# Distinguishing quantum channels by restricted testers

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Symposium KCIK, Sopot, May 23-25, 2013

# Discrimination of quantum states

$M_n = M_n(\mathbb{C})$, $D_n$ be the set of states (= density matrices), $\rho_0, \rho_1 \in D_n$

- $\rho$ unknown, $\rho \in \{\rho_0, \rho_1\}$ with some a priori probablity $(\lambda, 1-\lambda)$. Decide which is $\rho$.
- tests: $E \in M_n$, $0 \leq E \leq I$ effects, $\mathrm{Tr}\, \rho E$ - probability of choosing $\rho_0$
- optimality: minimize the average error probability

$$P_\lambda(E) := \lambda \mathrm{Tr}\,(I - E)\rho_0 + (1-\lambda)\mathrm{Tr}\, E\rho_1$$

It is well known that

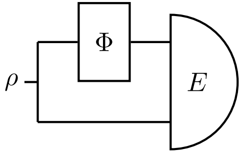
$$\min_{0 \leq E \leq I} P_\lambda(E) = \frac{1}{2}(1 - \|\lambda\rho_0 - (1-\lambda)\rho_1\|_1)$$

(Helstrom, 1969)

# Discrimination of quantum channels

$\mathcal{C}_{m,n} = \{\text{channels } M_n \to M_m\}$, $\Phi_0, \Phi_1 \in \mathcal{C}_{m,n}$.

- $\Phi$ unknown, $\Phi \in \{\Phi_0, \Phi_1\}$, with probability $(\lambda, 1-\lambda)$
- tests: triples $(l, \rho, E)$, $\rho \in D_{nl}$, $E$ an effect in $M_{ml}$,




$\mathrm{Tr}\, E(\Phi \otimes id_l)(\rho)$ - probability of choosing $\Phi_0$.

- minimal average error probability

$$\min_{(l,\rho,E)} P_\lambda(E) = \frac{1}{2}(1 - \|\lambda\Phi_0 - (1-\lambda)\Phi_1\|_\diamond)$$

$$\|\Phi\|_\diamond = \max_l \max_{\rho\in D_{nl}} \|(\Phi \otimes id_l)(\rho)\|_1 = \max_{\rho\in D_{nn}} \|\Phi \otimes id_n(\rho)\|_1$$

(Kitaev 1997)

# Positive cones, bases and order units

Let $\mathcal{V}$ be a real vector space, $\dim(\mathcal{V}) < \infty$, $\mathcal{V}^*$ its dual, with duality $\langle \cdot, \cdot \rangle$.

- $P \subset \mathcal{V}$ a positive cone: closed convex cone, $P \cap -P = \{0\}$, $\mathcal{V} = P - P$
- ordered vector space: $x \leq_P y$ if $y - x \in P$
- $P^* \subset \mathcal{V}^*$ the dual cone: $\{f \in \mathcal{V}^*, \langle f, p \rangle \geq 0,\ p \in P\}$,
- $B \subset P$ a base: for $p \in P$, $p \neq 0$, $\exists$ unique $s > 0, b \in B$: $p = sb$
- $e \in P$ an order unit: for $x \in \mathcal{V}$, $\exists r > 0$: $x \leq_P re$,

There is a 1-1 correspondence between bases of $P$ and order units $e \in P^*$:

$$B = \{p \in P,\ \langle e, p \rangle = 1\} =: B_e$$

# Order unit norms and base norms

Let $e \in P^*$ be an order unit, $B = B_e$ the corresponding base of $P$.
Let $[0, e]_{P^*} = \{f \in \mathcal{V}^*, 0 \leq_{P^*} f \leq_{P^*} e\}$.

- order unit norm: $f \in \mathcal{V}^*$:

$$\|f\|_e := \inf\{\lambda > 0, -\lambda e \leq_{P^*} f \leq_{P^*} \lambda e\}$$

- its dual is the base norm: $v \in \mathcal{V}$:

$$\begin{aligned} \|v\|_B &:= \inf\{t + s, v = tb_1 - sb_2, s, t \geq 0, b_1, b_2 \in B\} \\ &= \sup_{f \in [0,e]_{P^*}} \langle 2f - e, v \rangle \end{aligned}$$

# Base norms and discrimination

Formally, we can consider the discrimination problem in the base $B$: let $b_0, b_1 \in B$

- $b \in B$ unknown, $b \in \{b_0, b_1\}$
- tests: affine maps $B \to [0,1] \equiv t \in [0,e]_{P^*}$, $\langle t, b\rangle$ is the probability of choosing $b_0$
- Given $0 \leq \lambda \leq 1$, the average error probability is

$$P^B_\lambda(t) = \lambda - \langle t, \lambda b_0 - (1-\lambda) b_1\rangle$$

It follows that

$$\min_{t\in[0,e]_{P^*}} P^B_\lambda(t) = \frac{1}{2}(1 - \|\lambda b_0 - (1-\lambda)b_1\|_B)$$

# Discrimination of states

Let $\mathcal{V} = M_n^h = \{x \in M_n, x = x^*\}$, then

- $\mathcal{V}^* \equiv \mathcal{V}$, with duality $\langle a, b \rangle = \mathrm{Tr}\, ab$, (but we formally distinguish $\mathcal{V}$ - space of states and $\mathcal{V}^*$ - space of effects)
- $P = M_n^+ = P^*$ is a (self-dual) positive cone,
- $e = I_n$ is an order unit in $(\mathcal{V}^*, M_n^+)$, $B_e = D_n$ the corresponding base and

$$\| \cdot \|_e = \| \cdot \|, \quad \| \cdot \|_{D_n} = \| \cdot \|_1$$

- tests: elements in $[0, I]$ - effects

# Discrimination of states by restricted tests

Suppose the set of tests is restricted: let $\mathcal{E} \subset M_n^+$ be such that

(i) $\mathcal{E}$ is closed, convex and $int(\mathcal{E}) \neq \emptyset$

(ii) $0 \in \mathcal{E}$, $E \in \mathcal{E}$ implies $I - E \in \mathcal{E}$

We will call $\mathcal{E}$ an admissible set of effects. Then

- $P_{\mathcal{E}}^* := \cup_{t>0} t\mathcal{E}$ is a positive cone in $\mathcal{V}^*$, $P_{\mathcal{E}}^* \subseteq M_n^+$.
- $I$ is an order unit in $(\mathcal{V}^*, P_{\mathcal{E}}^*)$
- $\mathcal{E} \subseteq [0, I]_{P_{\mathcal{E}}^*} \subseteq [0, I]$
- $\|x\|_{(\mathcal{E})} := \sup_{E\in\mathcal{E}} \mathrm{Tr}\,(2E - I)x$ defines a norm in $M_n^h$.

We have

$$\min_{E\in\mathcal{E}} P_\lambda(E) = \frac{1}{2}(1 - \|\lambda\rho_0 - (1-\lambda)\rho_1\|_{(\mathcal{E})})$$

(Matthews, Werner, Winter 2009; Reeb, Kastoryano, Wolf 2011 )

# Discrimination of states by restricted tests

Let $P_{\mathcal{E}} = (P_{\mathcal{E}}^*)^* \subset \mathcal{V}$ and

$$D_{\mathcal{E}} := \{\rho \in P_{\mathcal{E}}, \mathrm{Tr}\,\rho = 1\}$$

is a base of $P_{\mathcal{E}}$, corresponding to $I$. Note that $[0, I]_{P_{\mathcal{E}}^*}$ is a admissible set of effects and

$$\|x\|_{(\mathcal{E})} \leq \|x\|_{([0,I]_{P_{\mathcal{E}}^*})} = \|x\|_{D_{\mathcal{E}}} \leq \|x\|_1$$

Since $M_n^+ \subseteq P_{\mathcal{E}}$, $\rho_0, \rho_1 \in D_n \subseteq D_{\mathcal{E}}$ elements of a larger base.

(Reeb, Kastoryano, Wolf 2011)

# Examples

Take $M_{mn}^h = M_m^h \otimes M_n^h$. Let

$$\mathcal{E}_{sep} = \{\sum_{i=1}^K E_i \otimes F_i, E_i \in M_m^+, F_i \in M_n^+, \sum_{i=1}^L E_i \otimes F_i = I, L \leq K\}$$

$$\mathcal{E}_{LOCC} = \text{ implemented by LOCC measurements}$$

$$\mathcal{E}_{LOCC\leftarrow} = \{\sum_i E_i \otimes F_i,\ 0 \leq E_i \leq I_m,\ 0 \leq F_i, \sum_i F_i \leq I_n\}$$

Then

$$P^*_{\mathcal{E}_{LOCC\leftarrow}} = P^*_{\mathcal{E}_{LOCC}} = P^*_{\mathcal{E}_{sep}} = Sep := M_m^+ \otimes M_n^+$$

and

$$\mathcal{E}_{LOCC\leftarrow} \subsetneq \mathcal{E}_{LOCC} \subsetneq [0, I]_{Sep} = \mathcal{E}_{Sep}$$

# Hermiticity-preserving linear maps

Let $\mathcal{L}_{m,n} := \{\Phi : M_n \to M_m \text{ linear}, \Phi(a^*) = \Phi(a)^*, a \in M_n\}$.

$$\mathcal{L}^*_{m,n} \equiv M^h_{mn} \equiv M^h_m \otimes M^h_n,$$

with duality defined by

$$\langle \Phi, a \otimes b \rangle = \mathrm{Tr}\, a^t \Phi(b), \quad a \in M_m, \; b \in M_n$$

Note that

$$\langle \Phi, A \rangle = \mathrm{Tr}\, C(\Phi)^t A, \qquad A \in M^h_{mn},$$

where $C : \mathcal{L}_{m,n} \to M^h_{mn}$ is the Choi isomorphism:

$$C(\Phi) = (\Phi \otimes id_n)(E_n), \quad E_n = |e_n\rangle\langle e_n|, \; |e_n\rangle = \sum_{i=1}^{n} |i_n \otimes i_n\rangle$$

# Positive cones in $\mathcal{L}_{m,n}$

Some natural positive cones:

- $\mathcal{P}_1$ = positive maps.
- $\mathcal{CP}$ = completely positive (cp) maps
- $\mathcal{P}_k$ = $k$-positive maps.
- $\mathcal{S}_1$ = entanglement breaking maps
- $\mathcal{S}_k$ = partially entanglement breaking maps

We have

$$\mathcal{S}_1 \subseteq \mathcal{S}_k \subseteq \mathcal{S}_{m\wedge n} = \mathcal{CP} = \mathcal{P}_{m\wedge n} \subseteq \mathcal{P}_k \subseteq \mathcal{P}_1$$

# Mapping cones

Mapping cones: Let $\mathcal{P}(m,n) \subset \mathcal{L}_{m,n}$ be postive cones such that if $\Phi \in \mathcal{P}(m,n)$ then $\psi_1 \circ \Phi \circ \psi_2 \in \mathcal{P}(j,l)$ for all cp maps $\psi_1 : M_m \to M_j$, $\psi_2 : M_l \to M_n$.

(Störmer 1986)

- $t \circ \mathcal{P}(m,n) \circ t$ are again mapping cones.
- All previous examples are mapping cones, invariant under this transformation.

# Dual cones

- $\mathcal{P}_k^* = S_k \equiv$ elements of Schmidt rank $\leq k$:

$$S_k := \{\sum_i |\psi_i\rangle\langle\psi_i|, |\psi_i\rangle = \sum_{j=1}^{k} |\xi_j^i \otimes \eta_j^i\rangle, \xi_j^i \in \mathbb{C}^m, \eta_j^i \in \mathbb{C}^n\}$$

- $\mathcal{P}_1^* = S_1 = Sep$
- $\mathcal{CP}^* = M_{mn}^+$
- $\mathcal{S}_k^* = k - BP$ $k$-block-positive operators
- $\mathcal{S}_1^* = BP$ block-positive operators

(Skowronek, Störmer,Zyczkowski 2009)

# Choi isomorphism on positive cones

- $C(\mathcal{CP}) = M_{mn}^{+} = CP$
- $C(\mathcal{P}_k) = k - BP$
- $C(\mathcal{S}_k) = S_k$

(Skowronek, Störmer,Zyczkowski 2009)

Let $\mathcal{P}(m,n)$ are mapping cones. Then

- $P(m,n) = \mathcal{P}^*(m,n)$ or $C(\mathcal{P}(m,n))$ satisfy:

$$(\psi_1 \otimes \psi_2)(P(n,k)) = P(m,l)$$

  for all cp maps $\psi_1 : M_n \to M_m,\ \psi_2 : M_k \to M_l$. We will call this again mapping cones.
- $P(m,n)^t$ are mapping cones as well.

# Quantum channels as a base of a cone

Fix the cone $\mathcal{P} = \mathcal{CP}$ in $\mathcal{L}_{m,n}$.
Let $\mathcal{V} = \{\Phi \in \mathcal{L}_{m,n},\ \mathrm{Tr}\,\Phi(a) = c\mathrm{Tr}\,a,\ \text{for some } c \in \mathbb{R}\}$.

- $\mathcal{V}$ is a subspace in $\mathcal{L}_{m,n}$, generated by the set $\mathcal{C}_{m,n}$ of channels
- $\mathcal{V}^* \equiv M^h_{mn}|_{\mathcal{V}^\perp}$, where

$$\mathcal{V}^\perp = \{X \in M^h_{mn}, \langle \Phi, X\rangle = 0, \Phi \in \mathcal{V}\}$$

- $\mathcal{Q} = \mathcal{V} \cap \mathcal{CP}$ is a positive cone in $\mathcal{V}$
- $\mathcal{Q}^* = \{A + \mathcal{V}^\perp, A \in M^+_{mn}\}$
- $\mathcal{C}_{m,n}$ is a base of $\mathcal{Q}$
- The corresponding order unit is

$$e = \frac{1}{n} I_{mn} + \mathcal{V}^\perp = \{I_m \otimes \sigma, \sigma \in D_n\}$$

# Discrimination of quantum channels

Let $\Phi_0, \Phi_1 \in \mathcal{C}_{m,n}$, unknown $\Phi \in \{\Phi_0, \Phi_1\}$, prior $(\lambda, 1-\lambda)$.

- tests: $\mathbf{t} \in [0, e]_{\mathcal{Q}^*}$
- minimum average error probability:

$$\min_{\mathbf{t}\in[0,e]_{\mathcal{Q}^*}} P_\lambda(\mathbf{t}) = \frac{1}{2}(1 - \|\lambda\Phi_0 - (1-\lambda)\Phi_1\|_{\mathcal{C}_{m,n}})$$

- $\mathbf{t} \in [0, e]_{\mathcal{Q}^*}$ iff there is some $\sigma \in D_n$ and $0 \leq T \leq I_m \otimes \sigma$ such that

$$\langle \Phi, \mathbf{t} \rangle_{\mathcal{V}} = \langle \Phi, T \rangle = \mathrm{Tr}\, C(\Phi)^t T, \quad \Phi \in \mathcal{V}$$

- We note that $(T, I \otimes \sigma - T)$ is a quantum 1-tester, or PPOVM

(Chiribella, D'Ariano, Perinotti 2008, Ziman 2008)

- such a tester $T$ is not unique for $\mathbf{t}$.

# Quantum 1-testers

**Theorem** (Ziman , Chiribella et al.)

*$T \in M_{mn}$ is a quantum 1-tester if and only if there is a triple $(l, \rho, E)$, where $\rho \in D_{nl}$ and $E$ an effect in $M_{ml}$, such that*

$$\langle \Phi, T \rangle = \mathrm{Tr}\, E(\Phi \otimes id_l)(\rho), \quad \Phi \text{ a channel}$$

- Note that there are many triples corresponding to the same tester.
- A test $\mathbf{t} \in [0, e]_{\mathcal{Q}^*}$ can be seen as an equivalence class of triples $(l, \rho, E)$.

It follows that $\| \cdot \|_{\mathcal{C}_{m,n}} = \| \cdot \|_{\diamond}$.

# Optimal triples

A triple $(l, \rho, E)$ is optimal if

$$P_\lambda(l, \rho, E) = \min_{(k,\rho',F)} P_\lambda(k, \rho', F)$$

Let $\tau_i = (\Phi_i \otimes id_k)(\rho)$, $i = 0, 1$. Then

- $\tau_i \in D_{mk}$ and $\mathrm{Tr}_{M_m}\tau_i = \mathrm{Tr}_{M_n}\rho$, $i = 0, 1$.
- If $(l, \rho, E)$ is optimal, then $E$ must be an optimal test for $\tau_0, \tau_1, \lambda$:
$$E = \mathrm{supp}(\lambda\tau_0 - (1-\lambda)\tau_1)_+$$

# Optimal triples

Let $(I, \rho, E)$ be a triple such that $\sigma := \mathrm{Tr}_{M_n}(\rho)$ is of rank $n$. Let $\tau_i = (\Phi_i \otimes id_k)(\rho)$, $i = 0, 1$. Then $(I, \rho, E)$ is an optimal triple if and only if

1. $E = \mathrm{supp}(\lambda\tau_0 - (1-\lambda)\tau_1)_+$
2. $\mathrm{Tr}_{M_m}|\lambda\tau_0 - (1-\lambda)\tau_1|$ is a multiple of $\sigma$.

In particular, there exists an optimal triple with $\rho$ a maximally entangled state if and only if

$$\mathrm{Tr}_{M_m}|\lambda C(\Phi_0) - (1-\lambda)C(\Phi_1)| = cI_n,$$

for some $c \in \mathbb{R}$.

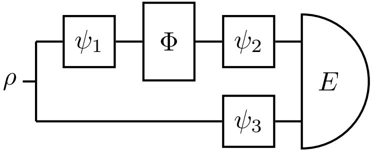
# Restricted tests for channels

An admissible family of tests: Let $\mathcal{T} \subset \mathcal{Q}^*$ be such that

(i) $\mathcal{T}$ is closed, convex and $int(\mathcal{T}) \neq \emptyset$

(ii) $0 \in \mathcal{T}$ and $\mathbf{t} \in \mathcal{T}$ implies $e - \mathbf{t} \in \mathcal{T}$

(iii) if $\mathbf{t} \in \mathcal{T}$ and $(k, \rho, E) \in \mathbf{t}$, then




defines a test in $\mathcal{T}$ for all channels $\psi_1, \psi_2, \psi_3$.

We will consider

- tests with the effect $E$ restricted to some admissible set of effects;
- tests with the input state $\rho$ restricted to some positive cone (mapping cone).

# Restricted tests for channels

Let $\mathcal{P}(m,n)$ be mapping cones,

$$\mathcal{CP}(m,n) \subseteq \mathcal{P}(m,n) \subseteq \mathcal{P}_1(m,n).$$

Let $\mathcal{V} = \{\Phi \in \mathcal{L}_{m,n}, \mathrm{Tr}\,\Phi(a) = c\mathrm{Tr}\,a\}$, put $\mathcal{Q} = \mathcal{V} \cap \mathcal{P}$.

- $\mathcal{Q}^* = \{A + \mathcal{V}^\perp, A \in \mathcal{P}^*\}$
- $e = \frac{1}{n} I + \mathcal{V}^\perp$ is an order unit in $(\mathcal{V}^*, \mathcal{Q}^*)$
- $B_e = \{\text{trace preserving elements in } \mathcal{P}\} =: \mathcal{C}_\mathcal{P} \supseteq \mathcal{C}_{m,n}$

Put $\mathcal{T} = [0, e]_{\mathcal{Q}^*}$: $\Phi_0, \Phi_1$ considered as elements of $\mathcal{C}_\mathcal{P}$. Then

$$\min_{\mathbf{t} \in \mathcal{T}} P_\lambda(\mathbf{t}) = \frac{1}{2}(1 - \|\lambda\Phi_0 - (1-\lambda)\Phi_1\|_{\mathcal{C}_\mathcal{P}})$$

# Restrictions on effects

**Theorem**
*Let $\tilde{P} = (\mathcal{P}^*)^t$. Then $\mathcal{T} = \{(I, \rho, E) \in \mathbf{t},\ E \in [0, I]_{\tilde{P}(m,l)}\}$.*

More generally, let $\mathcal{E}(m, l)$ be an admissible set of effects, for all $m, l$, such that

$$(\phi \otimes \psi)(\mathcal{E}) \subseteq \mathcal{E},\ \phi, \psi \text{ cp subunital maps}$$

Then

- $P(m, n) := P_{\mathcal{E}(m,n)}$ are mapping cones
- $\|A\|_{(\mathcal{E})} \geq \|(\phi \otimes \psi)(A)\|_{(\mathcal{E})}$, $\phi, \psi$ subunital cp maps

# Restriction on effects

Let $\mathcal{T} = \{\mathbf{t},\ (I, \rho, E) \in \mathbf{t} \text{ with } E \in \mathcal{E}(m, l)\}$. Then

$$\min_{\mathbf{t}\in\mathcal{T}} P_\lambda(\mathbf{t}) = \frac{1}{2}(1 - \|\lambda\Phi_0 - (1-\lambda)\Phi_1\|_{CP\to(\mathcal{E})})$$

where

$$\|\Phi\|_{CP\to(\mathcal{E})} = \max_{\rho\in D_{nn}} \|(\Phi \otimes id_n)(\rho)\|_{(\mathcal{E}(m,n))}$$

- $\|\phi \circ \Phi \circ \psi\|_{CP\to(\mathcal{E})} \leq \|\Phi\|_{CP\to(\mathcal{E})}$, $\phi, \psi$ channels
- Let $\tilde{\mathcal{P}}(m, n) = C^{-1}(P(m, n)^t)$, then

$$\|\Phi\|_{CP\to(\mathcal{E})} \leq \|\Phi\|_{CP\to([0,I]_P)} = \max_{\rho\in D_{nn}} \|(\Phi\otimes id_n)(\rho)\|_{D_P} = \|\Phi\|_{\mathcal{C}_{\tilde{\mathcal{P}}}}$$

# Restriction of input states

Let $P(m, n)$ be mapping cones. Let

$$\mathcal{T} = \{\mathbf{t},\ (l, \rho, E) \in \mathbf{t},\ \text{with } \rho \in P(n, l)\}$$

Then

- $\mathcal{T}$ is an admissible family of tests.
- $\min_{\mathbf{t}\in\mathcal{T}} P_\lambda(\mathbf{t}) = \frac{1}{2}(1 - \|\lambda\Phi_0 - (1-\lambda)\Phi_1\|_{P\to CP})$, where

$$\|\Phi\|_{P\to CP} = \max_{\rho\in D_{nn}\cap P} \|(\Phi \otimes id)(\rho)\|_1$$

# Restriction on input states by restriction on effects

The same can be obtained by a restriction on effects: Let $(l, \rho, E) \in \mathbf{t}$, $\rho \in P(n, l)$. Then we can find $(k, \rho', F) \in \mathbf{t}$, such that

$$F \in \mathcal{E}_P(m, k) := \{(id_m \otimes \psi)(E), \\ t \circ \psi \circ t \in P(k, l), \psi(I_l) \leq I_k, E \in [0, I_{ml}], l \in \mathbb{N}\}$$

- $\mathcal{E}_P$ is an admissible set of effects and $(\phi \otimes \psi)(\mathcal{E}_P) \subset \mathcal{E}_P$ for $\phi, \psi$ subunital cp maps
- $\mathbf{t} \in \mathcal{T}$ if and only if $(k, \rho, E) \in \mathbf{t}$, $E \in \mathcal{E}_P(m, k)$.

$$\|\Phi\|_{P \to CP} = \|\Phi\|_{CP \to (\mathcal{E}_P)} \leq \|\Phi\|_{\mathcal{C}_{\tilde{\mathcal{P}}}}$$

# Examples

Let $P = Sep$

- restrict $\rho \in Sep$: $\|\Phi\|_{Sep \to CP} = \max_{\rho \in D_n} \|\Phi(\rho)\|_1$,
- this is the same as restrict $E \in \mathcal{E}_{LOCC^{\leftarrow}}$
- restrict $E \in \mathcal{E}_{Sep}$: view $\Phi_0$, $\Phi_1$ as positive trace preserving maps
- $E \in \mathcal{E}_{LOCC}$: there are channels such that $\|\Phi_0 - \Phi_1\|_{CP \to (\mathcal{E}_{LOCC})} > \|\Phi_0 - \Phi_1\|_{Sep \to CP}$

(Matthews, Piani, Watrous, 2010)

Let $P = S_k$.

- restrict $\rho \in S_k$: $\|\Phi\|_{S_k \to CP} = \max_{\rho \in D_{nk}} \|(\Phi \otimes id_k)(\rho)\|_1$,

  (Johnston, Kribs, Paulsen, Pereira, 2010)
- this is the same as using triples $(kl, \rho, E)$, $E \in \mathcal{E}_{LOCC^{\leftarrow}(mk;l)}$
- restrict $E \in [0, I]_{S_k}$: $\Phi_0$, $\Phi_1$ $k$-positive trace preserving maps
- $\|\Phi_0 - \Phi_1\|_{CP \to S_k} > \|\Phi\|_{S_k \to CP}$ for some channels $\Phi_0, \Phi_1$.

# Conclusion

- Base norms appear in discrimination problems also for quantum channels (and for more general quantum protocols).
- Importance of mapping cones also in this context.
- For restricted tests, it is enough to consider restrictions only on effects and not on input states (but not other way round).