

$$1. \int \frac{dx}{x^2+2x}$$

$$\frac{1}{2} \ln \left| \frac{x}{x+2} \right| + C$$

$$2. \int \frac{dx}{x^2-1}$$

$$\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C$$

$$3. \int \frac{dx}{x^3+x}$$

$$\ln |x| - \frac{1}{2} \ln |1+x^2| + C$$

$$4. \int \frac{dx}{(x-1)(x-2)(x-3)}$$

$$\frac{1}{2} \ln |x-1| - \ln |x-2| + \frac{1}{2} \ln |x-3| + C$$

$$5. \int \frac{dx}{x(x+1)^2}$$

$$\ln |x| - \ln |1+x| + \frac{1}{1+x} + C$$

$$6. \int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} dx$$

$$\frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x-4} = \frac{A(x^2-x-12)+B(x^2-5x+4)+C(x^2+2x-3)}{(x-1)(x+3)(x-4)} =$$

$$= \frac{(A+B+C)x^2+x(-A-5B+2C)+(-12A+4B-3C)}{(x-1)(x+3)(x-4)}$$

$$A+B+C=2$$

$$-A-5B+2C=41 \quad A=4, B=-7, C=5$$

$$-12A+4B-3C=-91$$

$$\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} dx = \int \frac{4}{x-1} dx - \int \frac{7}{x+3} dx + \int \frac{5}{x-4} dx = 4 \ln |x-1| - 7 \ln |x+3| + 5 \ln |x-4| + C$$

$$4 \ln |x-1| - 7 \ln |x+3| + 5 \ln |x-4| + C$$

$$7. \int \frac{2dx}{x^2+2x+5}$$

$$\operatorname{arctg} \frac{x+1}{2} + C$$

$$8. \int \frac{dx}{3x^2+5}$$

$$\frac{1}{\sqrt{15}} \operatorname{arctg} \frac{\sqrt{3}x}{\sqrt{5}} + C$$

$$9. \int \frac{dx}{x^3+1}$$

$$\frac{1}{3} \ln |x+1| - \frac{1}{6} \ln (x^2-x+1) + \frac{\sqrt{3}}{3} \operatorname{arctg} \frac{2x-1}{3} + C$$

$$10. \int \frac{dx}{x^3+x^2+x}$$

$$\ln |x| - \frac{1}{2} \ln |x^2+x+1| - \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + C$$

$$11. \int \frac{x^2}{x^2-6x+10} dx$$

$$x + 3 \ln |x^2-6x+10| + 8 \operatorname{arctg} (x-3) + C$$

$$12. \int \frac{2x-3}{(x^2-3x+2)^2} dx$$

$$-\frac{1}{x^2-3x+2} + C$$

$$13. \int \frac{9x-14}{9x^2-24x+16} dx$$

$$\ln |3x-4| + \frac{2}{3} \frac{1}{3x-4} + C$$

$$14. \int \frac{dx}{2x^2+5x-12}$$

$$\frac{1}{11} (\ln |2x - 3| - \ln |x + 4|) + C$$

$$15. \int \frac{3x^2 + 32x - 120}{(x-2)(x+2)(x-5)} dx$$

$$\frac{11}{3} \ln |x - 2| - \frac{43}{7} \ln |x + 2| + \frac{125}{21} \ln |x - 5| + C$$

$$16. \int \frac{5x^3 - 15x^2 + 15x - 3}{x^3 - 8x^2 + 17x - 10} dx$$

$$5x + \frac{1}{2} \ln |x - 1| - \frac{7}{3} \ln |x - 2| + \frac{161}{6} \ln |x - 5| + C$$

$$17. \int \frac{5x^3 + 9x^2 - 22x - 8}{x^3 - 4x} dx$$

$$5x + 2 \ln |x| + 3 \ln |x - 2| + 4 \ln |x + 2| + C$$

$$18. \int \frac{x^3 + x + 1}{(x^2 + 1)^2} dx$$

$$\frac{1}{2} \ln(x^2 + 1) - \frac{1}{1+x^2} + C$$

$$19. \int \frac{2x^3 - 7x^2 + 12x - 10}{x^4 - 4x^3 + 8x^2 - 8x + 12} dx$$

$$\frac{1}{2} \ln(x^2 + 2) - \frac{\sqrt{2}}{2} \operatorname{arctg} \frac{x}{\sqrt{2}} + \frac{1}{2} \ln |x^2 - 4x + 6| + C$$

$$20. \int \frac{9x^4 + 3x^3 - 23x^2 + x}{9x^3 - 6x^2 - 5x + 2} dx$$

$$\frac{x^2}{2} + x - \ln |x - 1| + \frac{1}{3} \ln |3x - 1| - \frac{2}{3} \ln |3x + 2| + C$$

$$21. \int_1^2 (x^2 - 3x + 2) dx = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \right]_1^2 = \frac{2}{3} - \frac{5}{6}$$

$$-\frac{1}{6}$$

$$22. \int_0^3 |1 - 3x| dx = \int_0^{\frac{1}{3}} (1 - 3x) dx + \int_{\frac{1}{3}}^3 (3x - 1) dx = \left[x - \frac{3}{2}x^2 \right]_0^{\frac{1}{3}} + \left[\frac{3}{2}x^2 - x \right]_{\frac{1}{3}}^3 = \frac{1}{6} + \frac{21}{2} + \frac{1}{6} = \frac{65}{6}$$

$$\frac{65}{6}$$

$$23. \int_{-4}^{-2} \frac{1}{x} dx = [\ln |x|]_{-4}^{-2} = \ln 2 - \ln 4$$

$$-\ln 2$$

$$24. \int_0^1 \frac{dx}{1+x^2} = [\operatorname{arctg} x]_0^1 = \frac{\pi}{4}$$

$$\frac{\pi}{4}$$

$$25. \int_0^2 \frac{x}{x^2 + 3x + 2} dx = \frac{1}{2} \int_0^2 \frac{2x + 3 - 3}{x^2 + 3x + 2} dx = \frac{1}{2} \int_0^2 \frac{2x + 3}{x^2 + 3x + 2} - \frac{3}{2} \int_0^2 \frac{dx}{(x+1)(x+2)} = \frac{1}{2} [\ln |x^2 + 3x + 2|]_0^2 - \frac{3}{2} \int_0^2 \frac{(x+2) - (x+1)}{(x+1)(x+2)} dx = \frac{1}{2} \ln 6 - \frac{3}{2} [\ln |x + 1|]_0^2 + \frac{3}{2} [\ln |x + 2|]_0^2 = \frac{1}{2} \ln 6 - \frac{3}{2} \ln 3 + \frac{3}{2} (\ln 4 - \ln 2) = 2 \ln 2 - \ln 3 = \ln \frac{4}{3}$$

$$\ln \frac{4}{3}$$

$$26. \int_0^\pi \cos x dx = [\sin x]_0^\pi = 0$$

$$0$$

$$27. \int_0^\pi |\cos x| dx = \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^\pi \cos x dx = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^\pi = 2$$

$$2$$

$$28. \int_0^\pi \sin^3 x dx = \int_0^\pi \sin x (1 - \cos^2 x) dx = \int_0^\pi \sin x dx - \int_0^\pi \sin x \cos^2 x dx =$$

$$[-\cos x]_0^\pi - \int_0^\pi \sin x \cos^2 x dx = 2 - \int_0^\pi \sin x \cos^2 x dx = \left| \begin{array}{ll} t = \cos x & \pi \rightarrow -1 \\ dt = -\sin x dx & 0 \rightarrow 1 \end{array} \right|$$

$$= 2 + \int_1^{-1} t^2 dt = 2 + \left[\frac{1}{3}t^3\right]_1^{-1} = \frac{4}{3}$$

$$29. \int_3^7 \frac{x}{x^2-4} dx = \left| \begin{array}{l} t = x^2 \quad 7 \rightarrow 49 \\ dt = 2x dx \quad 3 \rightarrow 9 \end{array} \right| = \frac{1}{2} \int_9^{49} \frac{dt}{t-4} = \left[\frac{1}{2} \ln |t-4|\right]_9^{49} = \frac{1}{2} \ln 9$$

$$30. \int_0^{\frac{\pi}{2}} \cos x \cdot \sin^2 x dx = \left| \begin{array}{l} t = \sin x \quad \frac{\pi}{2} \rightarrow 1 \\ dt = \cos x dx \quad 0 \rightarrow 0 \end{array} \right| = \int_0^1 t^2 dt = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$$

$$31. \int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}} dx = \left| \begin{array}{l} t = \sqrt{x} \quad 1 \rightarrow 1, 0 \rightarrow 0 \\ dt = \frac{dx}{2\sqrt{x}} \quad dx = 2\sqrt{x} dt \end{array} \right| = \int_0^1 \frac{2\sqrt{x}\sqrt{x} dt}{1+\sqrt{x}} = 2 \int_0^1 \frac{t^2}{t+1} dt = 2 \int_0^1 \frac{t^2-1+1}{t+1} dt$$

$$= 2 \int_0^1 (t-1) dt + 2 \int_0^1 \frac{1}{t+1} dt = 2 \left[\frac{1}{2}t^2 - t + \ln |1+t|\right]_0^1 = 2 \ln 2 - 1$$

$$32. \int_{-1}^1 \frac{dx}{(1+x^2)^2}$$

$$\pi = [\arctg x]_{-1}^1 = \int_{-1}^1 \frac{dx}{1+x^2} = \left| \begin{array}{l} \frac{1}{1+x^2} \quad 1 \\ -\frac{2x}{(1+x^2)^2} \quad x \end{array} \right| = \left[\frac{x}{1+x^2}\right]_{-1}^1 + 2 \int_{-1}^1 \frac{x^2}{(1+x^2)^2} dx =$$

$$1 + 2 \int_{-1}^1 \frac{x^2+1-1}{(x^2+1)^2} dx = 1 + 2 \int_{-1}^1 \frac{dx}{1+x^2} - 2 \int_{-1}^1 \frac{dx}{(1+x^2)^2} = 1 + 2\pi - 2 \int_{-1}^1 \frac{dx}{(1+x^2)^2}$$

$$\int_{-1}^1 \frac{dx}{(1+x^2)^2} = \frac{1}{2}(1 + \pi)$$

$$33. \int_0^{\sqrt{2}} \sqrt{4-x^2} dx = \left| \begin{array}{l} x = 2 \sin t \quad t = \arcsin \frac{t}{2} \\ dx = 2 \cos t dt \quad \sqrt{2} \rightarrow \frac{\pi}{4}, 0 \rightarrow 0 \end{array} \right| = \int_0^{\frac{\pi}{4}} 2\sqrt{4-4\sin^2 t} \cos t dt =$$

$$= 4 \int_0^{\frac{\pi}{4}} \cos^2 t dt = 4 \int_0^{\frac{\pi}{4}} \frac{\cos 2t+1}{2} dt = 4 \left[\frac{1}{4} \sin 2t + \frac{t}{2}\right]_0^{\frac{\pi}{4}} = 1 + \frac{\pi}{2}$$

$$34. \int_0^{\ln 5} \frac{e^x \sqrt{e^x-1}}{e^x+3} dx = \left| \begin{array}{l} t = \sqrt{e^x-1} \quad \ln 5 \rightarrow 2 \\ dt = \frac{1}{2} \frac{e^x}{\sqrt{e^x-1}} dx \quad 0 \rightarrow 0 \end{array} \right| = \int_0^2 \frac{\sqrt{e^x-1}}{e^x+3} 2\sqrt{e^x-1} dt = 2 \int_0^2 \frac{e^x-1}{e^x-1+4} dt =$$

$$= 2 \int_0^2 \frac{t^2}{t^2+4} dt = 2 \int_0^2 \frac{t^2+4-4}{t^2+4} dt = 2 \int_0^2 1 dt - 8 \int_0^2 \frac{dt}{t^2+4} = \left| \begin{array}{l} t = 2s \quad s = \frac{t}{2} \\ dt = 2ds \quad 2 \rightarrow 1, 0 \rightarrow 0 \end{array} \right| =$$

$$= 4 - 16 \int_0^1 \frac{ds}{4+4s^2} = 4 - 4 \int_0^1 \frac{ds}{1+s^2} = 4 - 4[\arctg s]_0^1 = 4 - \pi$$

$$35. \int_1^2 \frac{dx}{\sqrt{3+2x-x^2}} = \int_1^2 \frac{dx}{\sqrt{4-(x-1)^2}} = \left| \begin{array}{l} t = x-1 \quad 2 \rightarrow 1 \\ dt = dx \quad 1 \rightarrow 0 \end{array} \right| = \int_0^1 \frac{dt}{\sqrt{4-t^2}} =$$

$$= \left| \begin{array}{l} t = 2 \sin s \quad s = \arcsin \frac{t}{2} \\ dt = 2 \cos s ds \quad 1 \rightarrow \frac{\pi}{6}, 0 \rightarrow 0 \end{array} \right| = \int_0^{\frac{\pi}{6}} \frac{2 \cos s ds}{\sqrt{4-4\sin^2 s}} = \int_0^{\frac{\pi}{6}} 1 ds = \frac{\pi}{6}$$

$$36. \int_0^{\frac{\pi}{2}} \frac{\sin \varphi}{6-5 \cos \varphi + \cos^2 \varphi} d\varphi = \left| \begin{array}{l} t = \cos \varphi \quad \frac{\pi}{2} \rightarrow 0 \\ dt = -\sin \varphi d\varphi \quad 0 \rightarrow 1 \end{array} \right| = \int_1^0 \frac{-dt}{6-5t+t^2} = \int_0^1 \frac{dt}{(t-3)(t-2)} =$$

$$= \int_0^1 \frac{(t-2)-(t-3)}{(t-3)(t-2)} dt = \int_0^1 \frac{dt}{t-3} - \int_0^1 \frac{dt}{t-2} = [\ln |t-3|]_0^1 - [\ln |t-2|]_0^1 = 2 \ln 2 - \ln 3$$

$$37. \int_0^1 x e^{-x} dx = \left| \begin{array}{l} x \quad e^{-x} \\ 1 \quad -e^{-x} \end{array} \right| = -[x e^{-x}]_0^1 + \int_0^1 e^{-x} dx = -\frac{1}{e} - [e^{-x}]_0^1 = 1 - \frac{2}{e}$$

$$\boxed{\frac{e-2}{e}}$$

38. $\int_1^e \ln x \, dx = \left| \frac{\ln x}{1} - \frac{1}{x} \right| = [x \ln x]_1^e - \int_1^e 1 \, dx = e - [x]_1^e = e - (e - 1) = 1$

$$\boxed{1}$$

39. $\int_0^{\frac{\pi}{2}} x \sin x \, dx = \left| \frac{x}{1} - \frac{\sin x}{-\cos x} \right| = -[x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_0^{\frac{\pi}{2}} = 1$

$$\boxed{1}$$

40. $\int_1^2 x \ln x \, dx = \left| \frac{\ln x}{\frac{1}{x}} - \frac{x}{\frac{1}{2}x^2} \right| = \frac{1}{2} [x^2 \ln x]_1^2 - \frac{1}{2} \int_1^2 x \, dx = 2 \ln 2 - \frac{1}{4} [x^2]_1^2 = 2 \ln 2 - \frac{3}{4}$

$$\boxed{2 \ln 2 - \frac{3}{4}}$$

41. $\int_0^1 x^3 e^{2x} \, dx = \left| \frac{x^3}{3x^2} - \frac{e^{2x}}{\frac{1}{2}e^{2x}} \right| = [\frac{1}{2}x^3 e^{2x}]_0^1 - \frac{3}{2} \int_0^1 x^2 e^{2x} \, dx = \frac{e^2}{2} - \frac{3}{2} \int_0^1 x^2 e^{2x} \, dx$
 $\int_0^1 x^2 e^{2x} \, dx = \left| \frac{x^2}{2x} - \frac{e^{2x}}{\frac{1}{2}e^{2x}} \right| = [\frac{1}{2}x^2 e^{2x}]_0^1 - \int_0^1 x e^{2x} \, dx = \frac{e^2}{2} - \int_0^1 x e^{2x} \, dx (= \frac{e^2-1}{4})$
 $\int_0^1 x e^{2x} \, dx = \left| \frac{x}{1} - \frac{e^{2x}}{\frac{1}{2}e^{2x}} \right| = [\frac{1}{2}x e^{2x}]_0^1 - \frac{1}{2} \int_0^1 e^{2x} \, dx = \frac{e^2}{2} - \frac{1}{2} [\frac{1}{2}e^{2x}]_0^1 = \frac{e^2+1}{4}$

$$\boxed{\frac{e^2+3}{8}}$$

42. $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx = \left| \frac{e^{2x}}{2e^{2x}} - \frac{\sin x}{-\cos x} \right| = [-e^{2x} \cos x]_0^{\frac{\pi}{2}} + 2 \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = 1 + 2 \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$
 $\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = \left| \frac{e^{2x}}{2e^{2x}} + \frac{\cos x}{\sin x} \right| = [e^{2x} \sin x]_0^{\frac{\pi}{2}} - 2 \int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx = e^\pi - 2 \int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx$
 $\int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx = 1 + 2e^\pi - 4 \int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx$

$$\boxed{\frac{2}{5}e^\pi + \frac{1}{5}}$$

43. $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} x \sin^{-2} x \, dx = \left| \frac{x}{1} - \frac{1}{\sin^2 x} \right| = [x \operatorname{tg} x]_{\frac{\pi}{3}}^{\frac{\pi}{4}} - \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \operatorname{tg} x \, dx = \frac{\pi}{4} - \frac{\pi}{3} \sqrt{3} + [\ln |\cos x|]_{\frac{\pi}{3}}^{\frac{\pi}{4}} =$
 $= \dots + \ln \frac{\sqrt{2}}{2} - \ln \frac{1}{2}$

$$\boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{3}\pi + \frac{1}{2} \ln 2}$$

44. $\int_{-1}^1 \arccos x \, dx = \left| \frac{\arccos x}{-\frac{1}{\sqrt{1-x^2}}} - \frac{1}{x} \right| = [x \arccos x]_{-1}^1 + \int_{-1}^1 \frac{x}{\sqrt{1-x^2}} \, dx = \pi - [\sqrt{1-x^2}]_{-1}^1 = \pi$

$$\boxed{\pi}$$

45. $\int_0^{\sqrt{3}} x \operatorname{arctg} x \, dx = \left| \frac{\operatorname{arctg} x}{\frac{1}{1+x^2}} - \frac{x}{\frac{x^2}{2}} \right| = [\frac{1}{2}x^2 \operatorname{arctg} x]_0^{\sqrt{3}} - \frac{1}{2} \int_0^{\sqrt{3}} \frac{x^2}{1+x^2} \, dx = \frac{3}{2} \operatorname{arctg} \sqrt{3} -$
 $-\frac{1}{2} \int_0^{\sqrt{3}} \left(1 - \frac{1}{1+x^2}\right) \, dx = \frac{3}{2} \frac{\pi}{3} - \frac{1}{2} [x]_0^{\sqrt{3}} + \frac{1}{2} \int_0^{\sqrt{3}} \frac{1}{1+x^2} \, dx = \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2} [\operatorname{arctg} x]_0^{\sqrt{3}} =$
 $= \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2} \frac{\pi}{3} = \frac{2}{3}\pi - \frac{\sqrt{3}}{2}$

$$\boxed{\frac{2}{3}\pi - \frac{\sqrt{3}}{2}}$$

46. $\int_0^{\ln 2} x \cosh x \, dx = \left| \frac{x}{1} - \frac{\cosh x}{\sinh x} \right| = [x \sinh x]_0^{\ln 2} - \int_0^{\ln 2} \sinh x \, dx = \ln 2 \sinh \ln 2 - [\cosh x]_0^{\ln 2} =$
 $= \ln 2 \frac{e^{\ln 2} - e^{-\ln 2}}{2} - \left(\frac{e^{\ln 2} + e^{-\ln 2}}{2} - \frac{e^0 + e^0}{2} \right) = \ln 2 \frac{2 - \frac{1}{2}}{2} - \left(\frac{2 + \frac{1}{2}}{2} - 1 \right) = \frac{1}{4}(3 \ln 2 - 1)$

$$\boxed{\frac{1}{4}(3 \ln 2 - 1)}$$

$$\begin{aligned}
47. \quad I_n &= \int_0^{\frac{\pi}{2}} \sin^n x \, dx = \left| \begin{array}{cc} \sin^{n-1} x & \sin x \\ (n-1) \sin^{n-2} x \cos x & -\cos x \end{array} \right|_0^{\frac{\pi}{2}} = [-\sin^{n-1} x \cos x]_0^{\frac{\pi}{2}} + \\
&+ (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x \, dx = (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x (1 - \sin^2 x) \, dx = (n-1)I_{n-2} - (n-1)I_n \\
\text{Teda } nI_n &= (n-1)I_{n-2}, n > 1, I_n = \frac{n-1}{n}I_{n-2}, n \geq 2 \\
I_0 &= \int_0^{\frac{\pi}{2}} \sin^0 x \, dx = \int_0^{\frac{\pi}{2}} 1 \, dx = \frac{\pi}{2}, I_1 = \int_0^{\frac{\pi}{2}} \sin x \, dx = [-\cos x]_0^{\frac{\pi}{2}} = 1
\end{aligned}$$

$$\boxed{I_0 = \frac{\pi}{2}, I_1 = 1, I_n = \frac{n-1}{n}I_{n-2}, n \geq 2}$$