1. $\lim_{(x,y)\to(0,0)}(1+xy)^{|x|+|y|}$ — 1

2. $f(x,y)=\frac{x+y}{x^2+y}, a=\infty, b=\infty$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y = 1$, $\lim_y \lim_x = 0$

3. $f(x,y)=\frac{y^x}{1+y^{2x}}, a=0, b=\infty$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y = 0$, $\lim_y \lim_x = \frac{1}{2}$

4. $f(x,y)=\frac{\sin(\pi x)}{2x+y}, a=\infty, b=\infty$ — $\lim_{xy} = 0$, $\lim_x \lim_y = 0$, $\lim_y \lim_x = 0$

5. $f(x,y)=\frac{x+y}{x-y}, a=0, b=0$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y = 1$, $\lim_y \lim_x = -1$

6. $f(x,y)=\frac{x^2+y^2}{1+(x-y)^4}, a=\infty, b=\infty$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y = 0$, $\lim_y \lim_x = 0$

7. $f(x,y)=(x+y)\operatorname{tg}\frac{1}{x}\operatorname{tg}\frac{1}{y}, a=0, b=0$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y$ neexistuje, $\lim_y \lim_x$ neexistuje

8. $f(x,y)=\begin{cases} 4-x-y & x\neq 2, y\neq 1 \\ 3 & x=2, y=1 \end{cases}, a=2, b=1$ — $\lim_{xy} = 1$, $\lim_x \lim_y = 1$, $\lim_y \lim_x = 1$

9. $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}, a=0, b=0$ — $\lim_{xy}$ neexistuje, $\lim_x \lim_y = 0$, $\lim_y \lim_x = 0$

Určte body nespojitosti:

10. $f(x,y)=\sin\frac{1}{x-y}$ — $D(f)=\{(x,y)\in R^2 : x\neq y\}$, spojitá na $D(f)$

11. $f(x,y)=\frac{x^2+3y^2+5}{y^2-2x}$ — $D(f)=\{(x,y)\in R^2 : y^2\neq 2x\}$, spojitá na $D(f)$

12. $f(x,y)=\begin{cases} \cos\frac{1}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$ — $D(f)=R^2$, spojitá na $R^2-\{(0,0)\}$

13. $z=\frac{x+y}{x-y}$ — $D(f)=\{(x,y)\in R^2 : x\neq y\}$, spojitá na $D(f)$

14. $z=\sin\frac{1}{|x|-|y|}$ — $D(f)=\{(x,y) : |x|\neq|y|\}$, spojitá na $D(f)$

15. $z=\ln|1-x^2-y^2|$ — $D(f)=\{(x,y)\in R^2 : x^2+y^2\neq 1\}$, spojitá na $D(f)$

16. $f(x,y,z)=\frac{3z}{x-2y+3z}$ — $D(f)=\{(x,y,z)\in R^3 : x-2y+3z\neq 0\}$, spojitá na $D(f)$

17. $f(x,y)=x^2-2xy-3y^2, A=(-1,1)$ — $\mathrm{D}f(\mathbf{x},A)=-4x-4y$

18. $f(x,y)=e^{xy}, A=(0,0)$ — $\mathrm{d}f(\mathbf{x},A)=0$

19. $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}, A=(0,0)$ — $\mathrm{d}f(\mathbf{x},A)=0$

20. $f(x,y,z)=x^5y^3z^3$ — $\mathrm{d}f(\mathbf{x},.)=5x^4y^3z^3\mathrm{d}x+3x^5y^2z^3\mathrm{d}y+3x^5y^3z^2\mathrm{d}z$

21. $f(x,y)=\ln\sqrt{x^2+y^2}$ — $\mathrm{d}f(\mathbf{x},.)=\frac{x}{x^2+y^2}\mathrm{d}x+\frac{y}{x^2+y^2}\mathrm{d}y$

22. $f(x,y)=e^{\alpha x}\cos\left(\frac{\beta y}{x}\right)$ — $\mathrm{d}f(\mathbf{x},.)=e^{\alpha x}\left(\alpha\cos\left(\frac{\beta y}{x}\right)+\beta\frac{y}{x^2}\sin\left(\frac{\beta y}{x}\right)\right)\mathrm{d}x-e^{\alpha x}\cdot\frac{\beta}{x}\cdot\sin\left(\frac{\beta y}{x}\right)\mathrm{d}y$

23. $f(x,y,z)=\operatorname{arctg}\left(\frac{yz}{x^2}\right)$ — $\mathrm{d}f(\mathbf{x},.)=-2\frac{x}{yz}\mathrm{d}x+\frac{x^2}{y^2z}\mathrm{d}y+\frac{x^2}{yz^2}\mathrm{d}z$

24. $\sqrt{3{,}03^2+9{,}01^2}$ — $\frac{30{,}06}{\sqrt{10}}\doteq 9{,}5058066-9{,}5058403$

25. $1{,}05^{2{,}01}$ — $1{,}1$ - $1{,}103038$

26. $\sin 151^\circ \operatorname{cotg} 41^\circ$ — $\frac{1}{2} + \frac{8-\sqrt{3}}{360}\pi \doteq 0{,}554698 - 0{,}5577097$

27. $\ln(\sqrt{0{,}96} + \sqrt[3]{1{,}02} + 2)$ — $2\ln 2 + \frac{1}{600} - 0{,}005 \doteq 1{,}38296 - 1{,}38279$

28. $f(x,y) = e^{xy}, A = (0,0)$ — $\mathrm{d}^2 f(\mathbf{x}, A) = 2xy$

29. $f(x,y,z) = xy + yz + xz,\ A = (1,1,0)$ — $\mathrm{d}^2 f(\mathbf{x}, A) = 2(xy + xz + yz - x - y - 2z + 1)$

30. $f(x,y,z) = \frac{z}{x^2+y^2}, A = (1,1-2)$ — $\mathrm{d}^2 f(\mathbf{x}, A) = -x^2 - y^2 + 4xy - xz - yz - 4x - 4y + 2z + 6$

31. $f(x,y) = x^2y^2, A = (1,1)$ — $\mathrm{d}^2 f(\mathbf{x}, A) = 2x^2 + 8xy + 2y^2 - 12x - 12y + 12$

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32. $f(x,y,z) = xyz, A = (7,11,-10)$ — $\mathrm{d}^3 f(\mathbf{x}, A) = 6(x-7)(y-11)(z+10)$

33. $f(x,y) = \cos(x^2+y^2), n = 5$ — $\mathrm{T}(f, 5, (0,0)) = 1 - \frac{1}{2}(x^2+y^2)^2$

34. $f(x,y) = e^x \sin y, n = 3$ — $\mathrm{T}(f, 3, (0,0)) = y + xy + \frac{1}{2}x^2y - \frac{1}{3!}y^3$

35. $f(x,y) = x^3 + y^3 - 2xy, n = 3$ — $\mathrm{T}(f, 3, (0,0)) = x^3 + y^3 - 2xy$

36. $f(x,y) = x^y,\ A = (1,1),\ n = 2$ — $\mathrm{T}(f, 2, A) = 1 + (x-1)^2 + (x-1)(y-1)$

37. $f(x,y) = \sin x \cos y,\ A = (\frac{\pi}{4}, \frac{\pi}{4}),\ n = 3$

$\mathrm{T}(f,3,A) = \frac{1}{2} + \frac{1}{2}(x-\frac{\pi}{4}) - \frac{1}{2}(y-\frac{\pi}{4}) - \frac{1}{4}(x-\frac{\pi}{4})^2 - \frac{1}{2}(x-\frac{\pi}{4})(y-\frac{\pi}{4}) - \frac{1}{4}(y-\frac{\pi}{4})^2 - \frac{1}{12}(x-\frac{\pi}{4})^3 + \frac{1}{4}(x-\frac{\pi}{4})^2(y-\frac{\pi}{4}) - \frac{1}{4}(x-\frac{\pi}{4})(y-\frac{\pi}{4})^2 + \frac{1}{12}(y-\frac{\pi}{4})^3$

38. $f(x,y) = 1 + 6x - y^2 - xy - x^2$ — $(4,-2)_{max}$

39. $f(z,t) = 5 + 6z - 4z^2 - 3t^2$ — $(\frac{4}{3}, 0)_{max}$

40. $f(x,y) = x^3 + y^3 - 18xy + 215$ — $(0,0)_{sedlo}, (6,6)_{min}$

41. $f(x,y) = \sqrt{(a-x)(a-y)(x+y-a)}$ — $(\frac{2}{3}a, \frac{2}{3}a)_{max}$

42. $f(x,y,z) = \frac{x}{y+z} + \frac{y}{x+z} + \frac{z}{x+y}$ — $(t,t,t)_{min},\ t \neq 0$

43. $z = xy - x + y - 1$, ak $x + y = 1$ — $(-\frac{1}{2}, \frac{3}{2})_{max}$

44. $z = x + y$, ak $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}, a > 0$ — $(-\sqrt{2}a, -\sqrt{2}a)_{max}, (\sqrt{2}a, \sqrt{2}a)_{min}$

45. $z = x^2 + y^2$, ak $\frac{x}{p} + \frac{y}{q} = 1$ — $\left(\frac{pq^2}{p^2+q^2}, \frac{p^2q}{p^2+q^2}\right)$ min

46. $u = \cos x \cos y \cos z$, ak $x + y + z = -\pi$

Maximá: $(-\frac{2\pi}{3} + k\pi, -\frac{2\pi}{3} + l\pi, -(k+l-\frac{1}{3})\pi)$, $(\frac{2\pi}{3} + k\pi, \frac{2\pi}{3} + l\pi, -(k+l+\frac{7}{3})\pi)$,
$(\frac{2\pi}{3} + 2k\pi, -\frac{\pi}{3} + 2l\pi, -(2k+2l+\frac{4}{3})\pi)$, $(-\frac{2\pi}{3} + 2k\pi, \frac{\pi}{3} + 2l\pi, -(2k+2l+\frac{2}{3})\pi)$,
$(-\frac{\pi}{3} + 2l\pi, \frac{2\pi}{3} + 2k\pi, -(2k+2l+\frac{2}{3})\pi)$, $(\frac{\pi}{3} + 2l\pi, -\frac{2\pi}{3} + 2k\pi, -(2k+2l+\frac{4}{3})\pi)$, $k, l \in Z$
Minimá: $(k\pi, l\pi, -(1+k+l)\pi)$, $k, l \in Z$

47. $u = xyz$, ak $x^2 + y^2 + z^2 = 3$

Maximá: $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, $(-1,-1,1)$, hodnota 1
Minimá: $(-1,-1,-1)$, $(-1,1,1)$, $(1,-1,1)$, $(1,1,-1)$, hodnota -1

48. $u = x^2 + y^2 + z^2$, ak $x + y - 3z + 7 = 0, x - y + z - 3 = 0$

$(0,-1,2)_{min}$

49. $f(x,y) = x^2 - 2y^2 + 4xy - 6x - 1$ na oblasti $x \geq 0$, $y \geq 0$, $y \leq -x + 3$

$(0,3)_{min}, f = -19$, $(0,0)_{max}, f = -1$

50. $f(x,y) = xy^2(4 - x - y)$ na oblasti ohraničenej krivkami $x = 0, y = 0, x + y = 6$

$(1,2)_{max}, f(1,2) = 4$; $(2,4)_{max}$, $f(2,4) = -64$

51. $f(x,y) = x^3 + y^3 - 3xy$ na obdĺžniku s vrcholmi $A = (0,-1)$, $B = (2,-1)$, $C = (2,2)$, $D = (0,2)$

Minimum: $(0,-1)$, $(1,1)$, hodnota -1; Maximum: $(2,-1)$, hodnota 13

52. $f(x,y) = \cos x \cos y \cos(x+y)$ na štvorci $A = (0,0)$, $B = (\pi,0)$, $C = (\pi,\pi)$, $D = (0,\pi)$

Minimum: $(\frac{\pi}{3}, \frac{\pi}{3})$, hodnota $-\frac{1}{8}$, Maximum: $(0,0)$, $(\pi,0)$, (π,π), $(0,\pi)$, hodnota 1

53. $f(x,y) = x^2 + y^2$, na kruhu $x^2 + y^2 \leq 4$

$\{(x,y) : x^2 + y^2 = 4\}_{max}$, $(0,0)_{min}$

54. $f(x,y,z) = x + y + z$, na oblasti $1 \geq x \geq y^2 + z^2$

Minimum: $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}), f = -\frac{1}{2}$, Maximum: $(1, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), f = 1 + \sqrt{2}$