$\int_1^2 (x^2-3x+2)\,\mathrm{d}x = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_1^2 = \frac{2}{3} - \frac{5}{6}$

$$\boxed{-\frac{1}{6}}$$

$\int_0^3 |1-3x|\mathrm{d}x = \int_0^{\frac{1}{3}} (1-3x)\,\mathrm{d}x + \int_{\frac{1}{3}}^3 (3x-1)\,\mathrm{d}x = \left[x - \frac{3}{2}x^2\right]_0^{\frac{1}{3}} + \left[\frac{3}{2}x^2 - x\right]_{\frac{1}{3}}^3 =$

$= \frac{1}{6} + \frac{21}{2} + \frac{1}{6} = \frac{65}{6}$

$$\boxed{\frac{65}{6}}$$

$\int_{-4}^{-2} \frac{1}{x}\mathrm{d}x = [\ln|x|]_{-4}^{-2} = \ln 2 - \ln 4$

$$\boxed{-\ln 2}$$

$\int_0^1 \frac{\mathrm{d}x}{1+x^2} = [\operatorname{arctg} x]_0^1 = \frac{\pi}{4}$

$$\boxed{\frac{\pi}{4}}$$

$\int_0^2 \frac{x}{x^2+3x+2}\mathrm{d}x = \frac{1}{2}\int_0^2 \frac{2x+3-3}{x^2+3x+2}\mathrm{d}x = \frac{1}{2}\int_0^2 \frac{2x+3}{x^2+3x+2} - \frac{3}{2}\int_0^2 \frac{\mathrm{d}x}{(x+1)(x+2)} = \frac{1}{2}\left[\ln|x^2+3x+2|\right]_0^2 -$

$-\frac{3}{2}\int_0^2 \frac{(x+2)-(x+1)}{(x+1)(x+2)}\mathrm{d}x = \frac{1}{2}\ln 6 - \frac{3}{2}[\ln|x+1|]_0^2 + \frac{3}{2}[\ln|x+2|]_0^2 = \frac{1}{2}\ln 6 - \frac{3}{2}\ln 3 + \frac{3}{2}(\ln 4 - \ln 2) =$

$= 2\ln 2 - \ln 3 = \ln\frac{4}{3}$

$$\boxed{\ln\frac{4}{3}}$$

$\int_0^\pi \cos x\,\mathrm{d}x = [\sin x]_0^\pi = 0$

$$\boxed{0}$$

$\int_0^\pi |\cos x|\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \cos x\,\mathrm{d}x - \int_{\frac{\pi}{2}}^\pi \cos x\,\mathrm{d}x = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^\pi = 2$

$$\boxed{2}$$

$\int_0^\pi \sin^3 x\,\mathrm{d}x = \int_0^\pi \sin x(1-\cos^2 x)\mathrm{d}x = \int_0^\pi \sin x\,\mathrm{d}x - \int_0^\pi \sin x\cos^2 x\,dx =$

$[-\cos x]_0^\pi - \int_0^\pi \sin x\cos^2 x\,\mathrm{d}x = 2 - \int_0^\pi \sin x\cos^2 x\,\mathrm{d}x = \begin{vmatrix} t = \cos x & \pi \to -1 \\ \mathrm{d}t = -\sin x\,\mathrm{d}x & 0 \to 1 \end{vmatrix}$

$= 2 + \int_1^{-1} t^2\,\mathrm{d}t = 2 + \left[\frac{1}{3}t^3\right]_1^{-1} = \frac{4}{3}$

$$\boxed{\frac{4}{3}}$$

$\int_3^7 \frac{x}{x^2-4}\,dx = \begin{vmatrix} t = x^2 & 7 \to 49 \\ \mathrm{d}t = 2x\,\mathrm{d}x & 3 \to 9 \end{vmatrix} = \frac{1}{2}\int_9^{49} \frac{\mathrm{d}t}{t-4} = \left[\frac{1}{2}\ln|t-4|\right]_9^{49} = \frac{1}{2}\ln 9$

$$\boxed{\ln 3}$$

$\int_0^{\frac{\pi}{2}} \cos x \cdot \sin^2 x\,dx = \begin{vmatrix} t = \sin x & \frac{\pi}{2} \to 1 \\ \mathrm{d}t = \cos x\,\mathrm{d}x & 0 \to 0 \end{vmatrix} = \int_0^1 t^2\mathrm{d}t = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$

$$\boxed{\frac{1}{3}}$$

$\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = \begin{vmatrix} t = \sqrt{x} & 1 \to 1, 0 \to 0 \\ \mathrm{d}t = \frac{\mathrm{d}x}{2\sqrt{x}} & \mathrm{d}x = 2\sqrt{x}\,\mathrm{d}t \end{vmatrix} = \int_0^1 \frac{2\sqrt{x}\sqrt{x}\,\mathrm{d}t}{1+\sqrt{x}} = 2\int_0^1 \frac{t^2}{t+1}\,\mathrm{d}t = 2\int_0^1 \frac{t^2-1+1}{t+1}\mathrm{d}t$

$= 2\int_0^1 (t-1)\,\mathrm{d}t + 2\int_0^1 \frac{1}{1+t}\mathrm{d}t = 2\left[\frac{1}{2}t^2 - t + \ln|1+t|\right]_0^1 = 2\ln 2 - 1$

$$\boxed{\ln 4 - 1}$$

$\int_{-1}^1 \frac{\mathrm{d}x}{(1+x^2)^2}$

$\pi = [\operatorname{arctg} x]_{-1}^1 = \int_{-1}^1 \frac{\mathrm{d}x}{1+x^2} = \begin{vmatrix} \frac{1}{1+x^2} & 1 \\ -\frac{2x}{(1+x^2)^2} & x \end{vmatrix} = \left[\frac{x}{1+x^2}\right]_{-1}^1 + 2\int_{-1}^1 \frac{x^2}{(1+x^2)^2} =$

$1 + 2\int_{-1}^1 \frac{x^2+1-1}{(x^2+1)^2}\mathrm{d}x = 1 + 2\int_{-1}^1 \frac{\mathrm{d}x}{1+x^2} - 2\int_{-1}^1 \frac{\mathrm{d}x}{(1+x^2)^2} = 1 + 2\pi - 2\int_{-1}^1 \frac{\mathrm{d}x}{(1+x^2)^2}$

$\int_{-1}^{1} \frac{\mathrm{d}x}{(1+x^2)^2} = \frac{1}{2}(1+\pi)$

$$\boxed{\frac{\pi+1}{2}}$$

$\int_0^{\sqrt{2}} \sqrt{4-x^2}\, dx = \left| \begin{matrix} x = 2\sin t & t = \arcsin\frac{t}{2} \\ \mathrm{d}x = 2\cos t\,\mathrm{d}t & \sqrt{2} \to \frac{\pi}{4}, 0 \to 0 \end{matrix} \right| = \int_0^{\frac{\pi}{4}} 2\sqrt{4-4\sin^2 t}\cos t\,\mathrm{d}t =$

$= 4\int_0^{\frac{\pi}{4}} \cos^2 t\,\mathrm{d}t = 4\int_0^{\frac{\pi}{4}} \frac{\cos 2t+1}{2}\mathrm{d}t = 4\left[\frac{1}{4}\sin 2t + \frac{t}{2}\right]_0^{\frac{\pi}{4}} = 1+\frac{\pi}{2}$

$$\boxed{1+\frac{\pi}{2}}$$

$\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\, dx = \left| \begin{matrix} t = \sqrt{e^x-1} & \ln 5 \to 2 \\ \mathrm{d}t = \frac{1}{2}\frac{e^x}{\sqrt{e^x-1}}\mathrm{d}x & 0 \to 0 \end{matrix} \right| = \int_0^2 \frac{\sqrt{e^x-1}}{e^x+3} 2\sqrt{e^x-1}\,\mathrm{d}t = 2\int_0^2 \frac{e^x-1}{e^x-1+4}\mathrm{d}t =$

$= 2\int_0^2 \frac{t^2}{t^2+4} = 2\int_0^2 \frac{t^2+4-4}{t^2+4}\mathrm{d}t = 2\int_0^2 1\, dt - 8\int_0^2 \frac{\mathrm{d}t}{t^2+4} = \left| \begin{matrix} t = 2s & s = \frac{t}{2} \\ \mathrm{d}t = 2\mathrm{d}s & 2 \to 1, 0 \to 0 \end{matrix} \right| =$

$= 4 - 16\int_0^1 \frac{\mathrm{d}s}{4+4s^2} = 4 - 4\int_0^1 \frac{\mathrm{d}s}{1+s^2} = 4 - 4\left[\operatorname{arctg} s\right]_0^1 = 4-\pi$

$$\boxed{4-\pi}$$

$\int_1^2 \frac{\mathrm{d}x}{\sqrt{3+2x-x^2}} = \int_1^2 \frac{\mathrm{d}x}{\sqrt{4-(x-1)^2}} = \left| \begin{matrix} t = x-1 & 2 \to 1 \\ \mathrm{d}t = \mathrm{d}x & 1 \to 0 \end{matrix} \right| = \int_0^1 \frac{\mathrm{d}t}{\sqrt{4-t^2}} =$

$= \left| \begin{matrix} t = 2\sin s & s = \arcsin\frac{t}{2} \\ \mathrm{d}t = 2\cos s\,\mathrm{d}s & 1 \to \frac{\pi}{6}, 0 \to 0 \end{matrix} \right| = \int_0^{\frac{\pi}{6}} \frac{2\cos s\,\mathrm{d}s}{\sqrt{4-4\sin^2 s}} = \int_0^{\frac{\pi}{6}} 1\,\mathrm{d}s = \frac{\pi}{6}$

$$\boxed{\frac{\pi}{6}}$$

$\int_0^{\frac{\pi}{2}} \frac{\sin\varphi}{6-5\cos\varphi+\cos^2\varphi}\mathrm{d}\varphi = \left| \begin{matrix} t = \cos\varphi & \frac{\pi}{2} \to 0 \\ \mathrm{d}t = -\sin\varphi\,\mathrm{d}\varphi & 0 \to 1 \end{matrix} \right| = \int_1^0 \frac{-\mathrm{d}t}{6-5t+t^2} = \int_0^1 \frac{\mathrm{d}t}{(t-3)(t-2)} =$

$= \int_0^1 \frac{(t-2)-(t-3)}{(t-3)(t-2)}\mathrm{d}t = \int_0^1 \frac{\mathrm{d}t}{t-3} - \int_0^1 \frac{\mathrm{d}t}{t-2} = \left[\ln|t-3|\right]_0^1 - \left[\ln|t-2|\right]_0^1 = 2\ln 2 - \ln 3$

$$\boxed{\ln\frac{4}{3}}$$

$\int_0^1 xe^{-x}\mathrm{d}x = \left| \begin{matrix} x & e^{-x} \\ 1 & -e^{-x} \end{matrix} \right| = -\left[xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\mathrm{d}x = -\frac{1}{e} - \left[e^{-x}\right]_0^1 = 1 - \frac{2}{e}$

$$\boxed{\frac{e-2}{e}}$$

$\int_1^e \ln x\, dx = \left| \begin{matrix} \ln x & 1 \\ \frac{1}{x} & x \end{matrix} \right| = \left[x\ln x\right]_1^e - \int_1^e 1\,\mathrm{d}x = e - [x]_1^e = e-(e-1) = 1$

$$\boxed{1}$$

$\int_0^{\frac{\pi}{2}} x\sin x\, dx = \left| \begin{matrix} x & \sin x \\ 1 & -\cos x \end{matrix} \right| = -\left[x\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\,\mathrm{d}x = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$

$$\boxed{1}$$

$\int_1^2 x\ln x\, dx = \left| \begin{matrix} \ln x & x \\ \frac{1}{x} & \frac{1}{2}x^2 \end{matrix} \right| = \frac{1}{2}\left[x^2\ln x\right]_1^2 - \frac{1}{2}\int_1^2 x\, dx = 2\ln 2 - \frac{1}{4}\left[x^2\right]_1^2 = 2\ln 2 - \frac{3}{4}$

$$\boxed{2\ln 2 - \frac{3}{4}}$$

$\int_0^1 x^3e^{2x}\mathrm{d}x = \left| \begin{matrix} x^3 & e^{2x} \\ 3x^2 & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}x^3e^{2x}\right]_0^1 - \frac{3}{2}\int_0^1 x^2e^{2x}\mathrm{d}x = \frac{e^2}{2} - \frac{3}{2}\int_0^1 x^2e^{2x}\mathrm{d}x$

$\int_0^1 x^2e^{2x}\mathrm{d}x = \left| \begin{matrix} x^2 & e^{2x} \\ 2x & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}x^2e^{2x}\right]_0^1 - \int_0^1 xe^{2x}\mathrm{d}x = \frac{e^2}{2} - \int_0^1 xe^{2x}\mathrm{d}x \left(= \frac{e^2-1}{4}\right)$

$\int_0^1 xe^{2x}\mathrm{d}x = \left| \begin{matrix} x & e^{2x} \\ 1 & \frac{1}{2}e^{2x} \end{matrix} \right| = \left[\frac{1}{2}xe^{2x}\right]_0^1 - \frac{1}{2}\int_0^1 e^{2x}\mathrm{d}x = \frac{e^2}{2} - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]_0^1 = \frac{e^2+1}{4}$

$$\boxed{\frac{e^2+3}{8}}$$

$\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x= \begin{vmatrix} e^{2x} & \sin x \\ 2e^{2x} & -\cos x \end{vmatrix} = \left[-e^{2x}\cos x\right]_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,dx = 1 + 2\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,dx$

$\int_0^{\frac{\pi}{2}} e^{2x}\cos x\,\mathrm{d}x = \begin{vmatrix} e^{2x} & \cos x \\ 2e^{2x} & \sin x \end{vmatrix} = \left[e^{2x}\sin x\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x = e^{\pi} - 2\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x$

$\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x = 1 + 2e^{\pi} - 4\int_0^{\frac{\pi}{2}} e^{2x}\sin x\,\mathrm{d}x$

$$\boxed{\frac{2}{5}e^{\pi} + \frac{1}{5}}$$

$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} x\sin^{-2}x\,dx= \begin{vmatrix} x & \frac{1}{\sin^2 x} \\ 1 & \mathrm{tg}\,x \end{vmatrix} = [x\mathrm{tg}\,x]_{\frac{\pi}{3}}^{\frac{\pi}{4}} - \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \mathrm{tg}\,x\,\mathrm{d}x = \frac{\pi}{4} - \frac{\pi}{3}\sqrt{3} + [\ln|\cos x|]_{\frac{\pi}{3}}^{\frac{\pi}{4}} =$

$= ... + \ln\frac{\sqrt{2}}{2} - \ln\frac{1}{2}$

$$\boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{3}\pi + \frac{1}{2}\ln 2}$$

$\int_{-1}^{1} \arccos x\,\mathrm{d}x= \begin{vmatrix} \arccos x & 1 \\ -\frac{1}{\sqrt{1-x^2}} & x \end{vmatrix} = [x\arccos x]_{-1}^{1} + \int_{-1}^{1}\frac{x}{\sqrt{1-x^2}}\,dx = \pi - \left[\sqrt{1-x^2}\right]_{-1}^{1} = \pi$

$$\boxed{\pi}$$

$\int_0^{\sqrt{3}} x\mathrm{arctg}\,x\,\mathrm{d}x = \begin{vmatrix} \mathrm{arctg}\,x & x \\ \frac{1}{1+x^2} & \frac{x^2}{2} \end{vmatrix} = \left[\frac{1}{2}x^2\mathrm{arctg}\,x\right]_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}}\frac{x^2}{1+x^2}dx = \frac{3}{2}\mathrm{arctg}\,\sqrt{3} -$

$-\frac{1}{2}\int_0^{\sqrt{3}}\left(1 - \frac{1}{1+x^2}\right)dx = \frac{3}{2}\frac{\pi}{3} - \frac{1}{2}[x]_0^{\sqrt{3}} + \frac{1}{2}\int_0^{\sqrt{3}}\frac{1}{1+x^2}\mathrm{d}x = \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}[\mathrm{arctg}\,x]_0^{\sqrt{3}} =$

$= \frac{\pi}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\pi}{3} = \frac{2}{3}\pi - \frac{\sqrt{3}}{2}$

$$\boxed{\frac{2}{3}\pi - \frac{\sqrt{3}}{2}}$$

$\int_0^{\ln 2} x\cosh x\,\mathrm{d}x= \begin{vmatrix} x & \cosh x \\ 1 & \sinh x \end{vmatrix} = [x\sinh x]_0^{\ln 2} - \int_0^{\ln 2}\sinh x\,\mathrm{d}x = \ln 2\sinh\ln 2 - [\cosh x]_0^{\ln 2} =$

$= \ln 2\frac{e^{\ln 2} - e^{-\ln 2}}{2} - \left(\frac{e^{\ln 2} + e^{-\ln 2}}{2} - \frac{e^0 + e^0}{2}\right) = \ln 2\frac{2-\frac{1}{2}}{2} - \left(\frac{2+\frac{1}{2}}{2} - 1\right) = \frac{1}{4}(3\ln 2 - 1)$

$$\boxed{\frac{1}{4}(3\ln 2 - 1)}$$

$I_n = \int_0^{\frac{\pi}{2}}\sin^n x\,\mathrm{d}x= \begin{vmatrix} \sin^{n-1}x & \sin x \\ (n-1)\sin^{n-2}x\cos x & -\cos x \end{vmatrix} = \left[-\sin^{n-1}x\cos x\right]_0^{\frac{\pi}{2}} +$

$+(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\cos^2 x\,dx = (n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x(1-\sin^2 x)\,\mathrm{d}x = (n-1)I_{n-2} - (n-1)I_n$

Teda $nI_n = (n-1)I_{n-2}, n > 1,\ I_n = \frac{n-1}{n}I_{n-2}, n \geq 2$

$I_0 = \int_0^{\frac{\pi}{2}}\sin^0 x\,dx = \int_0^{\frac{\pi}{2}} 1\,dx = \frac{\pi}{2},\ I_1 = \int_0^{\frac{\pi}{2}}\sin x\,\mathrm{d}x = [-\cos x]_0^{\frac{\pi}{2}} = 1$

$$\boxed{I_0 = \frac{\pi}{2},\ I_1 = 1,\ I_n = \frac{n-1}{n}I_{n-2}, n \geq 2}$$