x+1)-x}{x(x+1)}\,\mathrm{d}x + \frac{1}{x+1} = \int \frac{\mathrm{d}x}{x} - \int \frac{\mathrm{d}x}{x+1} + \frac{1}{x+1} =$

$= \ln|x| - \ln|x+1| + \frac{1}{x+1} + C = \ln\left|\frac{x}{x+1}\right| + \frac{1}{x+1} + C$

$$\boxed{\ln\left|\frac{x}{x+1}\right| + \frac{1}{x+1} + C}$$

(acn)

$\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}\,\mathrm{d}x = 4\int \frac{\mathrm{d}x}{x-1} - 7\int \frac{\mathrm{d}x}{x+3} + 5\int \frac{\mathrm{d}x}{x-4} = 4\ln|x-1| - 7\ln|x+3| + 5\ln|x-4| + C$

$$\left| \begin{array}{l} \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x-4} = \frac{A(x+3)(x-4)+B(x-1)(x-4)+C(x-1)(x+3)}{(x-1)(x+3)(x-4)} = \\ = \frac{A(x^2-x-12)+B(x^2-5x+4)+C(x^2+2x-3)}{(x-1)(x+3)(x-4)} = \frac{x^2(A+B+C)+x(-A-5B+2C)+(-12A+4B-3C)}{(x-1)(x+3)(x-4)} \Rightarrow \\ A = 4, \quad B = -7, \quad C = 5 \end{array} \right|$$

$$\boxed{4\ln|x-1| - 7\ln|x+3| + 5\ln|x-4| + C}$$

(aco)

$\int \frac{2\,\mathrm{d}x}{x^2+2x+5} = \int \frac{2\,\mathrm{d}x}{(x+1)^2+4} = \frac{1}{2}\int \frac{\mathrm{d}x}{\left(\frac{x+1}{2}\right)^2+1} = \left| \begin{array}{l} t = \frac{x+1}{2} \\ \mathrm{d}t = \frac{1}{2}\mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{1+t^2} = \mathrm{arctg}\, t + C = \mathrm{arctg}\left(\frac{x+1}{2}\right) + C$

$$\boxed{\operatorname{arctg}\left(\frac{x+1}{2}\right)+C}$$

(acp)

$$\int\frac{\mathrm{d}x}{3x^2+5}=\frac{1}{5}\int\frac{\mathrm{d}x}{\frac{3}{5}x^2+1}=\frac{1}{5}\int\frac{\mathrm{d}x}{\left(\frac{\sqrt{3}x}{\sqrt{5}}\right)^2+1}=\left|\begin{array}{c} t=\sqrt{\frac{3}{5}}x \\ \mathrm{d}t=\sqrt{\frac{3}{5}}\,\mathrm{d}x\end{array}\right|=\frac{1}{5}\sqrt{\frac{5}{3}}\int\frac{\mathrm{d}t}{1+t^2}=\frac{1}{\sqrt{15}}\operatorname{arctg}t+C=$$

$$=\frac{1}{\sqrt{15}}\operatorname{arctg}\sqrt{\frac{3}{5}}x+C=\frac{1}{\sqrt{15}}\operatorname{arctg}\frac{x}{\sqrt{15}}+C$$

$$\boxed{\frac{1}{\sqrt{15}}\operatorname{arctg}\frac{x}{\sqrt{15}}+C}$$

(acq)

$$\int\frac{\mathrm{d}x}{x^3+1}=\int\frac{\mathrm{d}x}{(x+1)(x^2-x+1)}=\frac{1}{3}\int\frac{\mathrm{d}x}{x+1}-\int\frac{\frac{1}{3}x-\frac{2}{3}}{x^2-x+1}=\frac{1}{3}\ln|x+1|-\int\frac{\frac{1}{6}(2x-1)-\frac{1}{2}}{x^2-x+1}=$$

$$=\ldots-\frac{1}{6}\int\frac{2x-1}{x^2-x+1}\mathrm{d}x+\frac{1}{2}\int\frac{\mathrm{d}x}{x^2-x+1}=\ldots-\frac{1}{6}\ln(x^2-x+1)+\frac{1}{2}\int\frac{\mathrm{d}x}{(x-\frac{1}{2})^2+\frac{3}{4}}=\ldots+\frac{2}{3}\int\frac{\mathrm{d}x}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}=$$

$$=\left|\begin{array}{c} t=\frac{2x-1}{\sqrt{3}} \\ \mathrm{d}t=\frac{2}{\sqrt{3}}\mathrm{d}x\end{array}\right|=\ldots+\frac{1}{\sqrt{3}}\int\frac{\mathrm{d}t}{1+t^2}=\ldots+\frac{1}{\sqrt{3}}\operatorname{arctg}t+C=\ldots+\frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right)+C$$

$$\left|\begin{array}{c}\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}=\frac{A(x^2-x+1)+Bx(x+1)+C(x+1)}{(x+1)(x^2-x+1)}= \\ =\frac{A(x^2-x+1)+B(x^2+x)+C(x+1)}{(x+1)(x^2-x+1)}=\frac{x^2(A+B)+x(-A+B+C)+(A+C)}{(x+1)(x^2-x+1)}\Rightarrow \\ A=\frac{1}{3},\quad B=-\frac{1}{3},\quad C=\frac{2}{3}\end{array}\right|$$

$$\boxed{\frac{1}{3}\ln|x+1|-\frac{1}{6}\ln(x^2-x+1)+\frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right)+C}$$

(acr)

$$\int\frac{\mathrm{d}x}{x^3+x^2+x}=\int\frac{\mathrm{d}x}{x(x^2+x+1)}=\int\frac{(x^2+x+1)-(x^2+x)}{x(x^2+x+1)}=\int\frac{\mathrm{d}x}{x}-\int\frac{x+1}{x^2+x+1}\mathrm{d}x=\ln|x|-\int\frac{\frac{1}{2}(2x+1)+\frac{1}{2}}{x^2+x+1}=$$

$$=\ln|x|-\frac{1}{2}\ln(x^2+x+1)-\frac{1}{2}\int\frac{\mathrm{d}x}{(x+\frac{1}{2})^2+\frac{3}{4}}=\ldots-\frac{1}{2}\frac{4}{3}\int\frac{\mathrm{d}x}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}=\ldots-\frac{2}{3}\frac{\sqrt{3}}{2}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)+C=$$

$$=\ln|x|-\frac{1}{2}\ln(x^2+x+1)-\frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)+C$$

$$\boxed{\ln|x|-\frac{1}{2}\ln(x^2+x+1)-\frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)+C}$$

(acs)

$$\int x^2\sin x\,\mathrm{d}x=\begin{vmatrix} x^2 & \sin x \\ 2x & -\cos x\end{vmatrix}=-x^2\cos x+2\int x\cos x\,\mathrm{d}x=\begin{vmatrix} x & \cos x \\ 1 & \sin x\end{vmatrix}=$$

$$=-x^2\cos x+2x\sin x-2\int\sin x\,\mathrm{d}x=-x^2\cos x+2x\sin x+2\cos x+C$$

$$\boxed{(2-x^2)\cos x+2x\sin x+C}$$

(act)

$$c:=\int e^x\cos 2x\,\mathrm{d}x=\begin{vmatrix} \cos 2x & e^x \\ -2\sin 2x & e^x\end{vmatrix}=e^x\cos 2x+2\int e^x\sin 2x\,\mathrm{d}x$$

$$s:=\int e^x\sin 2x\,\mathrm{d}x=\begin{vmatrix} \sin 2x & e^x \\ 2\cos 2x & e^x\end{vmatrix}=e^x\sin 2x-2\int e^x\cos 2x\,\mathrm{d}x$$

$$\begin{array}{rcrcl} c & - & 2s & = & e^x\cos 2x \\ 2c & + & s & = & e^x\sin 2x\end{array}\qquad\begin{array}{l} c=\frac{1}{5}e^x(\cos 2x+2\sin 2x) \\ s=\frac{1}{5}e^x(\sin 2x-2\cos 2x)\end{array}$$

$$\boxed{\frac{1}{5}e^x(\cos 2x+2\sin 2x)+C}$$

(acu)

$$\int(x^2+5)\cos x\,\mathrm{d}x=\begin{vmatrix} x^2+5 & \cos x \\ 2x & \sin x\end{vmatrix}=(x^2+5)\sin x-2\int x\sin x=\begin{vmatrix} x & \sin x \\ 1 & -\cos x\end{vmatrix}=$$

$$=(x^2+5)\sin x+2x\cos x-\int\cos x\,\mathrm{d}x=(x^2+5)\sin x+2x\cos x-\sin x+C=(x^2+4)\sin x+2x\cos x+C$$

$$\boxed{(x^2+4)\sin x + 2x\cos x + C}$$

(acv)

$\int x^2 \sinh x \,\mathrm{d}x = \begin{vmatrix} x^2 & \sinh x \\ 2x & \cosh x \end{vmatrix} = x^2\cosh x - 2\int x\cosh x = \begin{vmatrix} x & \cosh x \\ 1 & \sinh x \end{vmatrix} =$

$= x^2\cosh x - 2x\sinh x + 2\int \sinh x \,\mathrm{d}x = x^2\cosh x - 2x\sinh x + 2\cosh x + C$

$$\boxed{(x^2+2)\cosh x - 2x\sinh x + C}$$

(acw)

$\int (x^2-2x+5)e^{-x}\mathrm{d}x = \begin{vmatrix} x^2-2x+5 & e^{-x} \\ 2x-2 & -e^{-x} \end{vmatrix} = -(x^2-2x+5)e^{-x} + 2\int (x-1)e^{-x}\mathrm{d}x =$

$= \begin{vmatrix} x-1 & e^{-x} \\ 1 & -e^{-x} \end{vmatrix} = -(x^2-2x+5)e^{-x} - 2(x-1)e^{-x} + 2\int e^{-x}\mathrm{d}x = -(x^2+3)e^{-x} - 2e^{-x} + C$

$$\boxed{-(x^2+5)e^{-x} + C}$$

(acx)

$\int x\ln^2 x \,\mathrm{d}x = \begin{vmatrix} \ln^2 x & x \\ 2\frac{\ln x}{x} & \frac{x^2}{2} \end{vmatrix} = \frac{1}{2}x^2\ln^2 x - \int x\ln x \,\mathrm{d}x = \begin{vmatrix} \ln x & x \\ \frac{1}{x} & \frac{1}{2}x^2 \end{vmatrix} = \frac{1}{2}x^2(\ln^2 x - \ln x) + \frac{1}{2}\int x \,\mathrm{d}x =$

$= \frac{1}{2}x^2(\ln^2 x - \ln x) + \frac{1}{4}x^2 + C =$

$$\boxed{\tfrac{1}{4}(2\ln^2 x - 2\ln x + 1) + C}$$

(acy)

$s := \int e^{-2x}\sin\frac{x}{2}\,\mathrm{d}x = \begin{vmatrix} e^{-2x} & \sin\frac{x}{2} \\ -2e^{-2x} & -2\cos\frac{x}{2} \end{vmatrix} = -2e^{-2x}\cos\frac{x}{2} - 4\int e^{-2x}\cos\frac{x}{2}\mathrm{d}x$

$c := \int e^{-2x}\cos\frac{x}{2}\mathrm{d}x = \begin{vmatrix} e^{-2x} & \cos\frac{x}{2} \\ -2e^{-2x} & 2\sin\frac{x}{2} \end{vmatrix} = 2e^{-2x}\sin\frac{x}{2} + 4\int e^{-2x}\sin\frac{x}{2}\mathrm{d}x$

$$\begin{array}{rcrcl} s & + & 4c & = & -2e^{-2x}\cos\frac{x}{2} \\ -4s & + & c & = & 2e^{-2x}\sin\frac{x}{2} \end{array} \qquad \begin{array}{l} s = -\frac{2}{17}e^{-2x}(\cos\frac{x}{2} + 4\sin\frac{x}{2}) \\ c = \frac{2}{17}e^{-2x}(\sin\frac{x}{2} - 4\cos\frac{x}{2}) \end{array}$$

$$\boxed{-\tfrac{2}{17}e^{-2x}(\cos\tfrac{x}{2} + 4\sin\tfrac{x}{2}) + C}$$

(acz)

$\int \sin(\ln x)\,\mathrm{d}x = \begin{vmatrix} \sin(\ln x) & 1 \\ \cos(\ln x)\frac{1}{x} & x \end{vmatrix} = x\sin(\ln x) - \int \cos(\ln x)\,\mathrm{d}x$

$\int \cos(\ln x)\,\mathrm{d}x = \begin{vmatrix} \cos(\ln x) & 1 \\ -\sin(\ln x)\frac{1}{x} & x \end{vmatrix} = x\cos(\ln x) + \int \sin(\ln x)\,\mathrm{d}x$

$$\begin{array}{rl} S + C = x\sin(\ln x) & S = \frac{1}{2}x\big(\sin(\ln x) - \cos(\ln x)\big) \\ -S + C = x\cos(\ln x) & C = \frac{1}{2}x\big(\sin(\ln x) + \cos(\ln x)\big) \end{array}$$

$$\boxed{\tfrac{1}{2}x\big(\sin(\ln x) - \cos(\ln x)\big) + C}$$

(ada)

$\int x^2 e^{3x}\mathrm{d}x = \begin{vmatrix} x^2 & e^{3x} \\ 2x & \frac{1}{3}e^{3x} \end{vmatrix} = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}\mathrm{d}x = \begin{vmatrix} x & e^{3x} \\ 1 & \frac{1}{3}e^{3x} \end{vmatrix} = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\int e^{3x}\mathrm{d}x =$

$= \frac{1}{9}e^{3x}\left(3x^2 - 2x\right) + \frac{2}{27}e^{3x} + C = \frac{1}{27}e^{3x}(9x^2 - 6x + 2) + C$

$$\boxed{\tfrac{1}{27}e^{3x}(9x^2 - 6x + 2) + C}$$

(adb)

$\int (x^2+5x+6)\cos 2x \,\mathrm{d}x = \begin{vmatrix} x^2+5x+6 & \cos 2x \\ 2x+5 & \frac{1}{2}\sin 2x \end{vmatrix} = \frac{1}{2}(x^2+5x+6)\sin 2x - \frac{1}{2}\int (2x+5)\sin 2x \,\mathrm{d}x =$

$$= \begin{vmatrix} 2x+5 & \sin 2x \\ 2 & -\frac{1}{2}\cos 2x \end{vmatrix} = \frac{1}{2}(x^2+5x+6)\sin 2x + \frac{1}{4}(2x+5)\cos 2x - \frac{1}{2}\int \cos 2x \, \mathrm{d}x =$$

$$= \frac{1}{2}(x^2+5x+6)\sin 2x + \frac{1}{4}(2x+5)\cos 2x - \frac{1}{2}\sin 2x + C = \frac{1}{2}(x^2+5x+5)\sin 2x + \frac{1}{4}(2x+5)\cos 2x + C$$

$$\boxed{\frac{1}{2}(x^2+5x+5)\sin 2x + \frac{1}{4}(2x+5)\cos 2x + C}$$

(adc)

$$\int x^3 \cos x \, \mathrm{d}x = \begin{vmatrix} x^3 & \cos x \\ 3x^2 & \sin x \end{vmatrix} = x^3 \sin x - 3\int x^2 \sin x \, \mathrm{d}x (= (x^3-6x)\sin x - (6-3x^2)\cos x + C)$$

$$\int x^2 \sin x \, \mathrm{d}x = \begin{vmatrix} x^2 & \sin x \\ 2x & -\cos x \end{vmatrix} = -x^2 \cos x + 2\int x \cos x \, \mathrm{d}x (= (2-x^2)\cos x + 2x\sin x + C)$$

$$\int x \cos x \, \mathrm{d}x = \begin{vmatrix} x & \cos x \\ 1 & \sin x \end{vmatrix} = x \sin x - \int \sin x \, \mathrm{d}x = x\sin x + \cos x + C$$

$$\boxed{(x^3-6x)\sin x - 3(2-x^2)\cos x + C}$$

(add)

$$\int x \ln x \, \mathrm{d}x = \begin{vmatrix} \ln x & x \\ \frac{1}{x} & \frac{x^2}{2} \end{vmatrix} = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, \mathrm{d}x = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$\boxed{\frac{1}{4}x^2(2\ln x - 1) + C}$$

(ade)

$$\int x \sin 3x \, \mathrm{d}x = \begin{vmatrix} x & \sin 3x \\ 1 & -\frac{1}{3}\cos 3x \end{vmatrix} = -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x \, \mathrm{d}x = -\frac{1}{3}x \cos 3x + \frac{1}{9}\sin 3x + C$$

$$\boxed{\frac{1}{9}(\sin 3x - 3x\cos 3x) + C}$$

(adf)

$$\int 5xe^{-4x} \mathrm{d}x = 5\int xe^{-4x}\mathrm{d}x = \begin{vmatrix} x & e^{-4x} \\ 1 & -\frac{1}{4}e^{-4x} \end{vmatrix} = -\frac{5}{4}xe^{-4x} + \frac{5}{4}\int e^{-4x}\mathrm{d}x = -\frac{5}{4}xe^{-4x} - \frac{5}{16}e^{-4x} + C$$

$$\boxed{-\frac{5}{16}e^{-4x}(1+4x) + C}$$

(adg)

$$\int x \operatorname{arctg} x \, \mathrm{d}x = \begin{vmatrix} x \operatorname{arctg} x & 1 \\ \operatorname{arctg} x - \frac{x}{1+x^2} & x \end{vmatrix} = x^2 \operatorname{arctg} x - \int x \operatorname{arctg} x \, \mathrm{d}x + \int \frac{x^2}{1+x^2}\mathrm{d}x = \ldots + \int \frac{x^2+1-1}{1+x^2}\mathrm{d}x =$$

$$= \ldots + \int 1 \, \mathrm{d}x - \int \frac{\mathrm{d}x}{1+x^2} = x^2 \operatorname{arctg} x - \int x \operatorname{arctg} x \, \mathrm{d}x + x - \operatorname{arctg} x$$

$$\boxed{\frac{1}{2}\big((x^2-1)\operatorname{arctg} x + x\big) + C}$$

(adh)

$$\int \arccos x \, \mathrm{d}x = \begin{vmatrix} \arccos x & 1 \\ -\frac{1}{\sqrt{1-x^2}} & x \end{vmatrix} = x \arccos x + \int \frac{x}{\sqrt{1-x^2}}\mathrm{d}x = \begin{vmatrix} t = 1-x^2 \\ \mathrm{d}t = -2x \, \mathrm{d}x \end{vmatrix} = x \arccos x - \frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t}} =$$

$$= x\arccos x - \sqrt{t} + C = x \arccos x - \sqrt{1-x^2} + C$$

$$\boxed{x \arccos x - \sqrt{1-x^2} + C}$$

(adi)

$$\int x \cosh x \, \mathrm{d}x = \begin{vmatrix} x & \cosh x \\ 1 & \sinh x \end{vmatrix} = x \sinh x - \int \sinh x \, \mathrm{d}x = x\sinh x - \cosh x + C$$

$$\boxed{x \sinh x - \cosh x + C}$$

(adj)

$$\int (2x+1)\cos(\tfrac{\pi}{3} - 5x) \, \mathrm{d}x = \begin{vmatrix} 2x+1 & \cos(\frac{\pi}{3}-5x) \\ 2 & -\frac{1}{5}\sin(\frac{\pi}{3}-5x) \end{vmatrix} = -\frac{1}{5}(2x+1)\sin(\tfrac{\pi}{3}-5x) + \frac{2}{5}\int \sin(\tfrac{\pi}{3}-5x) \, \mathrm{d}x =$$

$= -\frac{1}{5}(2x+1)\sin(\frac{\pi}{3}-5x) + \frac{2}{25}\cos(\frac{\pi}{3}-5x) + C$

$$\boxed{-\frac{1}{5}(2x+1)\sin(\frac{\pi}{3}-5x) + \frac{2}{25}\cos(\frac{\pi}{3}-5x) + C}$$

(adk)

$\int \frac{x}{5^x}\mathrm{d}x = \begin{vmatrix} x & 5^{-x} \\ 1 & -\frac{1}{\ln 5}5^{-x} \end{vmatrix} = -\frac{1}{\ln 5}x5^{-x} + \frac{1}{\ln 5}\int 5^{-x}\mathrm{d}x = -\frac{1}{\ln 5}\frac{x}{5^x} - \frac{1}{\ln^2 5}5^{-x} + C$

$$\boxed{-\frac{1}{\ln 5}\frac{x}{5^x} - \frac{1}{\ln^2 5}5^{-x} + C}$$

(adl)

$\int \frac{\ln x}{\sqrt{x}}\mathrm{d}x = \begin{vmatrix} \ln x & \frac{1}{\sqrt{x}} \\ \frac{1}{x} & 2\sqrt{x} \end{vmatrix} = 2\sqrt{x}\ln x - 2\int \frac{\mathrm{d}x}{\sqrt{x}} = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

$$\boxed{2\sqrt{x}(\ln x - 2) + C}$$

(adm)

$\int 4x^3\ln(x^5)\,\mathrm{d}x = 5\int 4x^3\ln x\,\mathrm{d}x = \begin{vmatrix} \ln x & 4x^3 \\ \frac{1}{x} & x^4 \end{vmatrix} = 5x^4\ln x - 5\int x^3\mathrm{d}x = 5x^4\ln x - \frac{5}{4}x^4 + C$

$$\boxed{\frac{5}{4}x^4(4\ln x - 1) + C}$$

(adn)

$\int \ln x\,\mathrm{d}x = \begin{vmatrix} \ln x & 1 \\ \frac{1}{x} & x \end{vmatrix} = x\ln x - \int 1\,\mathrm{d}x = x\ln x - x + C = x(\ln x - 1) + C$

$$\boxed{x(\ln x - 1) + C}$$

(ado)

$\int \frac{\ln x}{x^2}\mathrm{d}x = \begin{vmatrix} \ln x & \frac{1}{x^2} \\ \frac{1}{x} & -\frac{1}{x} \end{vmatrix} = -\frac{1}{x}\ln x + \int \frac{\mathrm{d}x}{x^2} = -\frac{1}{x}\ln x - \frac{1}{x} + C$

$$\boxed{-\frac{1}{x}(\ln x + 1) + C}$$

(adp)

$\int x\cos x\,\mathrm{d}x = \begin{vmatrix} x & \cos x \\ 1 & \sin x \end{vmatrix} = x\sin x - \int \sin x\,\mathrm{d}x = x\sin x + \cos x + C$

$$\boxed{x\sin x + \cos x + C}$$

(adq)

$\int xe^{-2x}\mathrm{d}x = \begin{vmatrix} x & e^{-2x} \\ 1 & -\frac{1}{2}e^{-2x} \end{vmatrix} = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\mathrm{d}x = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$

$$\boxed{-\frac{1}{4}e^{-2x}(2x+1) + C}$$

(adr)

$\int \operatorname{arccotg} x\,\mathrm{d}x = \begin{vmatrix} \operatorname{arccotg} x & 1 \\ -\frac{1}{1+x^2} & x \end{vmatrix} = x\operatorname{arccotg} x + \int \frac{x}{1+x^2}\mathrm{d}x = x\operatorname{arccotg} x + \frac{1}{2}\ln(1+x^2) + C$

$$\boxed{x\operatorname{arccotg} x + \frac{1}{2}\ln(1+x^2) + C}$$

(ads)

$\int \frac{x}{\sin^2 x}\mathrm{d}x = \begin{vmatrix} x & \frac{1}{\sin^2 x} \\ 1 & -\operatorname{cotg} x \end{vmatrix} = -x\operatorname{cotg} x + \int \operatorname{cotg} x\,\mathrm{d}x = -x\operatorname{cotg} x + \int \frac{\cos x}{\sin x}\mathrm{d}x =$

$= -x\operatorname{cotg} x + \ln|\sin x| + C$

$$\boxed{-x\operatorname{cotg} x + \ln|\sin x| + C}$$

(adt)

$\int \frac{x\cos x}{\sin^3 x}\mathrm{d}x = \begin{vmatrix} x & \frac{\cos x}{\sin^3 x} \\ 1 & -\frac{1}{2}\frac{1}{\sin^2 x} \end{vmatrix} = -\frac{1}{2}\frac{x}{\sin^2 x} + \frac{1}{2}\int \frac{\mathrm{d}x}{\sin^2 x} = -\frac{1}{2}\frac{x}{\sin^2 x} - \operatorname{cotg} x + C$

$$\boxed{-\frac{1}{2}\frac{x}{\sin^2 x} - \operatorname{cotg} x + C}$$

(adu)

$\int x\sinh x\,\mathrm{d}x = \begin{vmatrix} x & \sinh x \\ 1 & \cosh x \end{vmatrix} = x\cosh x - \int \cosh x\,\mathrm{d}x = x\cosh x - \sinh x + C$

$$\boxed{x\cosh x - \sinh x + C}$$

(adv)

$\int \sqrt{1-x^2}\mathrm{d}x = \begin{vmatrix} \sqrt{1-x^2} & 1 \\ -\frac{x}{\sqrt{1-x^2}} & x \end{vmatrix} = x\sqrt{1-x^2} + \int \frac{x^2}{\sqrt{1-x^2}}\mathrm{d}x = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\mathrm{d}x =$

$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\mathrm{d}x + \int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = x\sqrt{1-x^2} + \arcsin x - \int \sqrt{1-x^2}\mathrm{d}x$

$$\boxed{\frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2} + C}$$

(adw)

$\int x\operatorname{tg}^2 x\,\mathrm{d}x = \begin{vmatrix} x & \operatorname{tg}^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} \\ 1 & \operatorname{tg} x - x \end{vmatrix} = x\operatorname{tg} x - x^2 - \int \operatorname{tg} x\,\mathrm{d}x + \int x\,dx =$

$= x\operatorname{tg} x - x^2 + \ln|\cos x| + \frac{1}{2}x^2 + C$

$$\boxed{x\operatorname{tg} x + \ln|\cos x| - \frac{1}{2}x^2 + C}$$

(adx)

$\int \sqrt{4x-11}\,\mathrm{d}x = \begin{vmatrix} t = 4x-11 \\ \mathrm{d}t = 4\,\mathrm{d}x \end{vmatrix} = \frac{1}{4}\int \sqrt{t}\,\mathrm{d}t = \frac{1}{4}\frac{2}{3}t^{\frac{3}{2}} + C = \frac{1}{6}(4x-11)^{\frac{3}{2}} + C$

$$\boxed{\frac{1}{6}(4x-11)^{\frac{3}{2}} + C}$$

(ady)

$\int \frac{6}{5-3x}\mathrm{d}x = \begin{vmatrix} t = 5-3x \\ \mathrm{d}t = -3\,\mathrm{d}x \end{vmatrix} = -2\int \frac{\mathrm{d}t}{t} = -2\ln|t| + C = -2\ln|5-3x| + C$

$$\boxed{-2\ln|5-3x| + C}$$

(adz)

$\int \frac{4x}{4+x^2}\mathrm{d}x = \begin{vmatrix} t = 4+x^2 \\ \mathrm{d}t = 2x\,\mathrm{d}x \end{vmatrix} = 2\int \frac{\mathrm{d}t}{t} = 2\ln|t| = 2\ln(4+x^2) + C$

$$\boxed{2\ln(4+x^2) + C}$$

(aea)

$\int \frac{14\mathrm{d}x}{(2x+3)^8} = \begin{vmatrix} t = 2x+3 \\ \mathrm{d}t = 2\mathrm{d}x \end{vmatrix} = 7\int \frac{\mathrm{d}t}{t^8} = 7\int t^{-8}\mathrm{d}t = -t^{-7} + C = -(2x+3)^{-7} + C$

$$\boxed{-(2x+3)^{-7} + C}$$

(aeb)

$\int 10x(x^2+7)^4\mathrm{d}x = \begin{vmatrix} t = x^2+7 \\ \mathrm{d}t = 2x\,\mathrm{d}x \end{vmatrix} = 5\int t^4\mathrm{d}t = t^5 + C = (x^2+7)^5 + C$

$$\boxed{(x^2+7)^5 + C}$$

(aec)

$\int \frac{x\,\mathrm{d}x}{\sqrt{3-x^2}} = \begin{vmatrix} t = 3-x^2 \\ \mathrm{d}t = -2x\,\mathrm{d}x \end{vmatrix} = -\frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t}} = -\sqrt{t} + C = \sqrt{3-x^2} + C$

$$\boxed{\sqrt{3-x^2}+C}$$

(aed)

$$\int \frac{x^2}{1+x^6}\mathrm{d}x = \int \frac{x^2}{1+(x^3)^2}\mathrm{d}x = \left| \begin{array}{c} t = x^3 \\ \mathrm{d}t = 3x^2\mathrm{d}x \end{array} \right| = \frac{1}{3}\int \frac{\mathrm{d}t}{1+t^2} = \frac{1}{3}\operatorname{arctg} t + C = \frac{1}{3}\operatorname{arctg} x^3 + C$$

$$\boxed{\frac{1}{3}\operatorname{arctg} x^3 + C}$$

(aee)

$$\int x\sqrt[5]{4-x^2}\,\mathrm{d}x = \left| \begin{array}{c} t = 4-x^2 \\ \mathrm{d}t = -2x\,\mathrm{d}x \end{array} \right| = -\frac{1}{2}\int \sqrt[5]{t}\,\mathrm{d}t = -\frac{1}{2}\int t^{\frac{1}{5}}\mathrm{d}t = -\frac{1}{2}\frac{5}{6}t^{\frac{6}{5}} + C = -\frac{5}{12}(4-x^2)^{\frac{6}{5}} + C$$

$$\boxed{-\frac{5}{12}(4-x^2)^{\frac{6}{5}} + C}$$

(aef)

$$\int \sin^6 x\cos x\,\mathrm{d}x = \left| \begin{array}{c} t = \sin x \\ \mathrm{d}t = \cos x\,\mathrm{d}x \end{array} \right| = \int t^6\mathrm{d}t = \frac{1}{7}t^7 + C = \frac{1}{7}\sin^7 x + C$$

$$\boxed{\frac{1}{7}\sin^7 x + C}$$

(aeg)

$$\int \frac{\sin x}{\sqrt{2+\cos x}}\mathrm{d}x = \left| \begin{array}{c} t = 2+\cos x \\ \mathrm{d}t = -\sin x\,\mathrm{d}x \end{array} \right| = -\int \frac{\mathrm{d}t}{\sqrt{t}} = -2\sqrt{t} + C = -2\sqrt{2+\cos x} + C$$

$$\boxed{-2\sqrt{2+\cos x} + C}$$

(aeh)

$$\int \frac{\mathrm{d}x}{x^2+2x+2} = \int \frac{\mathrm{d}x}{(x+1)^2+1} = \left| \begin{array}{c} t = x+1 \\ \mathrm{d}t = \mathrm{d}x \end{array} \right| = \int \frac{\mathrm{d}t}{1+t^2} = \operatorname{arctg} t + C = \operatorname{arctg}(x+1) + C$$

$$\boxed{\operatorname{arctg}(x+1) + C}$$

(aei)

$$\int \frac{\mathrm{d}x}{\sqrt{4x-4x^2}} = \int \frac{\mathrm{d}x}{\sqrt{1-(2x-1)^2}} = \left| \begin{array}{c} t = (2x-1) \\ \mathrm{d}t = 2\,\mathrm{d}x \end{array} \right| = \frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{1-t^2}} = \frac{1}{2}\arcsin t + C = \frac{1}{2}\arcsin(2x-1) + C$$

$$\boxed{\frac{1}{2}\arcsin(2x-1) + C}$$

(aej)

$$\int \frac{e^{\frac{1}{x}}}{x^2}\mathrm{d}x = \left| \begin{array}{c} t = \frac{1}{x} \\ \mathrm{d}t = \frac{\mathrm{d}x}{x^2} \end{array} \right| = \int e^t\mathrm{d}t = e^t + C = e^{\frac{1}{x}} + C$$

$$\boxed{e^{\frac{1}{x}} + C}$$

(aek)

$$\int (x+2)e^{x^2+4x-5}\mathrm{d}x = \left| \begin{array}{c} t = x^2+4x-5 \\ \mathrm{d}t = (2x+4)\,\mathrm{d}x \end{array} \right| = \frac{1}{2}\int e^t\mathrm{d}t = \frac{1}{2}e^t + C = \frac{1}{2}e^{x^2+4x-5} + C$$

$$\boxed{\frac{1}{2}e^{x^2+4x-5} + C}$$

(ael)

$$\int \frac{\ln^4 x}{x}\mathrm{d}x = \left| \begin{array}{c} t = \ln x \\ \mathrm{d}t = \frac{1}{x}\mathrm{d}x \end{array} \right| = \int t^4\mathrm{d}t = \frac{1}{5}t^5 + C = \frac{1}{5}\ln^5 x + C$$

$$\boxed{\frac{1}{5}\ln^5 x + C}$$

(aem)

$$\int \frac{\cos(\ln x)}{x}\mathrm{d}x = \left| \begin{array}{c} t = \ln x \\ \mathrm{d}t = \frac{1}{x}\mathrm{d}x \end{array} \right| = \int \cos t\,\mathrm{d}t = \sin t + C = \sin(\ln x) + C$$

$$\boxed{\sin(\ln x) + C}$$

(aen)

$$\int e^{\cos^2 x} \sin 2x \, \mathrm{d}x = \left| \begin{array}{c} t = \cos^2 x \\ \mathrm{d}t = -2\cos x \sin x \, \mathrm{d}x = -\sin 2x \, \mathrm{d}x \end{array} \right| = -\int e^t \mathrm{d}t = -e^t + C = -e^{\cos^2 x} + C$$

$$\boxed{-e^{\cos^2 x} + C}$$

(aeo)

$$\int \frac{\operatorname{cotg}\sqrt{x}}{\sqrt{x}} \mathrm{d}x = \left| \begin{array}{c} t = \sqrt{x} \\ \mathrm{d}t = \frac{\mathrm{d}x}{2\sqrt{x}} \end{array} \right| = 2\int \operatorname{cotg} t \, \mathrm{d}t = 2\int \frac{\cos t}{\sin t} \mathrm{d}t = \left| \begin{array}{c} s = \sin t \\ \mathrm{d}s = \cos t \, \mathrm{d}t \end{array} \right| = 2\int \frac{\mathrm{d}s}{s} = 2\ln|s| + C =$$
$$= 2\ln|\sin t| + C = 2\ln|\sin\sqrt{x}| + C$$

$$\boxed{2\ln|\sin\sqrt{x}| + C}$$

(aep)

$$\int \frac{\sqrt[3]{\operatorname{tg}^2 x}}{\cos^2 x} \mathrm{d}x = \left| \begin{array}{c} t = \operatorname{tg} x \\ \mathrm{d}t = \frac{\mathrm{d}x}{\cos^2 x} \end{array} \right| = \int t^{\frac{2}{3}} \mathrm{d}t = \frac{3}{5} t^{\frac{5}{3}} + C = \frac{3}{5} \operatorname{tg}^{\frac{5}{3}} + C$$

$$\boxed{\frac{3}{5} \operatorname{tg}^{\frac{5}{3}} + C}$$

(aeq)

$$\int \frac{\mathrm{d}x}{\sin^2 x \sqrt{\operatorname{cotg} x - 1}} = \left| \begin{array}{c} t = \operatorname{cotg} x - 1 \\ \mathrm{d}t = -\frac{\mathrm{d}x}{\sin^2 x} \end{array} \right| = -\int \frac{\mathrm{d}t}{\sqrt{t}} = -2\sqrt{t} + C = -2\sqrt{\operatorname{cotg} x - 1} + C$$

$$\boxed{-2\sqrt{\operatorname{cotg} x - 1} + C}$$

(aer)

$$\int \frac{2^x}{\sqrt{1-4^x}} \mathrm{d}x = \left| \begin{array}{c} t = 2^x \\ \mathrm{d}t = 2^x \ln 2 \, \mathrm{d}x \end{array} \right| = \frac{1}{\ln 2} \int \frac{\mathrm{d}t}{\sqrt{1-t^2}} = \frac{1}{\ln 2} \arcsin t + C = \frac{1}{\ln 2} \arcsin 2^x + C$$

$$\boxed{\frac{1}{\ln 2} \arcsin 2^x + C}$$

(aes)

$$\int \frac{e^{2x}}{4+e^x} \mathrm{d}x = \int \frac{e^x(e^x+4) - 4e^x}{4+e^x} = \int e^x \mathrm{d}x - 4\int \frac{e^x}{4+e^x} \mathrm{d}x = \left| \begin{array}{c} t = e^x \\ \mathrm{d}t = e^x \mathrm{d}x \end{array} \right| = e^x - 4\int \frac{\mathrm{d}t}{4+t} =$$
$$= e^x - 4\ln|4+t| + C = e^x - 4\ln|4+e^x| + C$$

$$\int \frac{e^{2x}}{4+e^x} \mathrm{d}x = \left| \begin{array}{c} t = e^x \\ \mathrm{d}t = e^x \mathrm{d}x \end{array} \right| = \int \frac{t \, \mathrm{d}t}{4+t} = \int \frac{4+t-4}{4+t} = \int 1 \, \mathrm{d}t - 4\int \frac{1}{4+t} \mathrm{d}t = t - 4\ln|4+t| + C = e^x - 4\ln|4+e^x| + C$$

$$\boxed{e^x - 4\ln|4+e^x| + C}$$

(aet)

$$\int \frac{\mathrm{d}x}{(1+x^2)\operatorname{arctg} x} = \left| \begin{array}{c} t = \operatorname{arctg} x \\ \mathrm{d}t = \frac{\mathrm{d}x}{1+x^2} \end{array} \right| = \int \frac{\mathrm{d}t}{t} = \ln|t| + C = \ln|\operatorname{arctg} x| + C$$

$$\boxed{\ln|\operatorname{arctg} x| + C}$$

(aeu)

$$\int \frac{3\,\mathrm{d}x}{x\sqrt{1-\ln^2 x}} = \left| \begin{array}{c} t = \ln x \\ \mathrm{d}t = \frac{1}{x}\mathrm{d}x \end{array} \right| = 3\int \frac{\mathrm{d}t}{\sqrt{1-t^2}} = 3\arcsin t + C = 3\arcsin(\ln x) + C$$

$$\boxed{3\arcsin(\ln x) + C}$$

(aev)

$$\int \frac{x}{\sqrt{x^2-4}} \mathrm{d}x = \left| \begin{array}{l} t = x^2 - 4 \\ \mathrm{d}t = 2x \, \mathrm{d}x \quad \mathrm{d}x = \frac{\mathrm{d}t}{2x} \end{array} \right| = \int \frac{x \frac{\mathrm{d}t}{2x}}{\sqrt{x^2-4}} \mathrm{d}t = \frac{1}{2} \int \frac{\mathrm{d}t}{\sqrt{t}} = \sqrt{t} + C = \sqrt{x^2-4} + C$$

$$\boxed{\sqrt{x^2-4}+C}$$

(aew)

$$\int \frac{\cos x}{1+\sin x}\mathrm{d}x = \left| \begin{array}{c} t=\sin x \\ \mathrm{d}t=\cos x\,\mathrm{d}x \quad \mathrm{d}x=\frac{\mathrm{d}t}{\cos x} \end{array} \right| = \int \frac{\cos x\frac{\mathrm{d}t}{\cos x}}{1+\sin x} = \int \frac{\mathrm{d}t}{1+t} = \ln|1+t|+C = \ln|1+\sin x|+C$$

$$\boxed{\ln|1+\sin x|+C}$$

(aex)

$$\int \sqrt{\cos^3 x}\sin x\,\mathrm{d}x = \left| \begin{array}{c} t=\cos x \\ \mathrm{d}t=-\sin x\,\mathrm{d}x \quad \mathrm{d}x=-\frac{\mathrm{d}t}{\sin x} \end{array} \right| = \int \sqrt{t^3}\sin x\frac{-\mathrm{d}t}{\sin x} = -\int t^{\frac{3}{2}}\mathrm{d}t =$$

$$= -\frac{2}{5}t^{\frac{5}{2}}+C = -\frac{2}{5}\cos^{\frac{5}{2}}x+C$$

$$\boxed{-\frac{2}{5}\cos^{\frac{5}{2}}x+C}$$

(aey)

$$\int xe^{x^2}\mathrm{d}x = \left| \begin{array}{c} t=x^2 \\ \mathrm{d}t=2x\,\mathrm{d}x \quad \mathrm{d}x=\frac{\mathrm{d}t}{2x} \end{array} \right| = \int xe^{x^2}\frac{\mathrm{d}t}{2x} = \frac{1}{2}\int e^t\,\mathrm{d}t = \frac{1}{2}e^t+C = \frac{1}{2}e^{x^2}+C$$

$$\boxed{\frac{1}{2}e^{x^2}+C}$$

(aez)

$$\int \frac{\mathrm{d}x}{x\ln x} = \left| \begin{array}{l} t=\ln x \\ \mathrm{d}t=\frac{\mathrm{d}x}{x} \quad \mathrm{d}x=x\,\mathrm{d}t \end{array} \right| = \int \frac{x\,\mathrm{d}t}{x\ln x} = \int \frac{\mathrm{d}t}{t} = \ln|t|+C = \ln|\ln x|+C$$

$$\boxed{\ln|\ln x|+C}$$

(afa)

$$\int x^2\sqrt{x^3+1}\,\mathrm{d}x = \left| \begin{array}{c} t=x^3+1 \\ \mathrm{d}t=3x^2\mathrm{d}x \quad \mathrm{d}x=\frac{\mathrm{d}t}{3x^2} \end{array} \right| = \int x^2\sqrt{x^3+1}\frac{\mathrm{d}t}{3x^2} = \frac{1}{3}\int\sqrt{t}\,\mathrm{d}t = \frac{1}{3}\cdot\frac{2}{3}t^{\frac{3}{2}}+C =$$

$$= \frac{2}{9}(x^3+1)^{\frac{3}{2}}+C$$

$$\boxed{\frac{2}{9}(x^3+1)^{\frac{3}{2}}+C}$$

(afb)

$$\int \frac{\mathrm{d}x}{\sqrt{x}(x+4)} = \left| \begin{array}{cc} t=\frac{\sqrt{x}}{2} & x=4t^2 \\ \mathrm{d}t=\frac{1}{4\sqrt{x}}\mathrm{d}x & \mathrm{d}x=4\sqrt{x}\,\mathrm{d}t \end{array} \right| = \int \frac{4\sqrt{x}\,\mathrm{d}t}{\sqrt{x}(x+4)} = \int \frac{4\,\mathrm{d}t}{4+4t^2} = \int \frac{\mathrm{d}t}{1+t^2} =$$

$$= \operatorname{arctg} t+C = \operatorname{arctg}\frac{\sqrt{x}}{2}+C$$

$$\boxed{\operatorname{arctg}\frac{\sqrt{x}}{2}+C}$$

(afc)

$$\int \frac{x}{1+x^4}\mathrm{d}x = \left| \begin{array}{c} t=x^2 \\ \mathrm{d}t=2x\,\mathrm{d}x \quad \mathrm{d}x=\frac{\mathrm{d}t}{2x} \end{array} \right| = \int \frac{x\frac{\mathrm{d}t}{2x}}{1+x^4} = \frac{1}{2}\int\frac{\mathrm{d}t}{1+t^2} = \frac{1}{2}\operatorname{arctg} t+C = \frac{1}{2}\operatorname{arctg} x^2+C$$

$$\boxed{\frac{1}{2}\operatorname{arctg} x^2+C}$$

(afd)

$$\int \frac{\mathrm{d}x}{e^x-1} = \left| \begin{array}{c} t=e^{-x} \\ \mathrm{d}t=-e^{-x}\mathrm{d}x \quad \mathrm{d}x=-e^x\mathrm{d}t \end{array} \right| = \int \frac{-e^x\mathrm{d}t}{e^x-1} = -\int\frac{\mathrm{d}t}{1-e^{-x}} = -\int\frac{\mathrm{d}t}{1-t} = \ln|1-t|+C =$$

$$= \ln|1-e^{-x}|+C = \ln|1-\frac{1}{e^x}|+C = \ln|e^x-1|-\ln|e^x|+C = \ln|e^x-1|-x+C$$

$$\boxed{x-\ln|1+e^x|+C}$$

(afe)

$$\int \frac{e^x\sqrt{\operatorname{arctg} e^x}}{1+e^{2x}}\mathrm{d}x = \left| \begin{array}{l} t=\operatorname{arctg} e^x \\ \mathrm{d}t=\frac{e^x\mathrm{d}x}{1+e^{2x}} \quad \mathrm{d}x=\frac{1+e^{2x}}{e^x}\mathrm{d}t \end{array} \right| = \int \frac{e^x\sqrt{\operatorname{arctg} e^x}}{1+e^{2x}}\frac{1+e^{2x}}{e^x}\mathrm{d}t = \int\sqrt{t} = \frac{2}{3}t^{\frac{3}{2}}+C =$$

$= \frac{2}{3}\operatorname{arctg}^{\frac{3}{2}} e^x + C$

$$\boxed{\frac{2}{3}\operatorname{arctg}^{\frac{3}{2}} e^x + C}$$

(aff)

$\int \frac{\mathrm{d}x}{x\sqrt{x^2-1}}\mathrm{d}x = \left| \begin{matrix} t = \frac{1}{x} & \\ \mathrm{d}t = -\frac{\mathrm{d}x}{x^2} & \mathrm{d}x = -x^2\,\mathrm{d}t \end{matrix} \right| = \int \frac{-x^2\,\mathrm{d}t}{x\sqrt{x^2-1}} = -\int \frac{\mathrm{d}t}{\sqrt{1-\frac{1}{x^2}}} = -\int \frac{\mathrm{d}t}{\sqrt{1-t^2}} =$

$= \arccos t + C = \arccos\frac{1}{x} + C (= -\arcsin\frac{1}{x} + C)$

$$\boxed{\arccos\frac{1}{x} + C}$$

(afg)

$\int \frac{x\,\mathrm{d}x}{\sqrt{x+1}} = \left| \begin{matrix} t = \sqrt{x+1} & x = t^2 - 1 \\ \mathrm{d}t = \frac{\mathrm{d}x}{2\sqrt{x+1}} & \mathrm{d}x = 2\sqrt{x+1}\,\mathrm{d}t \end{matrix} \right| = \int \frac{x \cdot 2\sqrt{x+1}\mathrm{d}t}{\sqrt{x+1}} = 2\int (t^2-1)\,\mathrm{d}t =$

$= \frac{2}{3}t^3 - 2t + C = \frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1} + C$

$$\boxed{\frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1} + C}$$

(afh)

$\int \sin 3x\,\mathrm{d}x = \left| \begin{matrix} t = 3x & \\ \mathrm{d}t = 3\,\mathrm{d}x & \mathrm{d}x = \frac{1}{3}\mathrm{d}t \end{matrix} \right| = \frac{1}{3}\int \sin t\,\mathrm{d}t = -\frac{1}{3}\cos t + C = -\frac{1}{3}\cos 3x + C$

$$\boxed{-\frac{1}{3}\cos 3x + C}$$

(afi)

$\int \frac{\mathrm{d}x}{5-3x} = \left| \begin{matrix} t = 5-3x \\ \mathrm{d}t = -3\mathrm{d}x \end{matrix} \right| = -\frac{1}{3}\int \frac{\mathrm{d}t}{t} = -\frac{1}{3}\ln|t| + C = -\frac{1}{3}\ln|5-3x| + C$

$$\boxed{-\frac{1}{3}\ln|5-3x| + C}$$

(afj)

$\int e^{3-2x}\mathrm{d}x = \left| \begin{matrix} t = 3-2x \\ \mathrm{d}t = -2\,\mathrm{d}x \end{matrix} \right| = -\frac{1}{2}\int e^t\mathrm{d}t = -\frac{1}{2}e^t + C = -\frac{1}{2}e^{3-2x} + C$

$$\boxed{-\frac{1}{2}e^{3-2x} + C}$$

(afk)

$\int \sqrt[3]{3x-2}\,\mathrm{d}x = \left| \begin{matrix} t = 3x-2 \\ \mathrm{d}t = 3\,\mathrm{d}x \end{matrix} \right| = \frac{1}{3}\int \sqrt[3]{t}\ \mathrm{d}t = \frac{1}{3}\int t^{\frac{1}{3}}\mathrm{d}t = \frac{1}{3}\frac{3}{4}t^{\frac{4}{3}} + C = \frac{1}{4}t^{\frac{4}{3}} + C = \frac{1}{4}(3x-2)^{\frac{4}{3}} + C$

$$\boxed{\frac{1}{4}(3x-2)^{\frac{4}{3}} + C}$$

(afl)

$\int (4-7x)^{11} = \left| \begin{matrix} t = 4x-7 \\ \mathrm{d}t = 4\,\mathrm{d}x \end{matrix} \right| = \frac{1}{4}\int t^{11}\mathrm{d}t = \frac{1}{48}t^{12} + C = \frac{1}{48}(4x-7)^{12} + C$

$$\boxed{\frac{1}{48}(4x-7)^{12} + C}$$

(afm)

$\int \frac{\mathrm{d}x}{\cos^2 5x} = \left| \begin{matrix} t = 5x \\ \mathrm{d}t = 5\,\mathrm{d}x \end{matrix} \right| = \frac{1}{5}\int \frac{\mathrm{d}t}{\cos^2 t} = \frac{1}{5}\operatorname{tg} t + C = \frac{1}{5}\operatorname{tg} 5x + C$

$$\boxed{\frac{1}{5}\operatorname{tg} 5x + C}$$

(afn)

$\int \frac{\mathrm{d}x}{\sqrt{9-x^2}} = \frac{1}{3}\int \frac{\mathrm{d}x}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \left| \begin{matrix} t = \frac{x}{3} \\ \mathrm{d}t = \frac{1}{3}\mathrm{d}x \end{matrix} \right| = \int \frac{\mathrm{d}t}{\sqrt{1-t^2}} = \arcsin t + C = \arcsin\frac{x}{3} + C$

$$\boxed{\arcsin\frac{x}{3}+C}$$

(afo)

$$\int\frac{\mathrm{d}x}{x^2+16}=\left|\begin{array}{ll} t=\frac{x}{4} & x=4t \\ \mathrm{d}t=\frac{\mathrm{d}x}{4} & \mathrm{d}x=4\,\mathrm{d}t\end{array}\right|=\int\frac{4\,\mathrm{d}t}{(4t)^2+16}=\frac{1}{4}\int\frac{\mathrm{d}t}{1+t^2}=\frac{1}{4}\operatorname{arctg}t+C=\frac{1}{4}\operatorname{arctg}\frac{x}{4}+C$$

$$\boxed{\frac{1}{4}\operatorname{arctg}\frac{x}{4}+C}$$

(afp)

$$\int(3x^2+2x-1)\,\mathrm{d}x=\int 3x^2\mathrm{d}x+2\int x\,\mathrm{d}x-\int 1\,\mathrm{d}x=x^3+x^2-x+C$$

$$\boxed{x^3+x^2-x+C}$$

(afq)

$$\int\left(\frac{2}{x\sqrt{x}}-\frac{5}{x^2}\right)\mathrm{d}x=\int\left(2x^{-\frac{3}{2}}-5x^{-2}\right)\mathrm{d}x=2\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}-5\frac{x^{-2+1}}{-2+1}+C=-4x^{-\frac{1}{2}}+\frac{5}{x}+C$$

$$\boxed{-\frac{4}{\sqrt{x}}+\frac{5}{x}+C}$$

(afr)

$$\int x^2(x^2+1)\,\mathrm{d}x=\int(x^4+x^2)\,\mathrm{d}x=\int x^4\mathrm{d}x+\int x^2\mathrm{d}x=\frac{1}{5}x^5+\frac{1}{3}x^3+C$$

$$\boxed{\frac{1}{5}x^5+\frac{1}{3}x^3+C}$$

(afs)

$$\int(x^3+1)^2\,\mathrm{d}x=\int(x^6+2x^3+1)\,\mathrm{d}x=\int x^6\mathrm{d}x+2\int x^3\mathrm{d}x+\int 1\,\mathrm{d}x=\frac{1}{7}x^7+\frac{2}{4}x^4+x+C=\frac{1}{7}x^2+\frac{1}{2}x^4+x+C$$

$$\boxed{\frac{1}{7}x^2+\frac{1}{2}x^4+x+C}$$

(aft)

$$\int\frac{x^3+3x-1}{x}\mathrm{d}x=\int\left(x^2+3-\frac{1}{x}\right)\mathrm{d}x=\int x^2\mathrm{d}x+\int 3\,\mathrm{d}x-\int\frac{1}{x}\mathrm{d}x=\frac{1}{3}x^3+3x-\ln|x|+C$$

$$\boxed{\frac{1}{3}x^3+3x-\ln|x|+C}$$

(afu)

$$\int\frac{x^2-3x+4}{\sqrt{x}}\mathrm{d}x=\int\frac{x^2-3x+4}{x^{\frac{1}{2}}}\mathrm{d}x=\int(x^{\frac{3}{2}}-3x^{\frac{1}{2}}+4x^{-\frac{1}{2}})\,\mathrm{d}x=\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}-\frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=$$
$$=\frac{2}{5}x^{\frac{5}{2}}-2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C=\frac{2}{5}\sqrt{x^5}-2\sqrt{x^3}+8\sqrt{x}+C$$

$$\boxed{\frac{2}{5}\sqrt{x^5}-2\sqrt{x^3}+8\sqrt{x}+C}$$

(afv)

$$\int\frac{(x-1)^3}{\sqrt{x}}\mathrm{d}x=\int\frac{x^3-3x^2+3x-1}{x^{\frac{1}{2}}}\mathrm{d}x=\int(x^{\frac{5}{2}}-3x^{\frac{3}{2}}+3x^{\frac{1}{2}}-x^{-\frac{1}{2}})\,\mathrm{d}x=\frac{2}{7}x^{\frac{7}{2}}-\frac{6}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}-2x^{\frac{1}{2}}+C$$

$$\boxed{\frac{2}{7}x^{\frac{7}{2}}-\frac{6}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}-2x^{\frac{1}{2}}+C}$$

(afw)

$$\int\frac{(\sqrt{x}+2)^3}{x}\mathrm{d}x=\int\frac{(\sqrt{x})^3+3\cdot2\cdot(\sqrt{x})^2+3\cdot2^2\sqrt{x}+2^3}{x}\mathrm{d}x=\int\frac{x^{\frac{3}{2}}+6x+12x^{\frac{1}{2}}+8}{x}\mathrm{d}x=$$
$$=\int x^{\frac{1}{2}}\mathrm{d}x+\int 6\,\mathrm{d}x+12\int x^{-\frac{1}{2}}\mathrm{d}x+8\int\frac{\mathrm{d}x}{x}=\frac{2}{3}x^{\frac{3}{2}}+6x+24\sqrt{x}+8\ln|x|+C$$

$$\boxed{\frac{2}{3}x^{\frac{3}{2}}+6x+24\sqrt{x}+8\ln|x|+C}$$

(afx)

$$\int(\cos x+2\sqrt[5]{x^3})\,\mathrm{d}x=\int\cos x\,\mathrm{d}x+2\int x^{\frac{3}{5}}\mathrm{d}x=\sin x+\frac{5}{4}x^{\frac{8}{5}}+C$$

$$\boxed{\sin x+\frac{5}{4}x^{\frac{8}{5}}+C}$$

(afy)

$$\int\left(\sin x+\frac{3}{\sqrt{4-4x^2}}\right)\mathrm{d}x=\int\sin x\,dx+\frac{3}{2}\int\frac{\mathrm{d}x}{\sqrt{1-x^2}}=-\cos x+\frac{3}{2}\arcsin x+C$$

$$\boxed{-\cos x + \frac{3}{2}\arcsin x + C}$$

(afz)

$\int \left(2^x + \sqrt{\frac{1}{x}}\right)\,\mathrm{d}x = \int 2^x \mathrm{d}x + \int \sqrt{x^{-1}}\mathrm{d}x = \frac{2^x}{\ln 2} + \int x^{-\frac{1}{2}}\mathrm{d}x = \frac{2^x}{\ln 2} + 2\sqrt{x} + C$

$$\boxed{\frac{2^x}{\ln 2} + 2\sqrt{x} + C}$$

(aga)

$\int \left(10^{-x} + \frac{x^2+2}{x^2+1}\right)\,\mathrm{d}x = \int 10^{-x}\mathrm{d}x + \int \frac{x^2+1+1}{x^2+1}\mathrm{d}x = \int \frac{1}{10^x}\mathrm{d}x + \int 1\,\mathrm{d}x + \int \frac{\mathrm{d}x}{1+x^2} = \int \left(\frac{1}{10}\right)^x \mathrm{d}x + x + \operatorname{arctg} x =$

$= \frac{\left(\frac{1}{10}\right)^x}{\ln\frac{1}{10}} + x + \operatorname{arctg} x + C = \frac{10^{-x}}{-\ln 10} + x + \operatorname{arctg} x + C = -\frac{10^{-x}}{\ln 10} + x + \operatorname{arctg} x + C$

$$\boxed{-\frac{10^{-x}}{\ln 10} + x + \operatorname{arctg} x + C}$$

(agb)

$\int \frac{x^2}{3(1+x^2)}\mathrm{d}x = \frac{1}{3}\int \frac{x^2+1-1}{x^2+1}\mathrm{d}x = \frac{1}{3}\int \left(1 - \frac{1}{1+x^2}\right)\,\mathrm{d}x = \frac{1}{3}\int 1\,\mathrm{d}x - \frac{1}{3}\int \frac{\mathrm{d}x}{1+x^2} = \frac{1}{3}x - \frac{1}{3}\operatorname{arctg} x + C$

$$\boxed{\frac{1}{3}x - \frac{1}{3}\operatorname{arctg} x + C}$$

(agc)

$\int \operatorname{cotg}^2 x\,\mathrm{d}x = \int \frac{\cos^2 x}{\sin^2 x}\mathrm{d}x = \int \frac{1-\sin^2 x}{\sin^2 x}\mathrm{d}x = \int \frac{\mathrm{d}x}{\sin^2 x} - \int 1\,\mathrm{d}x = -\operatorname{cotg} x - x + C$

$$\boxed{-\operatorname{cotg} x - x + C}$$

(agd)

$\int (\sqrt{x}+1)(x-\sqrt{x}+1)\,\mathrm{d}x = \int \big(x(\sqrt{x}+1) - (\sqrt{x}+1)(\sqrt{x}-1)\big)\,\mathrm{d}x = \int (x^{\frac{3}{2}} + x - x + 1)\,\mathrm{d}x =$

$= \int x^{\frac{3}{2}}\mathrm{d}x + \int 1\,\mathrm{d}x = \frac{2}{5}x^{\frac{5}{2}} + x + C$

$$\boxed{\frac{2}{5}x^{\frac{5}{2}} + x + C}$$

(age)

$\int \frac{\mathrm{d}x}{x^2+7} =$ /vzorec/ $= \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C$

$$\boxed{\frac{1}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C}$$

(agf)

$\int 4^{2-3x}\mathrm{d}x = \int 4^2 \cdot 4^{-3x}\mathrm{d}x = 16\int (4^{-3})^x \mathrm{d}x = 16\int \left(\frac{1}{64}\right)^x \mathrm{d}x = 16\frac{\left(\frac{1}{64}\right)^x}{\ln\frac{1}{64}} + C =$

$= 16\frac{64^{-x}}{-\ln 2^6} + C = -\frac{16}{6\ln 2}64^{-x} + C = -\frac{8}{3\ln 2}64^{-x} + C = -\frac{8}{3\ln 2}4^{-3x} + C$

$$\boxed{-\frac{8}{3\ln 2}4^{-3x} + C}$$

(agg)

$\int \frac{x}{(x+1)^2}\mathrm{d}x = \int \frac{x+1-1}{(x+1)^2}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{x+1} - \int \frac{\mathrm{d}x}{(x+1)^2} = \ln|x+1| + \frac{1}{x+1} + C$

$$\boxed{\ln|x+1| + \frac{1}{x+1} + C}$$