V nasledujúcich úlohách určte dĺžku kriviek:

$y = \frac{1}{3}(x^2+2)^{\frac{3}{2}},\ x \in \langle 0,3\rangle$

$y' = \frac{1}{3}\frac{3}{2}(x^2+2)^{\frac{1}{2}} \cdot 2x = x(x^2+2)^{\frac{1}{2}}$

$\int_0^3 \sqrt{1+x^2(x^2+2)}\,\mathrm{d}x = \int_0^3 \sqrt{x^4+2x^2+1}\,\mathrm{d}x = \int_0^3 (1+x^2)\,dx = \left[x+\frac{x^3}{3}\right]_0^3 = 12$

$$\boxed{12}$$

$y = \frac{x^2}{4}, x \in \langle 0, 2\sqrt{2}\rangle$

$y' = \frac{x}{2},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{x^2}{4}} = \frac{1}{2}\sqrt{4+x^2}$

$\int_0^{2\sqrt{2}} \frac{1}{2}\sqrt{4+x^2}\,\mathrm{d}x = \frac{1}{2}\int_0^{2\sqrt{2}} \sqrt{4+x^2}\mathrm{d}x = \left| \begin{array}{cc} x = 2\,\mathrm{tg}\,t & 2\sqrt{2} \mapsto \mathrm{arctg}\,\sqrt{2} \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{\cos^2 t} & 0 \mapsto 0 \end{array} \right| =$

$= \frac{1}{2}\int_0^{\mathrm{arctg}\,\sqrt{2}} \sqrt{4+4\,\mathrm{tg}^2\,t}\,\frac{2\,\mathrm{d}t}{\cos^2 t} = 2\int_0^{\mathrm{arctg}\,\sqrt{2}} \frac{\mathrm{d}t}{\cos^3 t} = \left| \begin{array}{cc} s = \sin t & \mathrm{arctg}\,\sqrt{2} \mapsto \sqrt{\frac{2}{3}} \\ \mathrm{d}s = \cos t\,\mathrm{d}t & 0 \mapsto 0 \end{array} \right| = 2\int_0^{\sqrt{\frac{2}{3}}} \frac{\mathrm{d}s}{\cos^4 t} =$

$= 2\int_0^{\sqrt{\frac{2}{3}}} \frac{\mathrm{d}s}{(1-s^2)^2} = 2\int_0^{\sqrt{\frac{2}{3}}} \frac{\mathrm{d}s}{(1-s)^2(1+s)^2} = \frac{1}{2}\int_0^{\sqrt{\frac{2}{3}}}(-\frac{1}{s-1}+\frac{1}{(s-1)^2}+\frac{1}{s+1}+\frac{1}{(s+1)^2}) = \frac{1}{2}\left[-\ln|s-1|\right]_0^{\sqrt{\frac{2}{3}}} +$

$+\frac{1}{2}\left[-\frac{1}{s-1}+\ln|s+1|-\frac{1}{s+1}\right]_0^{\sqrt{\frac{2}{3}}} = \frac{1}{2}(-\ln(1-\sqrt{\frac{2}{3}}) - \frac{1}{\sqrt{\frac{2}{3}}-1} - 1 + \ln(1+\sqrt{\frac{2}{3}}) - \frac{1}{\sqrt{\frac{2}{3}}+1}) =$

$= \ln(\sqrt{3}+\sqrt{2})+\sqrt{6}$

Rozklad na parciálne zlomky:

$\frac{1}{(t-1)^2(t+1)^2} = \frac{A}{t-1}+\frac{B}{(t-1)^2}+\frac{C}{t+1}+\frac{D}{(t+1)^2} = \frac{A(t-1)(t+1)^2+B(t+1)^2+C(t+1)(t-1)^2+D(t-1)^2}{(t-1)^2(t+1)^2} \Rightarrow$

$1 = A(t-1)(t+1)^2+B(t+1)^2+C(t+1)(t-1)^2+D(t-1)^2$

$t=1 \Rightarrow 1 = 4B, B = \frac{1}{4},\ t=-1 \Rightarrow 1 = 4D, D = \frac{1}{4}$

Po zderivovaní obidvoch strán:

$0 = A(t+1)^2+2A(t-1)(t+1)+2B(t+1)+C(t-1)^2+2C(t-1)(t+1)+2D(t-1)$

$t=1 \Rightarrow 0 = 4A+4B = 4A+1, A = -\frac{1}{4},\ t=-1 \Rightarrow 0 = 4C-4D = 4C-1, C = \frac{1}{4}$

$$\boxed{\ln(\sqrt{3}+\sqrt{2})+\sqrt{6}}$$

$y = \ln\sin x,\ x \in \langle \frac{\pi}{3}, \frac{\pi}{2}\rangle$

$y' = \frac{1}{\sin x}\cos x = \frac{\cos x}{\sin x}, \sqrt{1+(y')^2} = \sqrt{1+\frac{\cos^2 x}{\sin^2 x}} = \frac{1}{\sin x}$

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\mathrm{d}x}{\sin x} = \left| \begin{array}{cc} t = \cos x & \frac{\pi}{2} \mapsto 0 \\ \mathrm{d}t = -\sin x\,\mathrm{d}x & \frac{\pi}{3} \mapsto \frac{1}{2} \end{array} \right| = \int_{\frac{1}{2}}^0 -\frac{\mathrm{d}t}{\sin^2 x} = \int_{\frac{1}{2}}^0 -\frac{\mathrm{d}t}{1-t^2} = \int_0^{\frac{1}{2}} \frac{\mathrm{d}t}{1-t^2} = \frac{1}{2}\int_0^{\frac{1}{2}} \frac{(1-t)+(1+t)}{(1-t)(1+t)}\mathrm{d}t =$

$= \frac{1}{2}\int_0^{\frac{1}{2}} \frac{\mathrm{d}t}{1+t} + \frac{1}{2}\int_0^{\frac{1}{2}} \frac{\mathrm{d}t}{1-t} = \frac{1}{2}\left[\ln|1+t|\right]_0^{\frac{1}{2}} - \frac{1}{2}\left[\ln|1-t|\right]_0^{\frac{1}{2}} = \frac{1}{2}\ln\frac{3}{2} - \frac{1}{2}\ln\frac{1}{2} =$

$= \frac{1}{2}(\ln 3 - \ln 2 + \ln 2) = \frac{1}{2}\ln 3 = \ln\sqrt{3}$

$$\boxed{\ln\sqrt{3}}$$

$y = \frac{2}{3}x\sqrt{x},\ x \in \langle 0,1\rangle$

$y' = (\frac{2}{3}x^{\frac{3}{2}})' = \frac{2}{3}\frac{3}{2}x^{\frac{1}{2}} = \sqrt{x},\ \sqrt{1+(y')^2} = \sqrt{1+x}$

$\int_0^1 \sqrt{1+x}\,\mathrm{d}x = \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}(2\sqrt{2}-1)$

$$\boxed{\frac{2}{3}(2\sqrt{2}-1)}$$

$y = \frac{x^2}{4} - \frac{1}{2}\ln x, x \in \langle 1,2\rangle$

$y' = \frac{x}{2} - \frac{1}{2x} = \frac{1}{2}(x-\frac{1}{x}),\ \sqrt{1+(y')^2} = \sqrt{1+\frac{1}{4}(x-\frac{1}{x})^2} = \frac{1}{2}(x+\frac{1}{x})$

$\frac{1}{2}\int_1^2 (x+\frac{1}{x})\mathrm{d}x = \frac{1}{2}\left[\frac{x^2}{2} - \ln|x|\right]_1^2 = \frac{1}{2}(2-\frac{1}{2}+\ln 2 - \ln 1) = \frac{3}{4}+\frac{1}{2}\ln 2 = \frac{3}{4}+\ln\sqrt{2}$

$$\boxed{\frac{3}{4}+\ln\sqrt{2}}$$

$y = \ln \frac{e^x+1}{e^x-1},\ y \in \langle \ln 2, \ln 5 \rangle$

$$y' = \frac{e^x-1}{e^x+1} \frac{e^x(e^x-1)-e^x(e^x+1)}{(e^x-1)^2} = \frac{-2e^x}{e^{2x}-1},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{4e^{2x}}{(e^{2x}-1)^2}} = \sqrt{\frac{e^{4x}-2e^{2x}+1+4e^{2x}}{(e^{2x}-1)^2}} =$$

$$= \sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}} = \frac{e^{2x}+1}{e^{2x}-1}$$

$$\int_{\ln 2}^{\ln 5} \frac{e^{2x}-1+2}{e^{2x}-1}\mathrm{d}x = \int_{\ln 2}^{\ln 5} 1\,\mathrm{d}x + 2\int_{\ln 2}^{\ln 5} \frac{\mathrm{d}x}{e^{2x}-1} = \left| \begin{array}{cc} t = e^x & \ln 5 \mapsto 5 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto 2 \end{array} \right| = [x]_{\ln 2}^{\ln 5} + 2\int_2^5 \frac{\mathrm{d}t}{t(t^2-1)} =$$

$$= \ln\frac{5}{2} + 2\int_2^5 \frac{t^2-(t^2-1)}{t(t^2-1)}\mathrm{d}t = \ln\frac{5}{2} + \int_2^5 \frac{2t}{t^2-1}\mathrm{d}t - 2\int_2^5 \frac{\mathrm{d}t}{t} = \ln\frac{5}{2} + \left[\ln|t^2-1|\right]_2^5 - 2\left[\ln|t|\right]_2^5 =$$

$$= -\ln\frac{5}{2} + \ln 24 - \ln 3 = \ln 8 - \ln\frac{5}{2} = \ln\frac{16}{5}$$

$$\boxed{\ln\frac{16}{5}}$$

$y = \ln \frac{e^x+1}{e^x-1},\ y \in \langle a, b \rangle$

$$y' = \frac{e^x-1}{e^x+1} \frac{e^x(e^x-1)-e^x(e^x+1)}{(e^x-1)^2} = \frac{-2e^x}{e^{2x}-1},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{4e^{2x}}{(e^{2x}-1)^2}} = \sqrt{\frac{e^{4x}-2e^{2x}+1+4e^{2x}}{(e^{2x}-1)^2}} =$$

$$= \sqrt{\frac{(e^{2x}+1)^2}{(e^{2x}-1)^2}} = \frac{e^{2x}+1}{e^{2x}-1}$$

$$\int_a^b \frac{e^{2x}+1}{e^{2x}-1}\mathrm{d}x = \int_a^b \frac{e^{2x}-1+2}{e^{2x}-1}\mathrm{d}x = \int_a^b (1+\frac{2}{e^{2x}-1})\mathrm{d}x = [x]_a^b + 2\int_a^b \frac{\mathrm{d}x}{e^{2x}-1} = \left| \begin{array}{cc} t = e^x & b \mapsto e^b \\ \mathrm{d}t = e^x\,\mathrm{d}x & a \mapsto e^a \end{array} \right| =$$

$$= (b-a) + 2\int_{e^a}^{e^b} \frac{\mathrm{d}t}{t(t^2-1)} = (b-a) + \int_{e^a}^{e^b} \frac{2t}{t^2-1}\mathrm{d}t - 2\int_{e^a}^{e^b} \frac{\mathrm{d}t}{t} = (a-b) + \left[\ln|t^2-1|\right]_{e^a}^{e^b} =$$

$$= (a-b) + \ln\frac{e^{2b}-1}{e^{2a}-1}$$

$$\boxed{(a-b) + \ln\frac{e^{2b}-1}{e^{2a}-1}}$$

$y = \cosh x,\ x \in \langle 0, 1 \rangle$

$$y' = \sinh x, \sqrt{1+(y')^2} = \sqrt{1+\sinh^2 x} = \sqrt{\cosh^2 x} = \cosh x$$

$$\int_0^1 \cosh x\,\mathrm{d}x = [\sinh x]_0^1 = \sinh 1 = \frac{1}{2}(e - \frac{1}{e})$$

$$\boxed{\sinh 1 = \frac{1}{2}(e - \frac{1}{e})}$$

$y = \ln x,\ x \in \langle \sqrt{3}, \sqrt{8} \rangle$

$$y' = \frac{1}{x},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{1}{x^2}} = \frac{\sqrt{1+x^2}}{x}$$

$$\int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{1+x^2}}{x}\mathrm{d}x = \left| \begin{array}{cc} t = \sqrt{1+x^2} & \sqrt{8} \mapsto 3 \\ \mathrm{d}t = \frac{x}{\sqrt{1+x^2}}\mathrm{d}x & \sqrt{3} \mapsto 2 \end{array} \right| = \int_2^3 \frac{1+x^2}{x^2}\mathrm{d}t = \int_2^3 \frac{t^2}{t^2-1}\mathrm{d}t = \int_2^3 \frac{t^2-1+1}{t^2-1} =$$

$$= \int_2^3 1\,\mathrm{d}t + \int_2^3 \frac{\mathrm{d}t}{t^2-1} = [x]_2^3 + \left[\frac{1}{2}\ln\frac{|t-1|}{|t+1|}\right]_2^3 = 1 + \frac{1}{2}(\ln\frac{2}{4} - \ln\frac{1}{3}) = 1 + \frac{1}{2}\ln\frac{3}{2}$$

$$\boxed{1 + \frac{1}{2}\ln\frac{3}{2}}$$

$y = \arcsin x + \sqrt{1-x^2},\ x \in \langle 0, 1 \rangle$

$$y' = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \frac{1-x}{\sqrt{1-x^2}} = \frac{\sqrt{1-x}\sqrt{1-x}}{\sqrt{1-x}\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{1-x}{1+x}} = \sqrt{\frac{2}{1+x}}$$

$$\int_0^1 \sqrt{\frac{2}{1+x}}\mathrm{d}x = \left| \begin{array}{ccc} t = \sqrt{\frac{2}{1+x}} & x = \frac{2-t^2}{t^2} & 0 \mapsto \sqrt{2}, 1 \mapsto 1 \\ \mathrm{d}t = \frac{1}{2}\sqrt{\frac{1+x}{2}}\frac{-2}{(1+x)^2}\mathrm{d}x & \mathrm{d}x = -\frac{4}{t^3}\mathrm{d}t & 1+x = \frac{2}{t^2} \end{array} \right| = \int_{\sqrt{2}}^1 \frac{-4}{t^2}\mathrm{d}t =$$

$$= \int_1^{\sqrt{2}} \frac{4}{t^2}\mathrm{d}t = \left[\frac{-4}{t}\right]_1^{\sqrt{2}} = 4 - \frac{4}{\sqrt{2}} = 4 - 2\sqrt{2} = 2(2-\sqrt{2})$$

$$\boxed{2(2-\sqrt{2})}$$

$y = 2\sqrt{x},\ x \in \langle 1, 2 \rangle$

$$y' = 2\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}},\ \sqrt{1+(y')^2} = \sqrt{1+\frac{1}{x}} = \sqrt{\frac{x+1}{x}}$$

$$\int_1^2 \sqrt{\frac{1+x}{x}}\mathrm{d}x = \left| \begin{matrix} t=\sqrt{\frac{1+x}{x}} & x=\frac{1}{t^2-1} & 2\mapsto\sqrt{\frac{3}{2}} \\ \mathrm{d}t=\frac{1}{2}\sqrt{\frac{x}{1+x}}\frac{-\mathrm{d}x}{x^2} & \mathrm{d}x=\frac{-2t}{(t^2-1)^2}\mathrm{d}t & 1\mapsto\sqrt{2} \end{matrix} \right| = \int_{\sqrt{2}}^{\sqrt{\frac{3}{2}}} -2\frac{t^2}{(t^2-1)^2}\mathrm{d}t =$$

$$= 2\int_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} \frac{t^2}{(t^2-1)^2}\mathrm{d}t = 2\int_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} \frac{t^2}{(t-1)^2(t+1)^2}\mathrm{d}t = \frac{1}{2}\int_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} \left(\frac{1}{t-1}+\frac{1}{(t-1)^2}-\frac{1}{t+1}+\frac{1}{(t+1)^2}\right)\mathrm{d}t =$$

$$= \frac{1}{2}\left[\ln|t-1|\right]_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} - \frac{1}{2}\left[\frac{1}{t-1}\right]_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} - \frac{1}{2}\left[\ln|t+1|\right]_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} - \frac{1}{2}\left[\frac{1}{x+1}\right]_{\sqrt{\frac{3}{2}}}^{\sqrt{2}} =$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{2}-1}{\sqrt{\frac{3}{2}}-1}\right| - \frac{1}{2}\left(\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{\frac{3}{2}}-1}\right) - \frac{1}{2}\ln\left|\frac{\sqrt{2}+1}{\sqrt{\frac{3}{2}}+1}\right| - \frac{1}{2}\left(\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{\frac{3}{2}}+1}\right) = \frac{1}{2}\ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| + \frac{1}{2}\ln\left|\frac{\sqrt{\frac{3}{2}}+1}{\sqrt{\frac{3}{2}}-1}\right| -$$

$$- \frac{1}{2}\frac{\sqrt{2}+1+\sqrt{2}-1}{(\sqrt{2}-1)(\sqrt{2}+1)} + \frac{1}{2}\frac{\sqrt{\frac{3}{2}}+1+\sqrt{\frac{3}{2}}-1}{(\sqrt{\frac{3}{2}}+1)(\sqrt{\frac{3}{2}}-1)} = \frac{1}{2}\ln\left|\frac{(\sqrt{2}-1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}\right| + \frac{1}{2}\ln\left|\frac{(\sqrt{\frac{3}{2}}+1)^2}{(\sqrt{\frac{3}{2}}+1)(\sqrt{\frac{3}{2}}-1)}\right| - \sqrt{2} + 2\sqrt{\frac{3}{2}} =$$

$$= \ln(\sqrt{2}-1) + \ln\left(\sqrt{\frac{3}{2}}-1\right) - \frac{1}{2}\ln\frac{1}{2} - \sqrt{2} + \sqrt{6} = \ln(\sqrt{2}-1) + \ln\left(\sqrt{\frac{3}{2}}-1\right) + \frac{1}{2}\ln 2 - \sqrt{2} + \sqrt{6}$$

Rozklad na parciálne zlomky:

$$\frac{t^2}{(t-1)^2(t+1)^2} = \frac{A}{t-1} + \frac{B}{(t-1)^2} + \frac{C}{t+1} + \frac{D}{(t+1)^2} = \frac{A(t-1)(t+1)^2+B(t+1)^2+C(t+1)(t-1)^2+D(t-1)^2}{(t-1)^2(t+1)^2} =$$

$$= \frac{A(t^3+t^2-t-1)+B(t^2+2t+1)+C(t^3-t^2-t+1)+D(t^2-2t+1)}{(t-1)^2(t+1)^2} =$$

$$= \frac{t^3(A+C)+t^2(A+B-C+D)+t(-A+2B-C-2D)+(-A+B+C+D)}{(t-1)^2(t+1)^2},\ A=-C,\ B=D,\ B=-C,\ A+B=\frac{1}{2}$$

$$A=B=D=\frac{1}{4}, C=-\frac{1}{4}$$

Iný spôsob rozkladu na parciálne zlomky:

$t^2 = A(t-1)(t+1)^2 + B(t+1)^2 + C(t+1)(t-1)^2 + D(t-1)^2$

$t=1 \Rightarrow 1=4B, B=\frac{1}{4},\quad t=-1 \Rightarrow 1=4D, D=\frac{1}{4}$

Po zderivovaní oboch strán dostávame:

$2t = A(t+1)^2 + 2A(t-1)(t+1) + 2B(t+1) + C(t-1)^2 + 2C(t+1)(t-1) + 2D(t-1)$

$t=1 \Rightarrow 2=4A+4B=4A+1, A=\frac{1}{4},\quad t=-1 \Rightarrow -2=4C-4D=4C-1, C=-\frac{1}{4}$

$$\boxed{\ln(\sqrt{2}-1) + \ln\left(\sqrt{\frac{3}{2}}-1\right) + \frac{1}{2}\ln 2 - \sqrt{2} + \sqrt{6}}$$

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, $a>0$

$y^{\frac{2}{3}} = a^{\frac{2}{3}} - x^{\frac{2}{3}}$, $y = (a^{\frac{2}{3}} - x^{\frac{2}{3}})^{\frac{3}{2}}$, keďže odmocňujeme, tak $a^{\frac{2}{3}} - x^{\frac{2}{3}} \geq 0$, t.j. $x^2 \leq a^2$, $|x| \leq |a|$. Podobne aj $|y| \leq |a|$. Zo symetrie $(x,y) \in K \Rightarrow (-x,y),(x,-y),(-x,-y) \in K$ vyplýva, že sa stačí obmedziť iba na prvý kvadrant $0 \leq x \leq a, 0 \leq y \leq a$ a celková dĺžka bude štvornásobkom.

$y' = \frac{3}{2}(a^{\frac{2}{3}} - x^{\frac{2}{3}})^{\frac{1}{2}} \cdot \frac{-2}{3}x^{-\frac{1}{3}} = -\frac{\sqrt{a^{\frac{2}{3}}-x^{\frac{2}{3}}}}{x^{\frac{1}{3}}}$, $\sqrt{1+(y')^2} = \sqrt{1+\frac{a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x^{\frac{2}{3}}}} = \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$l = 4\int_0^a \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}\mathrm{d}x = 4a^{\frac{1}{3}}\int_0^a x^{-\frac{1}{3}}\mathrm{d}x = 4a^{\frac{1}{3}}\left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right]_0^a = 6a^{\frac{1}{3}}a^{\frac{2}{3}} = 6a$

$$\boxed{6a}$$

$y^2 = 4x^3$, $y>0$, $x \in \langle 0,2\rangle$

$y = 2x^{\frac{3}{2}}$, $y' = 2\frac{3}{2}x^{\frac{1}{2}} = 3\sqrt{x}$, $\sqrt{1+(y')^2} = \sqrt{1+9x}$

$\int_0^2 \sqrt{1+9x}\,\mathrm{d}x = \left[\frac{2}{3}(1+9x)^{\frac{3}{2}} \cdot \frac{1}{9}\right]_0^2 = \frac{2}{27}\left[(1+9x)^{\frac{3}{2}}\right]_0^2 = \frac{2}{27}(19\sqrt{19}-1)$

$$\boxed{\frac{2}{27}(19\sqrt{19}-1)}$$

$y = 2x - x^2$, $x \in \langle 0,1\rangle$

$y' = 2-2x = 2(1-x)$, $\sqrt{1+(y')^2} = \sqrt{1+4(1-x)^2}$

$\int_0^1 \sqrt{1+4(1-x)^2}\,\mathrm{d}x = \left| \begin{matrix} t=1-x & 1\mapsto 0 \\ \mathrm{d}t=-\mathrm{d}x & 0\mapsto 1 \end{matrix} \right| = \int_1^0 -\sqrt{1+4t^2}\,\mathrm{d}t = \int_0^1 \sqrt{1+4t^2}\,\mathrm{d}t =$

$= \left| \begin{matrix} t=\frac{1}{2}\operatorname{tg} s & 1\mapsto \operatorname{arctg} 2 \\ \mathrm{d}t=\frac{\mathrm{d}s}{2\cos^2 s} & 0\mapsto 0 \end{matrix} \right| = \int_0^{\operatorname{arctg} 2} \sqrt{1+4\frac{1}{4}\operatorname{tg}^2 s}\frac{\mathrm{d}s}{2\cos^2 s} = \frac{1}{2}\int_0^{\operatorname{arctg} 2} \frac{\mathrm{d}s}{\cos^3 s} =$

$$= \left| \begin{array}{ll} u = \sin s & u = \frac{\operatorname{tg} s}{\sqrt{1+\operatorname{tg}^2 s}} \\ \mathrm{d}u = \cos s\,\mathrm{d}s & \operatorname{arctg} 2 \mapsto \frac{2}{\sqrt{5}}, 0 \mapsto 0 \end{array} \right| = \frac{1}{2}\int_0^{\frac{2}{\sqrt{5}}} \frac{\mathrm{d}u}{\cos^4 s} = \frac{1}{2}\int_0^{\frac{2}{\sqrt{5}}} \frac{\mathrm{d}u}{(1-u^2)^2} = \frac{1}{2}\int_0^{\frac{2}{\sqrt{5}}} \frac{\mathrm{d}u}{(1-u)^2(1+u)^2} =$$

$$= \frac{1}{8}\int_0^{\frac{2}{\sqrt{5}}} \left(-\frac{1}{u-1} + \frac{1}{(u-1)^2} + \frac{1}{u+1} + \frac{1}{(u+1)^2}\right) \mathrm{d}u = \frac{1}{8}\left[-\ln|u-1| - \frac{1}{u-1} + \ln|u+1| - \frac{1}{u+1}\right]_0^{\frac{2}{\sqrt{5}}} =$$

$$= \frac{1}{8}\left(-\ln\left(1-\frac{2}{\sqrt{5}}\right) - \frac{1}{\frac{2}{\sqrt{5}}-1} - 1 + \ln\left(\frac{2}{\sqrt{5}}+1\right) - \frac{1}{\frac{2}{\sqrt{5}}+1} + 1\right) = \frac{\sqrt{5}}{2} + \frac{1}{4}\ln(\sqrt{5}+2)$$

Rozklad na parciálne zlomky:

$$\frac{1}{(t-1)^2(t+1)^2} = \frac{A}{t-1} + \frac{B}{(t-1)^2} + \frac{C}{t+1} + \frac{D}{(t+1)^2} = \frac{A(t-1)(t+1)^2 + B(t+1)^2 + C(t+1)(t-1)^2 + D(t-1)^2}{(t-1)^2(t+1)^2} \Rightarrow$$

$$1 = A(t-1)(t+1)^2 + B(t+1)^2 + C(t+1)(t-1)^2 + D(t-1)^2$$

$$t = 1 \Rightarrow 1 = 4B, B = \frac{1}{4},\ t = -1 \Rightarrow 1 = 4D, D = \frac{1}{4}$$

Po zderivovaní obidvoch strán:

$$0 = A(t+1)^2 + 2A(t-1)(t+1) + 2B(t+1) + C(t-1)^2 + 2C(t-1)(t+1) + 2D(t-1)$$

$$t = 1 \Rightarrow 0 = 4A + 4B = 4A + 1, A = -\frac{1}{4},\ t = -1 \Rightarrow 0 = 4C - 4D = 4C - 1, C = \frac{1}{4}$$

$$\boxed{\frac{\sqrt{5}}{2} + \frac{1}{4}\ln(\sqrt{5}+2)}$$

$y = \frac{2+x^6}{8x^2},\ x \in \langle 1, 2 \rangle$

$$y' = \frac{1}{4}(-2)x^{-3} + \frac{1}{8}4x^3 = \frac{1}{2}(x^3 - x^{-3}),\ \sqrt{1+(y')^2} = \sqrt{1 + \frac{1}{4}(x^3 - x^{-3})^2} = \sqrt{1 + \frac{1}{4}(x^6 - 2 + x^{-6})} =$$

$$= \sqrt{\frac{1}{4}(x^6 + 2 + x^{-6})} = \frac{1}{2}(x^3 + x^{-3})$$

$$\int_1^2 \frac{1}{2}(x^3 + x^{-3}) = \frac{1}{2}\left[\frac{x^4}{4} - \frac{1}{2}x^{-2}\right]_1^2 = \frac{1}{2}\left(4 - \frac{1}{4} - \frac{1}{8} + \frac{1}{2}\right) = \frac{1}{2}\left(4 + \frac{1}{8}\right) = \frac{33}{16}$$

$$\boxed{\frac{33}{16}}$$