1. $\int \cos^5 2x \sin 2x \, \mathrm{d}x = \left| \begin{matrix} t = \cos 2x \\ \mathrm{d}t = -2\sin 2x \, \mathrm{d}x \end{matrix} \right| = -\frac{1}{2}\int t^5 \mathrm{d}t = -\frac{1}{12}t^6 + C$

$$\boxed{-\frac{1}{12}\cos^6 2x + C}$$

2. $\int \cos^5 x \, \mathrm{d}x = \int \cos x (1-\sin^2 x)^2 \mathrm{d}x = \left| \begin{matrix} t = \sin x \\ \mathrm{d}t = \cos x \, \mathrm{d}x \end{matrix} \right| = \int (1-t^2)^2 \mathrm{d}t = \int (1-2t^2+t^4)\mathrm{d}t =$

$= t - \frac{2}{3}t^3 + \frac{1}{5}t^5 + C$

$$\boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C}$$

3. $\int \frac{\sin^3 x}{\cos^4 x}\mathrm{d}x = \int \frac{\sin^2 x}{\cos^4 x}\sin x \, \mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^4 x}\sin x \, \mathrm{d}x = \left| \begin{matrix} t = \cos x \\ \mathrm{d}t = -\sin x \, \mathrm{d}x \end{matrix} \right| = -\int \frac{1-t^2}{t^4}\mathrm{d}t =$

$= -\int t^{-4}\mathrm{d}t + \int t^{-2}\mathrm{d}t = \frac{1}{3}\frac{1}{t^3} - \frac{1}{t} + C$

$$\boxed{\frac{1}{3\cos^3 x} - \frac{1}{\cos x} + C}$$

4. $\int \frac{\mathrm{d}x}{\sin x \cos^3 x} = \int \frac{\frac{\mathrm{d}x}{\cos^2 x}}{\frac{\sin x}{\cos x}\cos^2 x} = \left| \begin{matrix} t = \operatorname{tg} x & \\ \mathrm{d}t = \frac{\mathrm{d}x}{\cos^2 x} & \frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x \end{matrix} \right| = \int \frac{(1+t^2)\mathrm{d}t}{t} = \ln|t| + \frac{1}{2}t^2 + C$

$$\boxed{\ln|\operatorname{tg} x| + \frac{1}{2}\operatorname{tg}^2 x + C}$$

5. $\int \operatorname{cotg}^3 x \, \mathrm{d}x = \int \frac{\cos^3 x}{\sin^3 x}\mathrm{d}x = \left| \begin{matrix} \cos^2 x & \frac{\cos x}{\sin^3 x} \\ -2\cos x \sin x & -\frac{1}{2}\frac{1}{\sin^2 x} \end{matrix} \right| = -\frac{1}{2}\operatorname{cotg}^2 x - \int \frac{\cos x}{\sin x}\mathrm{d}x = \left| \begin{matrix} t = \sin x \\ \mathrm{d}t = \cos x \, \mathrm{d}x \end{matrix} \right| =$

$= -\frac{1}{2}\operatorname{cotg}^2 x - \int \frac{\mathrm{d}t}{t} = -\frac{1}{2}\operatorname{cotg}^2 x - \ln|t| + C$

$$\boxed{-\frac{1}{2}\operatorname{cotg}^2 x - \ln|\sin x| + C}$$

6. $\int \frac{\sin x - \cos x}{\sin x + \cos x}\mathrm{d}x = \left| \begin{matrix} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{matrix} \right| = \int \frac{\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \frac{2\,\mathrm{d}t}{1+t^2} = 2\int \frac{t^2+2t-1}{(1+2t-t^2)(1+t^2)}\mathrm{d}t =$

$= -2\int \frac{-t^2+2t+1-4t}{(1+2t-t^2)(1+t^2)}\mathrm{d}t = -2\int \frac{\mathrm{d}t}{1+t^2} + \int \frac{8t\,\mathrm{d}t}{(1+2t-t^2)(1+t^2)} = -2\operatorname{arctg} t + \int \frac{2t-2}{1+2t-t^2}\mathrm{d}t + \int \frac{2t+2}{1+t^2}\mathrm{d}t =$

$\left| \frac{8t}{(1+2t-t^2)(1+t^2)} = \frac{At+B}{1+2t-t^2} + \frac{Ct+D}{1+t^2} = \frac{t^3(A-C)+t^2(B+2C-D)+t(A+C-2D)+B+D}{(1+2t-t^2)(1+t^2)} \Rightarrow \begin{matrix} A = C = D = 2 \\ B = -2 \end{matrix} \right|$

$= -2\operatorname{arctg} t - \int \frac{2-2t}{1+2t-t^2}\mathrm{d}t + \int \frac{2t\,\mathrm{d}t}{1+t^2} + 2\int \frac{\mathrm{d}t}{1+t^2} = -2\operatorname{arctg} t - \ln|1+2t-t^2| + \ln|1+t^2| + 2\operatorname{arctg} t + C =$

$= \ln\left|\frac{1+t^2}{1+2t-t^2}\right| + C = \ln\left|\frac{1+\operatorname{tg}^2\frac{x}{2}}{1+2\operatorname{tg} x - \operatorname{tg}^2 x}\right| + C = \ln\left|\frac{\frac{1}{\cos^2\frac{x}{2}}}{1+2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} - \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}\right| + C = \ln\left|\frac{1}{\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} - \sin^2\frac{x}{2}}\right| + C =$

$= \ln\left|\frac{1}{\cos x + \sin x}\right| + C = -\ln|\sin x + \cos x| + C$

$\int \frac{\sin x - \cos x}{\sin x + \cos x}\mathrm{d}x = \left| \begin{matrix} t = \sin x + \cos x \\ \mathrm{d}t = \cos x - \sin x \end{matrix} \right| = -\int \frac{\mathrm{d}t}{t} = -\ln|t| + C = -\ln|\sin x + \cos x| + C$

$$\boxed{-\ln|\sin x + \cos x| + C}$$

7. $\int \frac{\mathrm{d}x}{5-3\cos x} = \left| \begin{matrix} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{matrix} \right| = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{5-3\frac{1-t^2}{1+t^2}} = \int \frac{2\,\mathrm{d}t}{2+8t^2} = \int \frac{\mathrm{d}t}{1+4t^2} = \left| \begin{matrix} 2t = s \\ 2\mathrm{d}t = \mathrm{d}s \end{matrix} \right| =$

$\frac{1}{2}\int \frac{\mathrm{d}s}{1+s^2} = \frac{1}{2}\operatorname{arctg} s + C = \frac{1}{2}\operatorname{arctg} 2t + C$

$$\boxed{\frac{1}{2}\operatorname{arctg}(2\operatorname{tg}\frac{x}{2}) + C}$$

8. $\int \frac{\cos x}{1+\cos x}\mathrm{d}x = \left| \begin{matrix} t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{1+t^2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2} & \cos x = \frac{1-t^2}{1+t^2} \end{matrix} \right| = \int \frac{\frac{1-t^2}{1+t^2}}{1+\frac{1-t^2}{1+t^2}} \frac{2\,\mathrm{d}t}{1+t^2} = \int \frac{1-t^2}{1+t^2}\mathrm{d}t = -\int \frac{t^2+1-2}{1+t^2}\mathrm{d}t =$

$= -\int 1\,\mathrm{d}t + 2\int \frac{\mathrm{d}t}{1+t^2} = -t + 2\operatorname{arctg} t + C = x - \operatorname{tg}\frac{x}{2} + C = x - \frac{\sin x}{1+\cos x} + C$

$$\boxed{x - \frac{\sin x}{1+\cos x} + C}$$

9. $\int \frac{\mathrm{d}x}{\sin x+\cos x}= \begin{vmatrix} t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{1+t^2} \\ \mathrm{d}x=\frac{2\,\mathrm{d}t}{1+t^2} & \cos x=\frac{1-t^2}{1+t^2} \end{vmatrix} = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{2\,\mathrm{d}t}{1+2t-t^2}$

$$\frac{2}{1+2t-t^2} = \frac{-2}{t^2-2t-1} = \frac{A}{t-1+\sqrt{2}} + \frac{B}{t-1-\sqrt{2}} = \frac{(A+B)t-(A+B)+\sqrt{2}(-A+B)}{t^2-2t-1} \Rightarrow A=\frac{\sqrt{2}}{2}, B=-\frac{\sqrt{2}}{2}$$

$= \frac{\sqrt{2}}{2}\int \frac{\mathrm{d}t}{t-1+\sqrt{2}} - \frac{\sqrt{2}}{2}\int \frac{\mathrm{d}t}{t-1-\sqrt{2}} = \frac{\sqrt{2}}{2}\ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}-1+\sqrt{2}}{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}-1-\sqrt{2}}\right| + C =$

$= \frac{\sqrt{2}}{2}\ln\left|\frac{\sin\frac{x}{2}+(\sqrt{2}-1)\cos\frac{x}{2}}{\sin\frac{x}{2}-(\sqrt{2}+1)\cos\frac{x}{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{\left(\sin\frac{x}{2}+(\sqrt{2}-1)\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+(\sqrt{2}+1)\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}-(\sqrt{2}+1)\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+(\sqrt{2}+1)\cos\frac{x}{2}\right)}\right| + C =$

$= \frac{\sqrt{2}}{2}\ln\left|\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sqrt{2}\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}-(3+2\sqrt{2})\cos^2\frac{x}{2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{1+\sqrt{2}\sin x}{\sqrt{2}+1+(2+\sqrt{2})\cos x}\right| + C$

$$\boxed{\frac{\sqrt{2}}{2}\ln\left|\frac{1+\sqrt{2}\sin x}{\sqrt{2}+1+(2+\sqrt{2})\cos x}\right| + C}$$

10. $\int \frac{\mathrm{d}x}{\cos x+2\sin x+3}= \begin{vmatrix} t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{1+t^2} \\ \mathrm{d}x=\frac{2\,\mathrm{d}t}{1+t^2} & \cos x=\frac{1-t^2}{1+t^2} \end{vmatrix} = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{\frac{1-t^2}{1+t^2}+\frac{4t}{1+t^2}+3} = \int \frac{2\,\mathrm{d}t}{2t^2+4t+4} = \int \frac{\mathrm{d}t}{t^2+2t+2} =$

$= \int \frac{\mathrm{d}t}{(t+1)^2+1} = \begin{vmatrix} s=t+1 \\ \mathrm{d}s=\mathrm{d}t \end{vmatrix} = \int \frac{\mathrm{d}s}{1+s^2} = \operatorname{arctg} s + C = \operatorname{arctg}(t+1) + C = \operatorname{arctg}(1+\operatorname{tg}\frac{x}{2}) + C$

$$\boxed{\operatorname{arctg}(1+\operatorname{tg}\tfrac{x}{2}) + C}$$

11. $\int \sin x\sin 2x\sin 3x\,\mathrm{d}x= \frac{1}{2}\int \sin x(\cos x-\cos 5x)\,\mathrm{d}x = \frac{1}{4}\int \sin 2x\,\mathrm{d}x - \frac{1}{4}\int(\sin 6x-\sin 4x)\mathrm{d}x =$
$= -\frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x + C$

$$\boxed{-\tfrac{1}{8}\cos 2x + \tfrac{1}{24}\cos 6x - \tfrac{1}{16}\cos 4x + C}$$

12. $\int \cosh^3 x\,\mathrm{d}x= \begin{vmatrix} \cosh^2 x & \cosh x \\ 2\cosh x\sinh x & \sinh x \end{vmatrix} = \cosh^2 x\sinh x - 2\int \cosh x\sinh^2 x\,\mathrm{d}x = \begin{vmatrix} t=\sinh x \\ \mathrm{d}t=\cosh x\,\mathrm{d}x \end{vmatrix} =$

$= \cosh^2 x\sinh x - 2\int t^2\mathrm{d}t = \cosh^2 x\sinh x - \frac{2}{3}t^3 + C = \frac{2}{3}(\cosh^2 x-\sinh^2 x)\sinh x + \frac{1}{3}\cosh^2 x\sinh x + C =$
$= \frac{2}{3}\sinh x - \frac{1}{3}\cosh^2 x\sinh x + C$

$$\boxed{\tfrac{2}{3}\sinh x - \tfrac{1}{3}\cosh^2 x\sinh x + C}$$

13. $\int \operatorname{tgh} x\,\mathrm{d}x= \int \frac{\sinh x}{\cosh x}\mathrm{d}x = \begin{vmatrix} t=\cosh x \\ \mathrm{d}t=\sinh x \end{vmatrix} = \int \frac{\mathrm{d}t}{t} = \ln\cosh x + C = -x + \ln(1+e^{2x}) + C$

$$\boxed{\ln\cosh x + C}$$

14. $\int \frac{\mathrm{d}x}{(2-x)\sqrt{1-x}}= \begin{vmatrix} t=\sqrt{1-x} & -2\sqrt{1-x}\,\mathrm{d}t=\mathrm{d}x \\ \mathrm{d}t=-\frac{1}{2}\frac{1}{\sqrt{1-x}}\mathrm{d}x & -2t\,\mathrm{d}t=\mathrm{d}x \end{vmatrix} = \int \frac{-2t\,\mathrm{d}t}{(1+t^2)t} = -2\int \frac{\mathrm{d}t}{1+t^2} = -2\operatorname{arctg} t + C$

$$\boxed{-2\operatorname{arctg}\sqrt{x-1} + C}$$

15. $\int \frac{\mathrm{d}x}{1+\sqrt[3]{x}}= \begin{vmatrix} t^3=x & t=\sqrt[3]{x} \\ 3t^2\mathrm{d}t=\mathrm{d}x & \end{vmatrix} = \int \frac{3t^2\mathrm{d}t}{1+t} = 3\int \frac{t^2-1+1}{1+t}\mathrm{d}t = 3\int(t-1)\,\mathrm{d}t + 3\int \frac{\mathrm{d}t}{1+t} =$
$= \frac{3}{2}t^2 - 3t + 3\ln|1+t| + C$

$$\boxed{\tfrac{3}{2}\sqrt[3]{x^2} - 3\sqrt[3]{x} + 3\ln|1+\sqrt[3]{x}| + C}$$

16. $\int \frac{\sqrt{x}}{1-\sqrt[3]{x}}\mathrm{d}x= \begin{vmatrix} x=t^6 & t=\sqrt[6]{x} \\ \mathrm{d}x=6t^5\mathrm{d}t & \end{vmatrix} = \int \frac{t^3}{1-t^2}6t^5\mathrm{d}t = -6\int \frac{t^8}{t^2-1}\mathrm{d}t =$

$|\, t^8 = t^6(t^2-1) + t^4(t^2-1) + t^2(t^2-1) + (t^2-1) + 1\,| = \int \left(t^6+t^4+t^2+1+\frac{1}{t^2-1}\right)\mathrm{d}t =$

$= \frac{t^7}{7} + \frac{t^5}{5} + \frac{t^3}{3} + t + \frac{1}{2}\int \frac{t+1-(t-1)}{t^2-1}\mathrm{d}t = \ldots + \int \frac{\mathrm{d}t}{t-1} - \int \frac{\mathrm{d}t}{t+1} = \ldots + \ln\left|\frac{t-1}{t+1}\right| + C$

$$\boxed{\tfrac{1}{7}x^{\frac{7}{6}} + \tfrac{1}{5}x^{\frac{5}{6}} + \tfrac{1}{3}x^{\frac{1}{2}} + x^{\frac{1}{6}} + \ln\left|\frac{x^{\frac{1}{6}}-1}{x^{\frac{1}{6}}+1}\right| + C}$$

17. $\int \frac{\mathrm{d}x}{x\sqrt{x-4}} = \left| \begin{matrix} t^2 = x-4 \quad t=\sqrt{x-4} \\ 2t\,\mathrm{d}t = \mathrm{d}x \end{matrix} \right| = \int \frac{2t\,\mathrm{d}t}{(4+t^2)t} = 2\int \frac{\mathrm{d}t}{4+t^2} = \frac{1}{2}\int \frac{\mathrm{d}t}{1+\left(\frac{t}{2}\right)^2} =$

$= \left| \begin{matrix} s = \frac{t}{2} \\ \mathrm{d}s = \frac{1}{2}\mathrm{d}t \quad 2\mathrm{d}s = \mathrm{d}t \end{matrix} \right| = \int \frac{\mathrm{d}s}{1+s^2} = \operatorname{arctg} s + C = \operatorname{arctg} \frac{t}{2} + C$

$$\boxed{\operatorname{arctg} \frac{\sqrt{x-4}}{2} + C}$$

18. $\int \sqrt{\frac{1+x}{1-x}}\,\mathrm{d}x = \left| \begin{matrix} t = \sqrt{\frac{1+x}{1-x}} & x = \frac{t^2-1}{t^2+1} & 1-x = \frac{2}{1+t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{1-x}{1+x}}\frac{2\,\mathrm{d}x}{(1-x)^2} & \mathrm{d}x = \frac{4t\,\mathrm{d}t}{(1+t^2)^2} & \end{matrix} \right| = \int t \frac{4t\,\mathrm{d}t}{(1+t^2)^2} = 4\int \frac{t^2+1-1}{(1+t^2)^2} =$

$= 4\int \frac{\mathrm{d}t}{1+t^2} - 4\int \frac{\mathrm{d}t}{(1+t^2)^2} = 4\operatorname{arctg} t - 2\operatorname{arctg} t - \frac{2t}{1+t^2} + C$

$$\boxed{-\frac{1}{2}\sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1+x}{1-x}} + \frac{1}{6}\left(\frac{1+x}{1-x}\right)^{\frac{3}{2}} + C}$$

19. $\int \frac{\mathrm{d}x}{\sqrt{3-2x-5x^2}} = \int \frac{\mathrm{d}x}{\sqrt{(3-5x)(x+1)}} = \left| \begin{matrix} t = \sqrt{\frac{3-5x}{x+1}} & x = \frac{3-t^2}{5+t^2} & x+1 = \frac{8}{5+t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{x+1}{3-5x}}\frac{-8\,\mathrm{d}x}{(x+1)^2} & \mathrm{d}x = \frac{-16t\,\mathrm{d}t}{(5+t^2)^2} & 3-5x = \frac{8t^2}{5+t^2} \end{matrix} \right| =$

$= \int \frac{-\frac{16t}{(5+t^2)^2}}{\frac{8t}{5+t^2}}\mathrm{d}t = -2\int \frac{\mathrm{d}t}{5+t^2} = -\frac{2}{5}\int \frac{\mathrm{d}t}{1+\left(\frac{t}{\sqrt{5}}\right)^2} = \left| \begin{matrix} s = \frac{t}{\sqrt{5}} \\ \mathrm{d}s = \frac{\mathrm{d}t}{\sqrt{5}} \quad \sqrt{5}\,\mathrm{d}s = \mathrm{d}t \end{matrix} \right| = -\frac{2}{\sqrt{5}}\int \frac{\mathrm{d}s}{1+s^2} =$

$= -\frac{2}{\sqrt{5}}\operatorname{arctg} s + C = -\frac{2}{\sqrt{5}}\operatorname{arctg} \frac{t}{\sqrt{5}} + C$

$$\boxed{-\frac{2}{\sqrt{5}}\operatorname{arctg}\sqrt{\frac{3-5x}{5+5x}} + C}$$

20. $\int \frac{x-1}{\sqrt{x^2-2x+2}}\mathrm{d}x = \int \frac{x-1}{\sqrt{(x-1)^2+1}}\mathrm{d}x = \left| \begin{matrix} t = x-1 \\ \mathrm{d}t = \mathrm{d}x \end{matrix} \right| = \int \frac{t}{\sqrt{t^2+1}}\mathrm{d}t = \left| \begin{matrix} s = t^2+1 \\ \mathrm{d}s = 2t\,\mathrm{d}t \end{matrix} \right| = \frac{1}{2}\int \frac{\mathrm{d}s}{\sqrt{s}} =$

$= \sqrt{s} + C = \sqrt{t^2+1} + C$

$\int \frac{x-1}{\sqrt{x^2-2x+2}}\mathrm{d}x = \left| \begin{matrix} t = x^2-2x+2 \\ \mathrm{d}t = (2x-2)\,\mathrm{d}x \end{matrix} \right| = \frac{1}{2}\int \frac{\mathrm{d}t}{\sqrt{t}} = \sqrt{t} + C$

$$\boxed{\sqrt{x^2-2x+2} + C}$$

21. $\int \frac{\mathrm{d}x}{(9+x^2)\sqrt{9+x^2}} = \left| \begin{matrix} x = 3\operatorname{tg} t \\ \mathrm{d}x = \frac{3}{\cos^2 t}\mathrm{d}t = 3(1+\operatorname{tg}^2 t)\,\mathrm{d}t \end{matrix} \right| = \int \frac{3(1+\operatorname{tg}^2 t)}{9(1+\operatorname{tg}^2 t)\cdot 3\sqrt{1+\operatorname{tg}^2 t}} = \frac{1}{9}\int \cos t\,\mathrm{d}t =$

$= \frac{1}{9}\sin t + C$

$$\boxed{\frac{1}{9}\frac{x}{\sqrt{9+x^2}} + C}$$

22. $\int \sqrt{3-2x-x^2}\,\mathrm{d}x = \int \sqrt{(x+3)(1-x)}\mathrm{d}x = \left| \begin{matrix} t = \sqrt{\frac{1-x}{x+3}} & x = \frac{1-3t^2}{1+t^2} & x+3 = \frac{4}{1+t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{x+3}{1-x}}\frac{-4\,\mathrm{d}x}{(x+3)^2} & \mathrm{d}x = \frac{-8t\,\mathrm{d}t}{(1+t^2)^2} & 1-x = \frac{4t^2}{1+t^2} \end{matrix} \right| =$

$= \int \frac{4t}{(1+t^2)}\frac{-8t}{(1+t^2)^2}\mathrm{d}t = -32\int \frac{t^2\mathrm{d}t}{(1+t^2)^3} = -32\int \frac{t^2+1-1}{(1+t^2)^3}\mathrm{d}t = -32\int \frac{\mathrm{d}t}{(1+t^2)^2} + 32\int \frac{\mathrm{d}t}{(1+t^2)^3} =$

$= -32\left(\frac{1}{2}\operatorname{arctg} t + \frac{1}{2}\frac{t}{1+t^2}\right) + 32\left(\frac{3}{8}\operatorname{arctg} t + \frac{3}{8}\frac{t}{1+t^2} + \frac{1}{4}\frac{t}{(1+t^2)^2}\right) + C = 8\frac{t}{(1+t^2)^2} - 4\frac{t}{1+t^2} - 4\operatorname{arctg} t + C$

$$\boxed{-4\operatorname{arctg}\sqrt{\frac{1-x}{x+3}} - \frac{x+1}{2}\sqrt{3-2x-x^2} + C}$$

23. $\int \frac{2x+1}{\sqrt{x^2+x}}\mathrm{d}x = \left| \begin{matrix} t = x^2+x \\ \mathrm{d}t = (2x+1)\,\mathrm{d}x \end{matrix} \right| = \int \frac{\mathrm{d}t}{\sqrt{t}} = 2\sqrt{t} + C$

$$\boxed{2\sqrt{x^2+x} + C}$$

24. $\int \frac{\sqrt{x^2+2x}}{x}\mathrm{d}x = \left| \begin{matrix} t = \sqrt{\frac{x}{x+2}} & x = \frac{2t^2}{1-t^2} & x+2 = \frac{2}{1-t^2} \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{x+2}{x}}\frac{2}{(x+2)^2} & \mathrm{d}x = \frac{4t\,\mathrm{d}t}{(1-t^2)^2} & \end{matrix} \right| = \int \frac{\frac{2t}{1-t^2}\frac{4t\,\mathrm{d}t}{(1-t^2)^2}}{\frac{2t^2}{1-t^2}} = 4\int \frac{\mathrm{d}t}{(1-t^2)^2} =$

$= 4J_2 = \frac{2t}{1-t^2} + \ln\left|\frac{1+t}{1-t}\right| + C$

$J_{n-1}=\int\frac{\mathrm{d}t}{(1-t^2)^{n-1}}=\begin{vmatrix}\frac{1}{(1-t^2)^{n-1}} & 1\\ -(n-1)(1-t^2)^{-n}(-2t) & t\end{vmatrix}=\frac{t}{(1-t^2)^{n-1}}+2(n-1)\int\frac{-t^2\mathrm{d}t}{(1-t^2)^n}=$

$=\frac{t}{(1-t^2)^{n-1}}+2(n-1)\int\frac{1-t^2-1}{(1-t^2)^n}\mathrm{d}t=\frac{t}{(1-t^2)^{n-1}}+2(n-1)\int\frac{\mathrm{d}t}{(1-t^2)^{n-1}}-2(n-1)\int\frac{\mathrm{d}t}{(1-t^2)^n}=$

$=\frac{t}{(1-t^2)^{n-1}}+(2n-2)J_{n-1}-(2n-2)J_n\Rightarrow J_n=\frac{1}{2n-2}\frac{t}{(1-t^2)^{n-1}}+\frac{2n-3}{2n-2}J_{n-1},\ n>1$

$J_1=\int\frac{1}{1-t^2}=\frac12\int\frac{1-t+1+t}{1-t^2}\mathrm{d}t=\frac12\int\frac{\mathrm{d}t}{1+t}+\frac12\int\frac{\mathrm{d}t}{1-t}=\frac12\ln|1+t|-\frac12\ln|1-t|+C=\frac12\ln|\frac{1+t}{1-t}|+C$

$J_2=\frac12\frac{t}{1-t^2}+\frac12J_1=\frac12\frac{t}{1-t^2}+\frac14\ln|\frac{1+t}{1-t}|+C$

$$\boxed{\ln|x+1+\sqrt{x^2+2x}|+\sqrt{x^2+2x}+C}$$

25. $\int\frac{\mathrm{d}x}{\sqrt{25+9x^2}}=\begin{vmatrix}x=\frac53\,\mathrm{tg}\,t\\ \mathrm{d}x=\frac53\frac{\mathrm{d}t}{\cos^2 t}\end{vmatrix}=\int\frac53\frac{\frac{\mathrm{d}t}{\cos^2 t}}{\sqrt{25+9\frac{25}{9}\,\mathrm{tg}^2\,t}}=\frac13\int\frac{\mathrm{d}t}{\cos t}=\frac13\int\frac{\cos t\,\mathrm{d}t}{1-\sin^2 t}=\begin{vmatrix}s=\sin t\\ \mathrm{d}s=\cos t\,\mathrm{d}t\end{vmatrix}=$

$=\frac13\int\frac{\mathrm{d}s}{1-s^2}=\frac16\ln|\frac{1+s}{1-s}|+C=\frac16\ln|\frac{1+\sin t}{1-\sin t}|+C=\begin{vmatrix}\mathrm{tg}\,t=\frac{3x}{5} & \cos t=\frac{1}{\sqrt{1+\mathrm{tg}^2\,t}}=\frac{5}{\sqrt{25+9x^2}}\\ \sin t=\frac{3x}{\sqrt{25+9x^2}} & \end{vmatrix}=$

$=\frac16\ln|\frac{1+\frac{3x}{\sqrt{25+9x^2}}}{1-\frac{3x}{\sqrt{25+9x^2}}}|+C=\frac16\ln|\frac{\sqrt{25+9x^2}+3x}{\sqrt{25+9x^2}-3x}|+C$

$$\boxed{\frac16\ln|\frac{\sqrt{25+9x^2}+3x}{\sqrt{25+9x^2}-3x}|+C}$$

26. $\int\frac{3\,\mathrm{d}x}{\sqrt{9x^2-1}}=\begin{vmatrix}t=3x\\ \mathrm{d}t=3\,\mathrm{d}x\end{vmatrix}=\int\frac{\mathrm{d}t}{\sqrt{t^2-1}}=\begin{vmatrix}t=\frac{1}{\cos s}\\ \mathrm{d}t=\mathrm{tg}\,s\,\mathrm{d}s\end{vmatrix}=\int\frac{\mathrm{tg}\,s\,\mathrm{d}s}{\sqrt{\frac{1}{\cos^2 s}-1}}=\int\frac{\mathrm{tg}\,s}{\mathrm{tg}\,s}\mathrm{d}s=\int 1\,\mathrm{d}s=s+C=$

$=\arccos\frac1t+C=\arccos\frac{1}{3x}+C$

$$\boxed{\arccos\frac{1}{3x}+C}$$

27. $\int e^{ax}\cos bx\,\mathrm{d}x=\begin{vmatrix}e^{ax} & \cos bx\\ ae^{ax} & \frac1b\sin bx\end{vmatrix}=\frac1be^{ax}\sin bx-\frac ab\int e^{ax}\sin bx\,\mathrm{d}x$

$\int e^{ax}\sin bx\,\mathrm{d}x=\begin{vmatrix}e^{ax} & \sin bx\\ ae^{ax} & -\frac1b\cos bx\end{vmatrix}=-\frac1be^{ax}\cos bx+\frac ab\int e^{ax}\cos bx\,\mathrm{d}x$

$$c+\frac abs=\frac1be^{ax}\sin bx\qquad c=\frac{e^{ax}}{a^2+b^2}\left(a\cos bx+b\sin bx\right)+C$$
$$-\frac abc+s=-\frac1be^{ax}\cos bx\qquad s=\frac{e^{ax}}{a^2+b^2}\left(a\sin bx-b\cos bx\right)+C$$

$$\boxed{\frac{e^{ax}}{a^2+b^2}\left(a\cos bx+b\sin bx\right)+C}$$

29. $\int(3x^2+2x+1)\sin\frac x3\mathrm{d}x=\begin{vmatrix}3x^2+2x+1 & \sin\frac x3\\ 6x+2 & -3\cos\frac x3\end{vmatrix}=-3(3x^2+2x+1)\cos\frac x3+6\int(3x+1)\cos\frac x3\mathrm{d}x=$

$=\begin{vmatrix}3x+1 & \cos\frac x3\\ 3 & 3\sin\frac x3\end{vmatrix}=-(9x^2+6x+3)\cos\frac x3+6(3x+1)\sin\frac x3-54\int\sin\frac x3\mathrm{d}x=$

$=-(9x^2+6x+3)\cos\frac x3+(18x+6)\sin\frac x3+162\cos\frac x3+C$

$$\boxed{-(9x^2+6x-159)\cos\frac x3+(18x+6)\sin\frac x3+C}$$

30. $\int(3x^2+1)\ln(x-4)\,\mathrm{d}x=\begin{vmatrix}\ln(x-4) & 3x^2+1\\ \frac{1}{x-4} & x^3+x\end{vmatrix}=x(x^2+1)\ln(x-4)-\int\frac{x^3+x}{x-4}\mathrm{d}x$

$$\left(x^3+x=x^2(x-4)+4x(x-4)+17(x-4)+68\right)$$

$=x(x^2+1)\ln(x-4)-\int\left(x^2+4x+17+\frac{68}{x-4}\right)\,\mathrm{d}x=x(x^2+1)\ln(x-4)-\frac13x^3+2x^2+17x+68\ln|x-4|+C$

$$\boxed{x(x^2+1)\ln(x-4)-\frac13x^3+2x^2+17x+68\ln|x-4|+C}$$

31. $\int\left(\frac{\ln x}{x}\right)^2\mathrm{d}x=\begin{vmatrix}\ln^2 x & \frac{1}{x^2}\\ 2\frac{\ln x}{x} & -\frac1x\end{vmatrix}=-\frac{\ln^2 x}{x}+2\int\frac{\ln x}{x^2}\mathrm{d}x$

$\int\frac{\ln x}{x^2}\mathrm{d}x=\begin{vmatrix}\ln x & \frac{1}{x^2}\\ \frac1x & -\frac1x\end{vmatrix}=-\frac{\ln x}{x}+\int\frac{\mathrm{d}x}{x^2}=-\frac{\ln x}{x}-\frac1x+C$

$$\boxed{-\tfrac{1}{x}(\ln^2 x - 2\ln x - 2) + C}$$

32. $\int x^2 \operatorname{arctg} 3x \, dx = \begin{vmatrix} \operatorname{arctg} 3x & x^2 \\ \frac{3}{1+9x^2} & \frac{x^3}{3} \end{vmatrix} = \frac{1}{3}x^3 \operatorname{arctg} 3x - \int \frac{x^3}{1+9x^2} dx = | \, x^3 : (9x^2+1) = \frac{1}{9}x \quad \text{zv.} \ -\frac{1}{9}x \, | =$

$= \frac{1}{3}x^3 \operatorname{arctg} 3x - \int \frac{1}{9}x \, dx + \frac{1}{9}\int \frac{x}{1+9x^2} dx = \frac{1}{3}x^3 \operatorname{arctg} 3x - \frac{1}{18}x^2 + \frac{1}{162}\ln(1+9x^2) + C$

$$\boxed{\tfrac{1}{3}x^3 \operatorname{arctg} 3x - \tfrac{1}{18}x^2 + \tfrac{1}{162}\ln(1+9x^2) + C}$$

33. $\int \arcsin^2 x \, dx = \begin{vmatrix} \arcsin^2 x & 1 \\ 2\frac{\arcsin x}{\sqrt{1-x^2}} & x \end{vmatrix} = x\arcsin^2 x - 2\int \arcsin x \frac{x}{\sqrt{1-x^2}} dx = \begin{vmatrix} \arcsin x & \frac{x}{\sqrt{1-x^2}} \\ \frac{1}{\sqrt{1-x^2}} & -\sqrt{1-x^2} \end{vmatrix} =$

$= x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2\int 1 \, dx = x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C$

$$\boxed{x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C}$$

34. $\int \sin x \sinh x \, dx = \begin{vmatrix} \sin x & \sinh x \\ \cos x & \cosh x \end{vmatrix} = \sin x \cosh x - \int \cos x \cosh x \, dx$

$\int \cos x \cosh x \, dx = \begin{vmatrix} \cos x & \cosh x \\ -\sin x & \sinh x \end{vmatrix} = \cos x \sinh x + \int \sin x \sinh x \, dx$

$$\begin{aligned} ss + cc &= \sin x \cosh x & \quad 2ss &= \sin x \cosh x - \cos x \sinh x \\ -ss + cc &= \cos x \sinh x & \quad ss &= \tfrac{1}{2}(\sin x \cosh x - \cos x \sinh x) \end{aligned}$$

$$\boxed{\tfrac{1}{2}(\sin x \cosh x - \cos x \sinh x) + C}$$

35. $\int (4x^3+2x)\operatorname{arctg} x \, dx = \begin{vmatrix} \operatorname{arctg} x & 4x^3+2x \\ \frac{1}{1+x^2} & x^4+x^2 \end{vmatrix} = (x^4+x^2)\operatorname{arctg} x - \int \frac{x^4+x^2}{1+x^2} dx =$

$= (x^4+x^2)\operatorname{arctg} x - \int x^2 dx = (x^4+x^2)\operatorname{arctg} x - \frac{1}{3}x^3 + C$

$$\boxed{(x^4+x^2)\operatorname{arctg} x - \tfrac{1}{3}x^3 + C}$$

36. $\int \frac{dx}{(2x^2+2)\sqrt{\operatorname{arccotg}^3 x}} = \begin{vmatrix} t = \operatorname{arccotg} x \\ dt = -\frac{1}{1+x^2}dx \end{vmatrix} = -\frac{1}{2}\int \frac{dt}{\sqrt{t^3}} = -\frac{1}{2}\int t^{-\frac{3}{2}} dt = -\frac{1}{2}(-2)t^{-\frac{1}{2}} + C = t^{-\frac{1}{2}} + C$

$$\boxed{\frac{1}{\sqrt{\operatorname{arccotg} x}} + C}$$

37. $\int (2x-1)\arccos x \, dx = \begin{vmatrix} \arccos x & 2x-1 \\ -\frac{1}{\sqrt{1-x^2}} & x^2-x \end{vmatrix} = (x^2-x)\arccos x + \int \frac{x^2-x}{\sqrt{1-x^2}} dx$

$= (x^2-x)\arccos x + \int x\sqrt{\frac{1-x}{1+x}} \, dx$

$\int x\sqrt{\frac{1-x}{1+x}} \, dx = \begin{vmatrix} t = \sqrt{\frac{1-x}{1+x}} & x = \frac{1-t^2}{1+t^2} \\ dx = \frac{(-2t)(1+t^2)-(1-t^2)(2t)}{(1+t^2)^2}dt & dx = \frac{-4t}{(1+t^2)^2}dt \end{vmatrix} = \int \frac{1-t^2}{1+t^2} \cdot t \cdot \frac{-4t \, dt}{(1+t^2)^2} = 4\int \frac{t^4-t2}{(1+t^2)^3} dt =$

$= 4\int \frac{t^2(1+t^2)-2(1+t^2)+2}{(1+t^2)^3} dt = 4\int \frac{t^2-2}{(1+t^2)^2} + 8\int \frac{dt}{(1+t^2)^3} = 4\int \frac{t^2+1-3}{(1+t^2)^2} dt + 8\int \frac{dt}{(1+t^2)^3} =$

$= 4\int \frac{dt}{1+t^2} - 12\int \frac{dt}{(1+t^2)^2} + 8\int \frac{dt}{(1+t^2)^3} =: I$

$\operatorname{arctg} t = \int \frac{1}{1+t^2} = \begin{vmatrix} \frac{1}{1+t^2} & 1 \\ -\frac{2t}{1+t^2} & t \end{vmatrix} = \frac{t}{1+t^2} + 2\int \frac{t^2}{1+t^2} dt = \frac{t}{1+t^2} + 2\int \frac{t^2+1-1}{(t^2+1)^2} dt =$

$= \frac{t}{1+t^2} + 2\int \frac{1}{1+t^2} dt - 2\int \frac{dt}{(1+t^2)^2} = \frac{t}{1+t^2} + 2\operatorname{arctg} t - 2\int \frac{dt}{(1+t^2)^2}$

$\int \frac{dt}{(1+t^2)^2} = \frac{1}{2}\frac{t}{1+t^2} + \frac{1}{2}\operatorname{arctg} t + C$

Podobne $\int \frac{dt}{(1+t^2)^3} = \frac{3}{4}\int \frac{dt}{(1+t^2)^2} + \frac{1}{4}\frac{t}{(1+t^2)^2} + C = \frac{3}{8}\operatorname{arctg} t + \frac{3}{8}\frac{t}{1+t^2} + \frac{1}{4}\frac{t}{(1+t^2)^2} + C$

$I = 4\operatorname{arctg} t - 6\operatorname{arctg} t - 6\frac{t}{1+t^2} + 3\operatorname{arctg} t + 3\frac{t}{1+t^2} + 2\frac{t}{(1+t^2)^2} + C = \operatorname{arctg} t - 3\frac{t}{1+t^2} + 2\frac{t}{(1+t^2)^2} + C =$

$= \operatorname{arctg}\sqrt{\frac{x-1}{x+1}} - 3\frac{\sqrt{\frac{x-1}{x+1}}}{1+\frac{1-x}{1+x}} + 2\frac{\sqrt{\frac{1-x}{1+x}}}{(1+\frac{1-x}{1+x})^2} + C = \operatorname{arctg}\sqrt{\frac{x-1}{x+1}} - \frac{3(1+x)}{2}\sqrt{\frac{1-x}{1+x}} + \frac{(1+x)^2}{2}\sqrt{\frac{x-1}{x+1}} + C$

$$\boxed{\operatorname{arctg}\sqrt{\tfrac{x-1}{x+1}}+\tfrac{1}{2}(x^2-x-2)\sqrt{\tfrac{x-1}{x+1}}+C}$$

38. $\int(x^2-3x+1)\cosh 2x\,\mathrm{d}x= \begin{vmatrix} x^2-3x+1 & \cosh 2x \\ 2x-3 & \frac{1}{2}\sinh 2x \end{vmatrix} = \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{2}\int(2x-3)\sinh 2x\,\mathrm{d}x =$

$= \begin{vmatrix} 2x-3 & \sinh 2x \\ 2 & \frac{1}{2}\cosh 2x \end{vmatrix} = \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{4}(2x-3)\cosh 2x + \frac{1}{2}\int\cosh 2x\,\mathrm{d}x =$

$= \frac{1}{2}(x^2-3x+1)\sinh 2x - \frac{1}{4}(2x-3)\cosh 2x + \frac{1}{4}\sinh 2x + C$

$$\boxed{\tfrac{1}{2}(x^2-3x+1)\sinh 2x - \tfrac{1}{4}(2x-3)\cosh 2x + \tfrac{1}{4}\sinh 2x + C}$$

39. $\int_0^3 |1-3x|\mathrm{d}x = \int_0^{\frac{1}{3}}(1-3x)\,\mathrm{d}x + \int_{\frac{1}{3}}^3(3x-1)\,\mathrm{d}x = \left[x-\frac{3}{2}x^2\right]_0^{\frac{1}{3}} + \left[\frac{3}{2}x^2-x\right]_{\frac{1}{3}}^3 =$

$= \frac{1}{6}+\frac{21}{2}+\frac{1}{6}=\frac{65}{6}$

$$\boxed{\tfrac{65}{6}}$$

40. $\int_{-4}^{-2}\frac{1}{x}\mathrm{d}x = [\ln|x|]_{-4}^{-2} = \ln 2 - \ln 4$

$$\boxed{-\ln 2}$$

41. $\int_0^{\pi}\cos x\,\mathrm{d}x = [\sin x]_0^{\pi} = 0$

$$\boxed{0}$$

42. $\int_0^{\pi}|\cos x|\,\mathrm{d}x = \int_0^{\frac{\pi}{2}}\cos x\,\mathrm{d}x - \int_{\frac{\pi}{2}}^{\pi}\cos x\,\mathrm{d}x = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^{\pi} = 2$

$$\boxed{2}$$

43. $\int_0^{\pi}\sin^3 x\,\mathrm{d}x = \int_0^{\pi}\sin x(1-\cos^2 x)\mathrm{d}x = \int_0^{\pi}\sin x\,\mathrm{d}x - \int_0^{\pi}\sin x\cos^2 x\,dx =$

$[-\cos x]_0^{\pi} - \int_0^{\pi}\sin x\cos^2 x\,\mathrm{d}x = 2 - \int_0^{\pi}\sin x\cos^2 x\,\mathrm{d}x = \begin{vmatrix} t=\cos x & \pi\to -1 \\ \mathrm{d}t=-\sin x\,\mathrm{d}x & 0\to 1 \end{vmatrix}$

$= 2+\int_1^{-1}t^2\,\mathrm{d}t = 2+\left[\frac{1}{3}t^3\right]_1^{-1} = \frac{4}{3}$

$$\boxed{\tfrac{4}{3}}$$

44. $\int_0^{\frac{\pi}{2}}\cos x\cdot\sin^2 x\,dx = \begin{vmatrix} t=\sin x & \frac{\pi}{2}\to 1 \\ \mathrm{d}t=\cos x\,\mathrm{d}x & 0\to 0 \end{vmatrix} = \int_0^1 t^2\mathrm{d}t = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$

$$\boxed{\tfrac{1}{3}}$$

45. $\int_0^1\frac{\sqrt{x}}{1+\sqrt{x}}\,dx = \begin{vmatrix} t=\sqrt{x} & 1\to 1, 0\to 0 \\ \mathrm{d}t=\frac{\mathrm{d}x}{2\sqrt{x}} & \mathrm{d}x=2\sqrt{x}\,\mathrm{d}t \end{vmatrix} = \int_0^1\frac{2\sqrt{x}\sqrt{x}\,\mathrm{d}t}{1+\sqrt{x}} = 2\int_0^1\frac{t^2}{t+1}\,\mathrm{d}t = 2\int_0^1\frac{t^2-1+1}{t+1}\mathrm{d}t$

$= 2\int_0^1(t-1)\,\mathrm{d}t + 2\int_0^1\frac{1}{1+t}\mathrm{d}t = 2\left[\frac{1}{2}t^2-t+\ln|1+t|\right]_0^1 = 2\ln 2 - 1$

$$\boxed{\ln 4 - 1}$$

46. $\int_{-1}^1\frac{\mathrm{d}x}{(1+x^2)^2}$

$\pi = [\operatorname{arctg} x]_{-1}^1 = \int_{-1}^1\frac{\mathrm{d}x}{1+x^2} = \begin{vmatrix} \frac{1}{1+x^2} & 1 \\ -\frac{2x}{(1+x^2)^2} & x \end{vmatrix} = \left[\frac{x}{1+x^2}\right]_{-1}^1 + 2\int_{-1}^1\frac{x^2}{(1+x^2)^2} =$

$1+2\int_{-1}^1\frac{x^2+1-1}{(x^2+1)^2}\mathrm{d}x = 1+2\int_{-1}^1\frac{\mathrm{d}x}{1+x^2} - 2\int_{-1}^1\frac{\mathrm{d}x}{(1+x^2)^2} = 1+2\pi-2\int_{-1}^1\frac{\mathrm{d}x}{(1+x^2)^2}$

$\int_{-1}^1\frac{\mathrm{d}x}{(1+x^2)^2} = \frac{1}{2}(1+\pi)$

$$\boxed{\tfrac{\pi+1}{2}}$$

47. $\int_0^{\sqrt{2}}\sqrt{4-x^2}\,dx = \begin{vmatrix} x=2\sin t & t=\arcsin\frac{t}{2} \\ \mathrm{d}x=2\cos t\,\mathrm{d}t & \sqrt{2}\to\frac{\pi}{4}, 0\to 0 \end{vmatrix} = \int_0^{\frac{\pi}{4}}2\sqrt{4-4\sin^2 t}\cos t\,\mathrm{d}t =$

$= 4\int_0^{\frac{\pi}{4}}\cos^2 t\,\mathrm{d}t = 4\int_0^{\frac{\pi}{4}}\frac{\cos 2t+1}{2}\mathrm{d}t = 4\left[\frac{1}{4}\sin 2t+\frac{t}{2}\right]_0^{\frac{\pi}{4}} = 1+\frac{\pi}{2}$

$$\boxed{1+\frac{\pi}{2}}$$

48. $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\,dx = \left| \begin{matrix} t=\sqrt{e^x-1} & \ln 5\to 2 \\ \mathrm{d}t=\frac{1}{2}\frac{e^x}{\sqrt{e^x-1}}\mathrm{d}x & 0\to 0 \end{matrix} \right| = \int_0^2 \frac{\sqrt{e^x-1}}{e^x+3} 2\sqrt{e^x-1}\,\mathrm{d}t = 2\int_0^2 \frac{e^x-1}{e^x-1+4}\mathrm{d}t =$

$= 2\int_0^2 \frac{t^2}{t^2+4} = 2\int_0^2 \frac{t^2+4-4}{t^2+4}\mathrm{d}t = 2\int_0^2 1\,dt - 8\int_0^2 \frac{\mathrm{d}t}{t^2+4} = \left| \begin{matrix} t=2s & s=\frac{t}{2} \\ \mathrm{d}t=2\mathrm{d}s & 2\to 1, 0\to 0 \end{matrix} \right| =$

$= 4 - 16\int_0^1 \frac{\mathrm{d}s}{4+4s^2} = 4 - 4\int_0^1 \frac{\mathrm{d}s}{1+s^2} = 4 - 4\left[\operatorname{arctg} s\right]_0^1 = 4-\pi$

$$\boxed{4-\pi}$$

49. $\int_1^2 \frac{\mathrm{d}x}{\sqrt{3+2x-x^2}} = \int_1^2 \frac{\mathrm{d}x}{\sqrt{4-(x-1)^2}} = \left| \begin{matrix} t=x-1 & 2\to 1 \\ \mathrm{d}t=\mathrm{d}x & 1\to 0 \end{matrix} \right| = \int_0^1 \frac{\mathrm{d}t}{\sqrt{4-t^2}} =$

$= \left| \begin{matrix} t=2\sin s & s=\arcsin\frac{t}{2} \\ \mathrm{d}t=2\cos s\,\mathrm{d}s & 1\to\frac{\pi}{6}, 0\to 0 \end{matrix} \right| = \int_0^{\frac{\pi}{6}} \frac{2\cos s\,\mathrm{d}s}{\sqrt{4-4\sin^2 s}} = \int_0^{\frac{\pi}{6}} 1\,\mathrm{d}s = \frac{\pi}{6}$

$$\boxed{\frac{\pi}{6}}$$

50. $\int_0^{\frac{\pi}{2}} \frac{\sin\varphi}{6-5\cos\varphi+\cos^2\varphi}\mathrm{d}\varphi = \left| \begin{matrix} t=\cos\varphi & \frac{\pi}{2}\to 0 \\ \mathrm{d}t=-\sin\varphi\,\mathrm{d}\varphi & 0\to 1 \end{matrix} \right| = \int_1^0 \frac{-\mathrm{d}t}{6-5t+t^2} = \int_0^1 \frac{\mathrm{d}t}{(t-3)(t-2)} =$

$= \int_0^1 \frac{(t-2)-(t-3)}{(t-3)(t-2)}\mathrm{d}t = \int_0^1 \frac{\mathrm{d}t}{t-3} - \int_0^1 \frac{\mathrm{d}t}{t-2} = \left[\ln|t-3|\right]_0^1 - \left[\ln|t-2|\right]_0^1 = 2\ln 2 - \ln 3$

$$\boxed{\ln\frac{4}{3}}$$

51. $\int_0^1 xe^{-x}\mathrm{d}x = \left| \begin{matrix} x & e^{-x} \\ 1 & -e^{-x} \end{matrix} \right| = -\left[xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\mathrm{d}x = -\frac{1}{e} - \left[e^{-x}\right]_0^1 = 1-\frac{2}{e}$

$$\boxed{\frac{e-2}{e}}$$

52. $\int_1^e \ln x\,dx = \left| \begin{matrix} \ln x & 1 \\ \frac{1}{x} & x \end{matrix} \right| = \left[x\ln x\right]_1^e - \int_1^e 1\,\mathrm{d}x = e - [x]_1^e = e-(e-1) = 1$

$$\boxed{1}$$

53. $\int_0^{\frac{\pi}{2}} x\sin x\,dx = \left| \begin{matrix} x & \sin x \\ 1 & -\cos x \end{matrix} \right| = -\left[x\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\,\mathrm{d}x = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$

$$\boxed{1}$$

54. $\int_1^2 x\ln x\,dx = \left| \begin{matrix} \ln x & x \\ \frac{1}{x} & \frac{1}{2}x^2 \end{matrix} \right| = \frac{1}{2}\left[x^2\ln x\right]_1^2 - \frac{1}{2}\int_1^2 x\,dx = 2\ln 2 - \frac{1}{4}\left[x^2\right]_1^2 = 2\ln 2 - \frac{3}{4}$

$$\boxed{2\ln 2-\frac{3}{4}}$$

55. $\int_0^1 x^3e^{2x}\mathrm{d}x = \left| \begin{matrix} x^3 & e^{2x} \\ 3x^2 & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}x^3e^{2x}\right]_0^1 - \frac{3}{2}\int_0^1 x^2e^{2x}\mathrm{d}x = \frac{e^2}{2} - \frac{3}{2}\int_0^1 x^2e^{2x}\mathrm{d}x$

$\int_0^1 x^2e^{2x}\mathrm{d}x = \left| \begin{matrix} x^2 & e^{2x} \\ 2x & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}x^2e^{2x}\right]_0^1 - \int_0^1 xe^{2x}\mathrm{d}x = \frac{e^2}{2} - \int_0^1 xe^{2x}\mathrm{d}x (= \frac{e^2-1}{4})$

$\int_0^1 xe^{2x}\mathrm{d}x = \left| \begin{matrix} x & e^{2x} \\ 1 & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}xe^{2x}\right]_0^1 - \frac{1}{2}\int_0^1 e^{2x}\mathrm{d}x = \frac{e^2}{2} - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]_0^1 = \frac{e^2+1}{4}$

$$\boxed{\frac{e^2+3}{8}}$$

56. $\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x = \left| \begin{matrix} e^{2x} & \sin x \\ 2e^{2x} & -\cos x \end{matrix} \right| = \left[-e^{2x}\cos x\right]_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,dx = 1 + 2\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,dx$

$\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,\mathrm{d}x = \left| \begin{matrix} e^{2x} & \cos x \\ 2e^{2x} & \sin x \end{matrix} \right| = \left[e^{2x}\sin x\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x = e^{\pi} - 2\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x$

$\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x = 1 + 2e^{\pi} - 4\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x$

$$\boxed{\frac{2}{5}e^{\pi}+\frac{1}{5}}$$

57. $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} x\sin^{-2}x\,dx = \begin{vmatrix} x & \frac{1}{\sin^2 x} \\ 1 & \operatorname{tg} x \end{vmatrix} = [x\operatorname{tg} x]_{\frac{\pi}{3}}^{\frac{\pi}{4}} - \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \operatorname{tg} x\,\mathrm{d}x = \frac{\pi}{4} - \frac{\pi}{3}\sqrt{3} + [\ln|\cos x|]_{\frac{\pi}{3}}^{\frac{\pi}{4}} =$

$= ... + \ln\frac{\sqrt{2}}{2} - \ln\frac{1}{2}$

$$\boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{3}\pi + \frac{1}{2}\ln 2}$$

58. $\int_{-1}^{1} \arccos x\,\mathrm{d}x = \begin{vmatrix} \arccos x & 1 \\ -\frac{1}{\sqrt{1-x^2}} & x \end{vmatrix} = [x\arccos x]_{-1}^{1} + \int_{-1}^{1} \frac{x}{\sqrt{1-x^2}}\,dx = \pi - \left[\sqrt{1-x^2}\right]_{-1}^{1} = \pi$

$$\boxed{\pi}$$

59. $\int_0^{\sqrt{3}} x\operatorname{arctg} x\,\mathrm{d}x = \begin{vmatrix} \operatorname{arctg} x & x \\ \frac{1}{1+x^2} & \frac{x^2}{2} \end{vmatrix} = \left[\frac{1}{2}x^2\operatorname{arctg} x\right]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{x^2}{1+x^2}dx = \frac{3}{2}\operatorname{arctg}\sqrt{3} -$

$-\frac{1}{2}\int_0^{\sqrt{3}}\left(1 - \frac{1}{1+x^2}\right)dx = \frac{3}{2}\frac{\pi}{3} - \frac{1}{2}[x]_0^{\sqrt{3}} + \frac{1}{2}\int_0^{\sqrt{3}} \frac{1}{1+x^2}\mathrm{d}x = \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}[\operatorname{arctg} x]_0^{\sqrt{3}} =$

$= \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\pi}{3} = \frac{2}{3}\pi - \frac{\sqrt{3}}{2}$

$$\boxed{\frac{2}{3}\pi - \frac{\sqrt{3}}{2}}$$

60. $\int_0^{\ln 2} x\cosh x\,\mathrm{d}x = \begin{vmatrix} x & \cosh x \\ 1 & \sinh x \end{vmatrix} = [x\sinh x]_0^{\ln 2} - \int_0^{\ln 2} \sinh x\,\mathrm{d}x = \ln 2\sinh\ln 2 - [\cosh x]_0^{\ln 2} =$

$= \ln 2\frac{e^{\ln 2} - e^{-\ln 2}}{2} - \left(\frac{e^{\ln 2} + e^{-\ln 2}}{2} - \frac{e^0 + e^0}{2}\right) = \ln 2\frac{2-\frac{1}{2}}{2} - (\frac{2+\frac{1}{2}}{2} - 1) = \frac{1}{4}(3\ln 2 - 1)$

$$\boxed{\frac{1}{4}(3\ln 2 - 1)}$$

61. $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,\mathrm{d}x = \begin{vmatrix} \sin^{n-1} x & \sin x \\ (n-1)\sin^{n-2} x\cos x & -\cos x \end{vmatrix} = \left[-\sin^{n-1} x\cos x\right]_0^{\frac{\pi}{2}} +$

$+(n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x\cos^2 x\,dx = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x(1-\sin^2 x)\,\mathrm{d}x = (n-1)I_{n-2} - (n-1)I_n$

Teda $nI_n = (n-1)I_{n-2}, n > 1,\ I_n = \frac{n-1}{n}I_{n-2}, n \geq 2$

$I_0 = \int_0^{\frac{\pi}{2}} \sin^0 x\,dx = \int_0^{\frac{\pi}{2}} 1\,dx = \frac{\pi}{2},\ I_1 = \int_0^{\frac{\pi}{2}} \sin x\,\mathrm{d}x = [-\cos x]_0^{\frac{\pi}{2}} = 1$

$$\boxed{I_0 = \frac{\pi}{2},\ I_1 = 1,\ I_n = \frac{n-1}{n}I_{n-2}, n \geq 2}$$