$y'' - y' - 2y = 0$

charakteristická rovnica: $x^2 - x - 2 = 0$

$D = 9,\ x_1 = 2, x_2 = -1$

všeobecné riešenie: $C_1e^{2x} + C_2e^{-x}$

$$\boxed{C_1e^{2x} + C_2e^{-x}}$$

$y'' + 25y = 0$

charakteristická rovnica: $x^2 + 25 = 0$

$D = -100,\ \frac{-b}{2a} = 0,\ \frac{\sqrt{-D}}{2a} = 5$

všeobecné riešenie: $C_1 \cos 5x + C_2 \sin 5x$

$$\boxed{C_1 \cos 5x + C_2 \sin 5x}$$

$y'' - y' = 0$

charakteristická rovnica: $x^2 - x = 0 = x(x-1)$

$x_1 = 0, x_2 = 1$

všeobecné riešenie: $C_1 + C_2e^x$

$$\boxed{C_1 + C_2e^x}$$

$y'' - 4y' + 4y = 0$

charakteristická rovnica: $x^2 - 4x + 4 = (x-2)^2$

$x_{1,2} = 2$

všeobecné riešenie: $C_1e^{2x} + C_2xe^{2x}$

$$\boxed{C_1e^{2x} + C_2xe^{2x}}$$

$y'' - 7y' + 6y = 0$

charakteristická rovnica: $x^2 - 7x + 6 = 0$

$D = 25,\ x_1 = 6,\ x_2 = 1$

všeobecné riešenie: $C_1e^x + C_2e^{6x}$

$$\boxed{C_1e^x + C_2e^{6x}}$$

$y'' + y' - 2y = 0$

charakteristická rovnica: $x^2 + x - 2 = 0$

$D = 9,\ x_1 = 1, x_2 = -2$

všeobecné riešenie: $C_1e^x + C_2e^{-2x}$

$$\boxed{C_1e^x + C_2e^{-2x}}$$

$y'' + y = 0$

charakteristická rovnica:$x^2 + 1 = 0$

$D = -4, \frac{-b}{2a} = 0, \frac{\sqrt{-D}}{2a} = 1$

všeobecné riešenie: $C_1 \cos x + C_2 \sin x$

$$\boxed{C_1 \cos x + C_2 \sin x}$$

$y'' - 2y' - y = 0$

charakteristická rovnica: $x^2 - 2x - 1 = 0$

$D = 8,\ x_1 = 1 + \sqrt{2}, x_2 = 1 - \sqrt{2}$

všeobecné riešenie: $C_1e^{(1+\sqrt{2})x} + C_2e^{(1-\sqrt{2})x}$

$$\boxed{C_1e^{(1+\sqrt{2})x} + C_2e^{(1-\sqrt{2})x}}$$

$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 20\frac{dx}{dt} + 25x = 0$

charakteristická rovnica: $4u^2 - 20u + 25 = 0 = (2u-5)^2$

$u_{1,2} = \frac{2}{5}$
všeobecné riešenie: $C_1 e^{\frac{2}{5}t} + C_2 t e^{\frac{2}{5}t}$

$$\boxed{x(t) = C_1 e^{\frac{2}{5}t} + C_2 t e^{\frac{2}{5}t}}$$

$y'' - 4y' + 13y = 0$
charakteristická rovnica: $x^2 - 4x + 13 = 0$
$D = -36,\ \frac{-b}{2a} = 2,\ \frac{\sqrt{-D}}{2a} = 3$
všeobecné riešenie: $e^{2x}(C_1 \cos 3x + C_2 \sin 3x)$

$$\boxed{e^{2x}(C_1 \cos 3x + C_2 \sin 3x)}$$

$y'' - 10y' + 25y = 0,\ y(0) = 0, y'(0) = 1$
charakteristická rovnica: $x^2 - 10x + 25 = 0 = (x-5)^2$
$x_{1,2} = 5$
všeobecné riešenie: $y = C_1 e^{5x} + C_2 x e^{5x},\ 0 = y(0) = C_1$
$y' = 5C_1 e^5 x + C_2 e^{5x} + 5C_2 x e^{5x},\ 1 = y'(0) = 5C_1 + C_2$
$C_1 = 0, C_2 = 1$
riešenie: xe^{5x}

$$\boxed{xe^{5x}}$$

$y'' - 2y' + 10y = 0,\ y(\frac{\pi}{6}) = 0,\ y'(\frac{\pi}{6}) = 2$
charakteristická rovnica: $x^2 - 2x + 10 = 0$
$D = -36, \frac{-b}{2a} = 1, \frac{\sqrt{-D}}{2a} = 3$
všeobecné riešenie: $y = e^x(C_1 \cos 3x + C_2 \sin 3x),\ y(\frac{\pi}{6}) = C_2 = 0$
$y' = (C_1 e^x \cos 3x)' = (C_1 e^x \cos 3x - 3C_1 e^x \sin 3x),\ y'(\frac{\pi}{6}) = -3C_1 e^{\frac{\pi}{6}} = 2,\ C_1 = -\frac{2}{3} e^{-\frac{\pi}{6}}$
riešenie: $-\frac{2}{3} e^{-\frac{\pi}{6}} e^x \cos 3x$

$$\boxed{-\frac{2}{3} e^{-\frac{\pi}{6}} e^x \cos 3x}$$

$y'' + 3y' = 0,\ y(0) = 1, y'(0) = 2$
charakteristická rovnica: $x^2 + 3x = 0 = x(x+3)$
$x_1 = 0, x_2 = -3$
všeobecné riešenie: $y = C_1 + C_2 e^{-3x},\ y(0) = C_1 + C_2 = 1$
$y' = -3C_2 e^{-3x},\ y'(0) = -3C_2 = 2,\ C_2 = -\frac{2}{3}, C_1 = \frac{8}{3}$
riešenie: $\frac{8}{3} - \frac{2}{3} e^{-3x}$

$$\boxed{\frac{8}{3} - \frac{2}{3} e^{-3x}}$$

$y'' + 4y' = 0,\ y(0) = 1,\ y'(0) = 2$
charakteristická rovnica: $x^2 + 4x = 0 = x(x+4)$
$x_1 = 0, x_2 = -4$
všeobecné riešenie: $y = C_1 + C_2 e^{-4x},\ y(0) = C_1 + C_2 = 1$
$y' = -4C_2 e^{-4x},\ y'(0) = -4C_2 = 2,\ C_2 = -\frac{1}{2},\ C_1 = \frac{3}{2}$
riešenie: $\frac{3}{2} - \frac{1}{2} e^{-4x}$

$$\boxed{\frac{3}{2} - \frac{1}{2} e^{-4x}}$$

$y'' - 12y = 0,\ y(\frac{1}{\sqrt{3}}) = \frac{4}{e^2}, y(0) = 4$
charakteristická rovnica: $x^2 - 12 = 0$
$x_1 = \sqrt{12} = 2\sqrt{3}, x_2 = -\sqrt{12} = -2\sqrt{3}$
všeobecné riešenie: $C_1 e^{2\sqrt{3}x} + C_2 e^{-2\sqrt{3}x}$
$y(\frac{1}{\sqrt{3}}) = C_1 e^2 + C_2 e^{-2} = \frac{4}{e^2}, y(0) = C_1 + C_2 = 4$
$C_1(e^2 - e^{-2}) = 0, C_1 = 0,\ C_2 = 4$

riešenie: $4e^{-2\sqrt{3}x}$

$$\boxed{4e^{-2\sqrt{3}x}}$$

$9\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 16y = 0, y(0) = -9, y'(0) = 12\frac{1}{2}$

charakteristická rovnica: $9u^2 + 16 = 0$

$D = -2^2 3^2 4^2$, $\frac{-b}{2a} = 0$, $\frac{\sqrt{-D}}{2a} = \frac{4}{3}$

všeobecné riešenie: $y = C_1 \cos\frac{4}{3}x + C_2 \sin\frac{4}{3}x$

$y' = -\frac{4}{3}C_1 \sin\frac{4}{3}x + \frac{4}{3}C_2 \cos\frac{4}{3}x$

$y(0) = C_1 = -9$, $y'(0) = \frac{4}{3}C_2 = 12\frac{1}{2}, C_2 = \frac{75}{8}$

riešenie: $-9\cos\frac{4}{3}x + \frac{75}{8}\sin\frac{4}{3}x$

$$\boxed{-9\cos\frac{4}{3}x + \frac{75}{8}\sin\frac{4}{3}x}$$

$y'' - 7y' + 6y = \sin x$

1. Riešenie homogénnej rovnice $y'' - 7y' + 6y = 0$:

charakteristická rovnica: $x^2 - 7x + 6 = 0$

$D = 25, x_1 = 1, x_2 = 6$

všeobecné riešenie: $y_v = C_1e^x + C_2e^{6x}$

2. Hľadanie partikulárneho riešenia: typ EPSC

Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde

$m = 0$, $n = 1$, $P(x) = 0$, $\mathrm{st}(P(x)) = -\infty$, $Q(x) = 1$, $\mathrm{st}(Q(x)) = 0$

$\max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0$, $k = 0$(korene sú 1 a 6), preto

$y_p = y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = C_1\cos x + C_2\sin x.$

$y_p' = C_2\cos x - C_1\sin x$

$y_p'' = -C_1\cos x - C_2\sin x$

$y_p'' - 7y_p' + 6y_p = \cos x(-C_1 - 7C_2 + 6C_1) + \sin x(-C_2 + 7C_1 + 6C_2) =$

$= (5C_1 - 7C_2)\cos x + (7C_1 + 5C_2)\sin x = \sin x$

$$\begin{matrix} 5C_1 - 7C_2 = 0 \\ 7C_1 + 5C_2 = 1 \end{matrix}, \; C_1 = \frac{7}{74}, C_2 = \frac{5}{74}$$

$y_p = \frac{7}{74}\cos x + \frac{5}{74}\sin x$

riešenie: $Y = y_p + y_v = \frac{7}{74}\cos x + \frac{5}{74}\sin x + C_1e^x + C_2e^{6x}$

$$\boxed{Y = \frac{7}{74}\cos x + \frac{5}{74}\sin x + C_1e^x + C_2e^{6x}}$$

$y'' + 2y' + 5y = -\frac{17}{2}\cos 2x$

1. Riešenie homogénnej rovnice $y'' + 2y' + 5y = 0$:

charakteristická rovnica: $x^2 + 2x + 5 = 0$

$D = -16$, $\frac{-b}{2a} = -1$, $\frac{\sqrt{-D}}{2a} = 2$

všeobecné riešenie: $y_v = e^{-x}(C_1\cos 2x + C_2\sin 2x)$

2. Hľadanie partikulárneho riešenia: typ EPSC

Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde

$m = 0$, $n = 2$, $P(x) = -\frac{17}{2}$, $\mathrm{st}(P(x)) = 0$, $Q(x) = 0$, $\mathrm{st}(Q(x)) = -\infty$

$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 0$, preto

$y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = C_1\cos 2x + C_2\sin 2x$

$y_p' = 2C_2\cos 2x - 2C_1\sin 2x$

$y_p'' = -4C_1\cos 2x - 4C_2\sin 2x$

$y_p'' + 2y_p' + 5y_p = \cos 2x(-4C_1 + 4C_2 + 5C_1) + \sin 2x(-4C_2 - 4C_1 + 5C_2) =$

$= \cos 2x\,(C_1 + 4C_2) + \sin 2x\,(-4C_1 + C_2) = -\frac{17}{2}\cos 2x$

$$\begin{matrix} C_1 + 4C_2 = -\frac{17}{2} \\ -4C_1 + C_2 = 0 \end{matrix}, \; C_1 = -\frac{1}{2}, C_2 = -2$$

$y_p = -\frac{1}{2}\cos 2x - 2\sin 2x$

riešenie: $Y = y_p + y_v = -\frac{1}{2}\cos 2x + 2\sin 2x + e^{-x}(C_1\cos 2x + C_2\sin 2x)$

$$\boxed{Y = -\frac{1}{2}\cos 2x + 2\sin 2x + e^{-x}(C_1 \cos 2x + C_2 \sin 2x)}$$

$2y'' + y' - y = 2e^x$

1. Riešenie homogénnej rovnice $2y'' + y' - y = 0$:
charakteristická rovnica: $2x^2 + x - 1 = 0$
$D = 9, x_1 = -1, x_2 = \frac{1}{2}$
všeobecné riešenie: $y_v = C_1 e^{-x} + C_2 e^{\frac{1}{2}x}$
2. Hľadanie partikulárneho riešenia: type EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 1, P(x) = 2$, $\mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0$, $k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = Ce^x$, $y_p' = Ce^x$, $y_p'' = Ce^x$
$2y_p'' + y_p' - y_p = e^x(2C + C - C) = 2Ce^x = 2e^x$, $C = 1$
$y_p = e^x$
riešenie: $Y = y_p + y_v = e^x + C_1 e^{-x} + C_2 e^{\frac{1}{2}x}$

$$\boxed{Y = e^x + C_1 e^{-x} + C_2 e^{\frac{1}{2}x}}$$

$y'' + a^2 y = e^x$

1. Riešenie homogénnej rovnice $y'' + a^2 y = 0$:
charakteristická rovnica: $x^2 + a^2 = 0$
$D = -4a^2$, $\frac{-b}{2a} = 0$, $\frac{\sqrt{-D}}{2a} = a$
všeobecné riešenie: $y_v = C_1 \cos ax + C_2 \sin ax$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x) = e^x$, kde
$m = 1$, $P(x) = 1$, $\mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0$, $k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = Ce^x$, $y_p' = Ce^x$, $y_p'' = Ce^x$
$y_p'' + a^2 y_p = Ce^x + Ca^2 e^x = C(1 + a^2)e^x = e^x$, $C = \frac{1}{1+a^2}$
$y_p = \frac{1}{1+a^2}e^x$
riešenie: $Y = y_p + y_v = \frac{1}{1+a^2}e^x + C_1 \cos ax + C_2 \sin ax$

$$\boxed{Y = \frac{1}{1+a^2}e^x + C_1 \cos ax + C_2 \sin ax}$$

$y'' - 6y' + 9y = 2x^2 - x + 3$

1. Riešenie homogénnej rovnice $y'' - 6y' + 9y = 0$:
charakteristická rovnica: $x^2 - 6x + 9 = (x - 3)^2 = 0$
$x_{1,2} = 3$
všeobecné riešenie: $y_v = C_1 e^{3x} + C_2 x e^{3x}$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 0$, $P(x) = 2x^2 - x + 3$, $\mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 2$, $k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = ax^2 + bx + c$, $y_p' = 2ax + b$, $y_p'' = 2a$
$y_p'' - 6y_p' + 9y_p = 2a - 12ax - 6b + 9ax^2 + 9bx + 9c = x^2(9a) + x(-12a + 9b) + (2a - 6b + 9c) = 2x^2 - x + 3$
$a = \frac{2}{9}, b = \frac{5}{27}, c = \frac{11}{27}$
$y_p = \frac{1}{27}(6x^2 + 5x + 11)$
riešenie: $Y = y_p + y_v = \frac{1}{27}(6x^2 + 5x + 11) + C_1 e^{3x} + C_2 x e^{3x}$

$$\boxed{Y = \frac{1}{27}(6x^2 + 5x + 11) + C_1 e^{3x} + C_2 x e^{3x}}$$

$y'' + 4y' - 5y = 1$

1. Riešenie homogénnej rovnice $y'' + 4y' - 5y = 0$:
charakteristická rovnica: $x^2 + 4x - 5 = 0$
$D = 36, x_1 = -1, x_2 = -5$
všeobecné riešenie: $y_v = C_1 e^x + C_2 e^{-5x}$
hpr typ EP

Pravá strana: $e^{mx}P(x)$, kde
$m = 0, P(x) = 1, \mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0, k = 0$
$y_p = C$, $y_p' = y_p'' = 0$, $y_p'' + 4y_p' - 5y_p = -5C = 1$, $C = -\frac{1}{5}$
$yp = -\frac{1}{5}$
riešenie: $Y = y_p + y_v = -\frac{1}{5} + C_1e^x + C_2e^{-5x}$

$$\boxed{Y = -\frac{1}{5} + C_1e^x + C_2e^{-5x}}$$

$y'' - 4y' + 4y = f(x)$
1. Riešenie homogénnej rovnice $y'' - 4y' + 4y = 0$:
charakteristická rovnica: $x^2 - 4x + 4 = (x-2)^2 = 0$
$x_{1,2} = 2$
všeobecné riešenie: $y_v = C_1e^{2x} + C_2xe^{2x}$

$f(x) = e^{-x}$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = -1$, $P(x) = 1$, $\mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0$, $k = 0$
$y_p = x^ke^{mx}\hat{P}(x) = Ce^{-x}$, $y_p' = -Ce^{-x}$, $y_p'' = Ce^{-x}$
$y_p'' - 4y_p' + 4y_p = e^{-x}(C + 4C + 4C) = 9Ce^{-x} = e^{-x}$, $C = \frac{1}{9}$
$y_p = \frac{1}{9}e^{-x}$
riešenie: $Y = y_p + y_v = \frac{1}{9}e^{-x} + C_1e^{2x} + C_2xe^{2x}$

$$\boxed{Y = \frac{1}{9}e^{-x} + C_1e^{2x} + C_2xe^{2x}}$$

$f(x) = 3e^{2x}$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 2$, $P(x) = 3$, $\mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0$, $k = 2$
$y_p = x^ke^{mx}\hat{P}(x) = Cx^2e^{2x}$
$y_p' = C(2xe^{2x} + 2x^2e^{2x}) = Ce^{2x}(2x + 2x^2)$
$y_p'' = Ce^{2x}(4x + 4x^2 + 2 + 4x) = Ce^{2x}(4x^2 + 8x + 2)$
$y_p'' - 4y_p' + 4y_p = Ce^{2x}(4x^2 + 8x + 2 - 8x^2 - 8x + 4x^2) = Ce^{2x}(2) = 3e^{2x}$, $C = \frac{3}{2}$
$y_p = \frac{3}{2}x^2e^{2x}$
riešenie: $Y = y_p + y_v = \frac{3}{2}x^2e^{2x} + C_1e^2x + C_2xe^2x = (\frac{3}{2}x^2 + C_2x + C_1)e^{2x}$

$$\boxed{Y = (\frac{3}{2}x^2 + C_2x + C_1)e^{2x}}$$

$f(x) = 2(\sin 2x + x)$
Použije sa princíp superpozície: $f(x) = f_1(x) + f_2(x)$, kde $f_1(x) = 2\sin 2x, f_2(x) = 2x$
$f_1(x) = 2\sin 2x$
2. Hľadanie partikulárneho riešenia: typ EPSC
Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde
$m = 0, n = 2, P(x) = 0, \mathrm{st}(P(x)) - \infty, Q(x) = 2, \mathrm{st}(Q(x)) = 0,$
$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 0$
$y_p = x^ke^{mx}\hat{P}(x) = C_1\cos 2x + C_2\sin 2x$
$y_p' = 2C_2\cos 2x - 2C_1\sin 2x$
$y_p'' = -4C_1\cos 2x - 4C_2\sin 2x$
$y_p'' - 4y_p' + 4y_p = (-4C_1 - 8C_2 + 4C_1)\cos 2x + (-4C_2 + 8C_1 + 4C_2)\sin 2x =$
$= -8C_1\cos 2x + 8C_1\sin 2x = 2\sin 2x$, $C_1 = 0, C_2 = \frac{1}{4}$
$y_{p_1} = \frac{1}{4}\cos 2x$
$f_2(x) = 2x$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 0, P(x) = 2x, \mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 1, k = 0$

$y_p = x^k e^{mx}\hat{P}(x) = ax + b,\ y_p' = a,\ y_p'' = 0$
$y_p'' - 4y_p' + 4y_p = -4a + 4b + 4ax = 2x,\ a = \frac{1}{2}, b = \frac{1}{2}$
$y_{p_2} = \frac{1}{2}(x+1)$
riešenie: $Y = y_{p_1} + y_{p_2} + y_v = \frac{1}{2}\cos 2x + \frac{1}{2}(x+1) + C_1 e^{2x} + C_2 x e^{2x}$

$$\boxed{Y = \frac{1}{2}\cos 2x + \frac{1}{2}(x+1) + C_1 e^{2x} + C_2 x e^{2x}}$$

$f(x) = 8(x^2 + e^{2x} + \sin 2x)$
Princíp superpozície: $f(x) = f_1(x) + f_2(x) + f_3(x)$, kde $f_1(x) = 8x^2, f_2(x) = 8e^{2x}, f_3(x) = 8\sin 2x$
$f_1(x) = 8x^2$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 0, P(x) = 8x^2, \mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 2, k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = ax^2 + bx + c,\ y_p' = 2ax + b,\ y_p'' = 2a$
$y_p'' - 4y_p' + 4y_p = 2a - 8ax - 4b + 4ax^2 + 4bx + 4c = x^2(4a) + x(-8a + 4b) + (2a - 4b + 4c)$
$a = 2, b = 4, c = 3$
$y_{p_1} = 2x^2 + 4x + 3$
$f_2(x) = 8e^{2x}$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 2, P(x) = 8, \mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0, k = 2$
$y_p = x^k e^{mx}\hat{P}(x) = Cx^2 e^{2x},\ y_p' = Ce^{2x}(2x + 2x^2),\ y_p'' = Ce^{2x}(4x + 4x^2 + 2 + 4x) = Ce^{2x}(4x^2 + 8x + 2)$
$y_p'' - 4y_p' + 4y_p = Ce^{2x}(4x^2 + 8x + 2 - 8x^2 - 8x + 4x^2) = Ce^{2x}(2) = 8e^{2x},\ C = 4$
$y_{p_2} = 4x^2 e^{2x}$
$f_3(x) = 8\sin 2x$
2. Hľadanie partikulárneho riešenia: typ EPSC
Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde
$m = 0, n = 2, P(x) = 0, \mathrm{st}(P(x)) = -\infty, Q(x) = 8, \mathrm{st}(Q(x)) = 0,$
$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 0$
$y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = C_1\cos 2x + C_2\sin 2x$
$y_p' = 2C_2\cos 2x - 2C_1\sin 2x,\ y_p'' = -4C_1\cos 2x - 4C_2\sin 2x$
$y_p'' - 4y_p' + 4y_p = (-4C_1 - 8C_2 + 4C_1)\cos 2x + (-4C_2 + 8C_1 + 4C_2)\sin 2x =$
$= -8C_2\cos 2x + 8C_1\sin 2x = 8\sin 2x,\ C_1 = 1, C_2 = 0$
$y_{p_3} = \cos 2x$
riešenie: $Y = y_{p_1} + y_{p_2} + y_{p_3} + y_v = 2x^2 + 4x + 3 + 4x^2e^{2x} + \cos 2x + C_1e^{2x} + C_2xe^{2x}$

$$\boxed{Y = 2x^2 + 4x + 3 + 4x^2e^{2x} + \cos 2x + C_1e^{2x} + C_2xe^{2x}}$$

$y'' + y = f(x)$
1. Riešenie homogénnej rovnice $y'' + y = 0$:
charakteristická rovnica: $x^2 + 1 = 0$
$D = -4, \frac{-b}{2a} = 0, \frac{\sqrt{-D}}{2a} = 1$
všeobecné riešenie: $y_v = C_1\cos x + C_2\sin x$
$f(x) = 2x^3 - x + 2$
2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = 0, P(x) = 2x^3 - x + 2, \mathrm{st}(P(x)) = 3, k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = ax^3 + bx^2 + cx + d, y_p' = 3ax^2 + 2bx + c, y_p'' = 6ax + 2b$
$y_p'' + y_p = ax^3 + bx^2 + x(6a + c) + (2b + d) = 2x^3 - x + 2,\ a = 2, b = 0, c = -13, d = 2$
$y_p = 2x^3 - 13x + 2$
riešenie: $Y = y_p + y_v = 2x^3 - 13x + 2 + C_1\cos x + C_2\sin x$

$$\boxed{Y = y_p + y_v = 2x^3 - 13x + 2 + C_1\cos x + C_2\sin x}$$

$f(x) = -8\cos 3x$

2. Hľadanie partikulárneho riešenia: typ EPSC
Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde
$m = 0, n = 3, P(x) = -8, \mathrm{st}(P(x)) = 0, Q(x) = 0, \mathrm{st}(Q(x)) = -\infty,$
$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 0$
$y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = C_1\cos 3x + C_2 \sin 3x$
$y_p' = 3C_2\cos 3x - 3C_1\sin 3x, y_p'' = -9C_1\cos 3x - 9C_2\sin 3x$
$y_p'' + y_p = (C_1 - 9C_1)\cos 3x + (C_2 - 9C_2)\sin 3x = -8C_1\cos 3x - 8C_2\sin 3x = -8\cos 3x$
$C_1 = 1, C_2 = 0$
$y_p = \cos 3x$
riešenie: $Y = y_p + y_v = \cos 3x + C_1\cos x + C_2\sin x$

$$\boxed{Y = \cos 3x + C_1\cos x + C_2\sin x}$$

$f(x) = \cos x$

2. Hľadanie partikulárneho riešenia: typ EPSC
Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde
$m = 0, n = 1, P(x) = 1, \mathrm{st}(P(x)) = 0, Q(x) = 0, \mathrm{st}(Q(x)) = -\infty,$
$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 1$
$y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = x(C_1\cos x + C_2\sin x)$
$y_p' = C_1\cos x + C_2\sin x + x(C_2\cos x - C_1\sin x)$
$y_p'' = C_2\cos x - C_1\sin x + C_2\cos x - C_1\sin x + x(-C_1\cos x - C_2\sin x)$
$y_p'' + y_p = 2C_2\cos x - 2C_1\sin x = \cos x$, $C_1 = 0, C_2 = \frac{1}{2}$
$y_p = \frac{1}{2}x\sin x$
riešenie: $Y = y_p + y_v = \frac{1}{2}x\sin x + C_1\cos x + C_2\sin x$

$$\boxed{Y = \frac{1}{2}x\sin x + C_1\cos x + C_2\sin x}$$

$f(x) = \sin x - 2e^{-x}$

Princíp superpozície: $f(x) = f_1(x) + f_2(x)$, kde $f_1(x) = \sin x$, $f_2(x) = -2e^{-x}$

$f_1(x) = \sin x$

2. Hľadanie partikulárneho riešenia: typ EPSC
Pravá strana: $e^{mx}(P(x)\cos nx + Q(x)\sin nx)$, kde
$m = 0, n = 1, P(x) = 0, \mathrm{st}(P(x)) = -\infty, Q(x) = 1, \mathrm{st}(Q(x)) = 0,$
$\mathrm{st}(\hat{P}(x)) = \mathrm{st}(\hat{Q}(x)) = \max(\mathrm{st}(P(x)), \mathrm{st}(Q(x))) = 0, k = 1$
$y_p = x^k e^{mx}(\hat{P}(x)\cos nx + \hat{Q}(x)\sin nx) = x(C_1\cos x + C_2\sin x)$
$y_p' = C_1\cos x + C_2\sin x + x(C_2\cos x - C_1\sin x)$
$y_p'' = C_2\cos x - C_1\sin x + C_2\cos x - C_1\sin x + x(-C_1\cos x - C_2\sin x)$
$y_p'' + y_p = 2C_2\cos x - 2C_1\sin x = \sin x$, $C_1 = -\frac{1}{2}, C_2 = 0,$
$y_{p_1} = -\frac{1}{2}x\cos x$

$f_2(x) = -2e^{-x}$

2. Hľadanie partikulárneho riešenia: typ EP
Pravá strana: $e^{mx}P(x)$, kde
$m = -1, P(x) = -2, \mathrm{st}(P(x)) = 0, \mathrm{st}(\hat{P}(x)) = \mathrm{st}(P(x)) = 0, k = 0$
$y_p = x^k e^{mx}\hat{P}(x) = Ce^{-x}$, $y_p'' = Ce^{-x}$
$y_p'' + y_p = 2Ce^{-x} = -2e^{-x}$, $C = -1$
$y_{p_2} = -e^{-x}$
riešenie: $Y = y_{p_1} + y_{p_2} + y_v = -\frac{1}{2}x\cos x - e^{-x} + C_1\cos x + C_2\sin x$

$$\boxed{Y = -\frac{1}{2}x\cos x - e^{-x} + C_1\cos x + C_2\sin x}$$

$y'' - y = \frac{2e^x}{e^x - 1}$

1. Riešenie homogénnej rovnice $y'' - y = 0$:
charakteristická rovnica: $x^2 - 1 = 0 = (x-1)(x+1)$

$x_1 = 1, x_2 = -1$
všeobecné riešenie: $y_v = C_1e^x + C_2e^{-x}$
2. Hľadanie partikulárneho riešenia: Lagrangeova metóda: $y = C_1(x)e^x + C_2(x)e^{-x}$
Wronského determinant(Wronskián): $W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -e^xe^{-x} - e^xe^{-x} = -2$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{2e^x}{e^x-1} & -e^{-x} \end{vmatrix} = \frac{-2}{e^x-1} \qquad W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{2e^x}{e^x-1} \end{vmatrix} = \frac{2e^{2x}}{e^x-1}$$

$C_1(x) = \int \frac{W_1}{W} = \int \frac{\mathrm{d}x}{e^x-1} = \int \frac{e^{-x}\mathrm{d}x}{1-e^{-x}} = |\, t = e^{-x}\mathrm{d}t = -e^{-x}\mathrm{d}x\,| = \int -\frac{\mathrm{d}t}{1-t} = \ln|1-t| + C =$
$= \ln|1-e^{-x}| + C = \ln|e^x - 1| - x + C$
$C_2(x) = \int \frac{W_2}{W} = \int \frac{-e^{2x}\mathrm{d}x}{e^x-1} = \begin{vmatrix} t = e^x \\ \mathrm{d}t = e^x\mathrm{d}x \end{vmatrix} = \int \frac{-t\,\mathrm{d}t}{t-1} = -\int(1 + \frac{1}{t-1})\,\mathrm{d}t = -t - \ln|t-1| + C =$
$= -e^x - \ln|e^x - 1| + C$
riešenie: $\ln|e^x - 1|(e^x - e^{-x}) - xe^x - 1 + C_1e^x + C_2e^{-x}$

$$\boxed{\ln|e^x - 1|(e^x - e^{-x}) - xe^x - 1 + C_1e^x + C_2e^{-x}}$$

$y'' - 2y' + y = \frac{e^x}{x}$
1. Riešenie homogénnej rovnice $y'' - 2y' + y = 0$:
charakteristická rovnica: $x^2 - 2x + 1 = 0 = (x-1)^2$
$x_{1,2} = 1$
všeobecné riešenie: $C_1e^x + C_2xe^x$
2. Hľadanie partikulárneho riešenia: Lagrangeova metóda $y = C_1(x)e^x + C_2(x)xe^x$
Wronskián: $W = \begin{vmatrix} e^x & xe^x \\ e^x & (x+1)e^x \end{vmatrix} = (x+1)e^{2x} - xe^{2x} = e^{2x}$

$$W_1 = \begin{vmatrix} 0 & xe^x \\ \frac{e^x}{x} & (1+x)e^x \end{vmatrix} = -e^{2x} \qquad W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{x} \end{vmatrix} = \frac{e^{2x}}{x}$$

$C_1 = \int \frac{W_1}{W} = \int -\mathrm{d}x = -x + C$
$C_2 = \int \frac{W_2}{W} = \int \frac{\mathrm{d}x}{x} = \ln|x| + C$
riešenie: $-xe^x + xe^x \ln|x| + C_1e^x + C_2xe^x$

$$\boxed{-xe^x + xe^x \ln|x| + C_1e^x + C_2xe^x}$$

$y'' + 2y' + y = \frac{e^{-2x}-1}{e^{-x}+1}$
1. Riešenie homogénnej rovnice $y'' + 2y' + y = 0$:
charakteristická rovnica: $x^2 + 2x + 1 = (x+1)^2 = 0$
$x_{1,2} = -1$
všeobecné riešenie: $C_1e^{-x} + C_2xe^{-x}$
2. Hľadanie partikulárneho riešenia: Lagrangeova metóda $y = C_1(x)e^{-x} + C_2(x)xe^{-x}$
Wronskián: $W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & (1-x)e^{-x} \end{vmatrix} = (1-x)e^{-2x} + xe^{-2x} = e^{-2x}$

$$W_1 = \begin{vmatrix} 0 & xe^{-x} \\ \frac{e^{-2x}-1}{e^{-x}+1} = e^{-x} - 1 & (1-x)e^{-x} \end{vmatrix} = xe^{-2x} - xe^{-x} \qquad W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x} - 1 \end{vmatrix} = e^{-2x} - e^{-x}$$

$C_1 = \int \frac{W_1}{W} = \int (x - xe^x)\,\mathrm{d}x = \frac{x^2}{2} - \int xe^x\mathrm{d}x = \begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix} = \frac{x^2}{2} - xe^x + e^x + C$
$C_2 = \int \frac{W_2}{W} = \int (1 - e^x)\,\mathrm{d}x = x - e^x + C$
riešenie: $C_1e^{-x} + C_2xe^{-x} = \frac{x^2}{2}e^{-x} - xe^xe^{-x} + e^xe^{-x} + Ce^{-x} + x^2e^{-x} - e^xxe^{-x} + Cxe^{-x} =$
$= \frac{3x^2}{2}e^{-x} - 2x + 1 + C_1e^{-x} + C_2xe^{-x}$

$$\boxed{\frac{3x^2}{2}e^{-x} - 2x + 1 + C_1e^{-x} + C_2xe^{-x}}$$

$y^{IV} - 2y''' + y'' = 0$
Substitúciou $z := y''$, prevedieme pôvodnú rovnicu na $z'' - 2z' + z = 0$,
ktorej všeobecné riešenie je $C_1e^x + C_2xe^x$.

$y' = \int (C_1e^x + C_2xe^x)\,\mathrm{d}x = C_1e^x + C_2xe^x - C_2e^x + C_3$
$y = \int (C_1e^x + C_2xe^x - C_2e^x + C_3)\,\mathrm{d}x = C_1e^x + C_2xe^x - C_2e^x - C_2e^x + C_3x + C_4 =$
$= (C_1 - 2C_2)e^x + C_2xe^x + C_3x + C_4$

$$\boxed{Ae^x + Bxe^x + Cx + D}$$

$y^{IV} + a^4y = 0$

charakteristická rovnica: $x^4 + a^4 = 0$
$x_{1,2,3,4} = \pm\frac{\sqrt{2}}{2}a \pm \frac{\sqrt{2}}{2}ai$
všeobecné riešenie: $e^{\frac{\sqrt{2}}{2}a}\left(C_1\cos\left(\frac{\sqrt{2}}{2}ax\right) + C_2\sin\left(\frac{\sqrt{2}}{2}ax\right)\right) + e^{-\frac{\sqrt{2}}{2}a}\left(C_3\cos\left(\frac{\sqrt{2}}{2}ax\right) + C_4\sin\left(\frac{\sqrt{2}}{2}ax\right)\right)$

$$\boxed{e^{\frac{\sqrt{2}}{2}a}\left(C_1\cos\left(\frac{\sqrt{2}}{2}ax\right) + C_2\sin\left(\frac{\sqrt{2}}{2}ax\right)\right) + e^{-\frac{\sqrt{2}}{2}a}\left(C_3\cos\left(\frac{\sqrt{2}}{2}ax\right) + C_4\sin\left(\frac{\sqrt{2}}{2}ax\right)\right)}$$

$y''' - 2y'' + y' = 0$

Označme $z := y'$, potom sa rovnica pretransformuje na rovnicu druhého stupňa s konštantnými koeficientami, ktorú vieme riešiť:
$z'' - 2z' + z = 0$
charakteristická rovnica: $x^2 - 2x + 1 = 0 = (x-1)^2$
$x_{1,2} = 1$
všeobecné riešenie: $z(x) = C_1e^x + C_2xe^x$
Potom $y = \int (C_1e^x + C_2xe^x) = \begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix} = C_1e^x + C_2xe^x - C_2e^x + C_3 = Cxe^x + De^x + E$

$$\boxed{Cxe^x + De^x + E}$$

$y''' = \frac{1}{x}$

$y'' = \int \frac{1}{x}\mathrm{d}x = \ln x + C,\ y' = \int (\ln x + C)\,\mathrm{d}x = \begin{vmatrix} \ln x & 1 \\ \frac{1}{x} & x \end{vmatrix} = x\ln x - x + Cx + D$
$y = \int (x\ln x - x + Cx + D)\,\mathrm{d}x = \begin{vmatrix} \ln x & x \\ \frac{1}{x} & \frac{x^2}{2} \end{vmatrix} = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C\frac{x^2}{2} + Dx + E =$
$= \frac{1}{2}x^2\ln x + Cx^2 + Dx + E$

$$\boxed{\frac{1}{2}x^2\ln x + Cx^2 + Dx + E}$$

$y''' = \cos 2x$

$y'' = \int \cos 2x\,\mathrm{d}x = \frac{\sin 2x}{2} + C,\ y' = \int (\frac{\sin 2x}{2} + C)\,\mathrm{d}x = -\frac{\cos 2x}{4} + Cx + D$
$y = \int (-\frac{\cos 2x}{4} + Cx + D)\,\mathrm{d}x = -\frac{\sin 2x}{8} + C\frac{x^2}{2} + Dx + E = -\frac{\sin 2x}{8} + Cx^2 + Dx + E$

$$\boxed{-\frac{\sin 2x}{8} + Cx^2 + Dx + E}$$