$$\int \frac{\ln x}{\sqrt[3]{x^2}}\mathrm{d}x = \begin{vmatrix} \ln x & \frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}} \\ \frac{1}{x} & \frac{x^{\frac{1}{3}}}{\frac{1}{3}} = 3x^{\frac{1}{3}} \end{vmatrix} = 3x^{\frac{1}{3}}\ln x - 3\int x^{-\frac{2}{3}}\mathrm{d}x = 3x^{\frac{1}{3}}\ln x - 3\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = 3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}} + C =$$

$$= 3x^{\frac{1}{3}}(\ln x - 3) + C$$

$$\boxed{3x^{\frac{1}{3}}(\ln x - 3) + C}$$

$$\int \frac{\ln x}{\sqrt[5]{x^4}}\mathrm{d}x = \begin{vmatrix} \ln x & \frac{1}{\sqrt[5]{x^4}} = x^{-\frac{4}{5}} \\ \frac{1}{x} & \frac{x^{\frac{1}{5}}}{\frac{1}{5}} = 5x^{\frac{1}{5}} \end{vmatrix} = 5x^{\frac{1}{5}}\ln x - 5\int x^{-\frac{4}{5}}\mathrm{d}x = 5x^{\frac{1}{5}}\ln x - 5\frac{x^{\frac{1}{5}}}{\frac{1}{5}} + C =$$

$$= 5x^{\frac{1}{5}}\ln x - 25x^{\frac{1}{5}} + C = 5x^{\frac{1}{5}}(\ln x - 5) + C$$

$$\boxed{5x^{\frac{1}{5}}(\ln x - 5) + C}$$

$$\int \frac{x^2+x-20}{(x-1)(x^2+2x+3)} = \int \frac{-3\,\mathrm{d}x}{x-1} + \int \frac{4x+11}{x^2+2x+3}\mathrm{d}x = -3\ln|x-1| + \int \frac{2(2x+2)+7}{x^2+2x+3}\mathrm{d}x = -3\ln|x-1| + 2\int \frac{2x+2}{x^2+2x+3}\mathrm{d}x +$$

$$+7\int \frac{\mathrm{d}x}{x^2+2x+3} = -3\ln|x-1| + 2\ln(x^2+2x+3) + 7\int \frac{\mathrm{d}x}{(x+1)^2+2} = \begin{vmatrix} t = x+1 \\ \mathrm{d}t = dx \end{vmatrix} = \ldots + 7\int \frac{dt}{2+t^2} =$$

$$= \ldots + \frac{7}{2}\int \frac{\mathrm{d}t}{1+\frac{t^2}{2}} = \ldots + \frac{7}{2}\int \frac{\mathrm{d}t}{\left(\frac{t}{\sqrt{2}}\right)^2+2} = \begin{vmatrix} s = \frac{t}{\sqrt{2}} & \\ \mathrm{d}s = \frac{\mathrm{d}t}{\sqrt{2}} & \mathrm{d}t = \sqrt{2}\,\mathrm{d}s \end{vmatrix} = \ldots + \frac{7}{2}\int \frac{\sqrt{2}\,\mathrm{d}s}{1+s^2} = \ldots + \frac{7}{\sqrt{2}}\int \frac{\mathrm{d}s}{1+s^2} =$$

$$\ldots + \frac{7}{\sqrt{2}}\operatorname{arctg} s + C = \ldots + \frac{7}{\sqrt{2}}\operatorname{arctg}\left(\frac{t}{\sqrt{2}}\right) = \ldots + \frac{7}{\sqrt{2}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + C$$

$$\frac{x^2+x-20}{(x-1)(x^2+2x+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+3} = \frac{A(x^2+2x+3)+(Bx+C)(x-1)}{(x-1)(x^2+2x+3)} = \frac{x^2(A+B)+x(2A-B+C)+(3A-C)}{(x-1)(x^2+2x+3)}$$

$$A+B=1, 2A-B+C=1, 3A-C=-20 \Rightarrow A+B=1, 5A-B=-19 \Rightarrow 6A=-18$$

$$A=-3, B=4, C=11$$

$$\boxed{-3\ln|x-1| + 2\ln(x^2+2x+3) + \frac{7}{\sqrt{2}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + C}$$

$$\int \frac{x^2+x-5}{(x+1)(x^2+6x+10)}\mathrm{d}x = \int \frac{-1}{x+1}\mathrm{d}x + \int \frac{2x+5}{x^2+6x+10}\mathrm{d}x = -\ln|x+1| + \int \frac{(2x+6)-1}{x^2+6x+10}\mathrm{d}x = -\ln|x+1| + \int \frac{2x+6}{x^2+6x+10}\mathrm{d}x -$$

$$-\int \frac{\mathrm{d}x}{x^2+6x+10} = -\ln|x+1| + \ln(x^2+6x+10) - \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{vmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{vmatrix} = \ldots - \int \frac{\mathrm{d}t}{1+t^2} = \ldots - \operatorname{arctg} t + C =$$

$$= -\ln|x+1| + \ln(x^2+6x+10) - \operatorname{arctg}(x+3) + C$$

$$\frac{x^2+x-5}{(x+1)(x^2+6x+10)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+6x+10} = \frac{A(x^2+6x+10)+(Bx+C)(x+1)}{(x+1)(x^2+6x+10)} = \frac{x^2(A+B)+x(6A+B+C)+(10A+C)}{(x+1)(x^2+6x+10)}$$

$$A+B=1, 6A+B+C=1, 10A+C=-5 \Rightarrow 5A+C=0, 10A+C=-5 \Rightarrow 5A=-5$$

$$A=-1, B=2, C=5$$

$$\boxed{-\ln|x+1| + \ln(x^2+6x+10) - \operatorname{arctg}(x+3) + C}$$

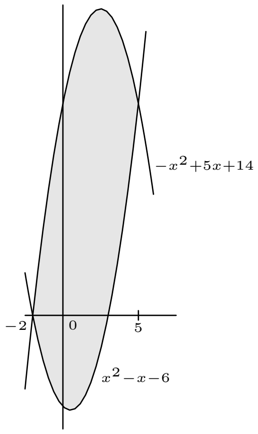
$$\int \frac{\mathrm{d}x}{(x-2)\sqrt{1-x}} = \begin{vmatrix} t = \sqrt{1-x} & x = 1-t^2 \\ \mathrm{d}t = \frac{1}{2}\frac{\mathrm{d}x}{1-x}(-1) & \mathrm{d}x = -2t\,\mathrm{d}t \end{vmatrix} = \int \frac{-2t\,\mathrm{d}t}{(-1-t^2)t} = \int \frac{2\,\mathrm{d}t}{1+t^2} = 2\operatorname{arctg} t + C = 2\operatorname{arctg}\sqrt{1-x} + C$$

$$\boxed{2\operatorname{arctg}\sqrt{1-x} + C}$$

$$\int \frac{\mathrm{d}x}{(x-2)\sqrt{x-3}} = \begin{vmatrix} t = \sqrt{x-3} & x = 3+t^2 \\ \mathrm{d}t = \frac{1}{2}\frac{\mathrm{d}x}{\sqrt{x-3}} & \mathrm{d}x = 2t\,\mathrm{d}t \end{vmatrix} = \int \frac{2t\,\mathrm{d}t}{(1+t^2)t} = \int \frac{2\,\mathrm{d}t}{1+t^2} = 2\operatorname{arctg} t + C = 2\operatorname{arctg}\sqrt{x-3} + C$$

$$\boxed{2\operatorname{arctg}\sqrt{x-3} + C}$$

$y = x^2 - x - 6, y = -x^2 + 5x + 14$



Prienik kriviek získame riešením kvadratickej rovnice:

$x^2 - x - 6 = -x^2 + 5x + 14$, $2x^2 - 6x - 20 = 0$, $x^2 - 3x - 10 = 0$, $x_1 = 5$, $x_2 = -2$

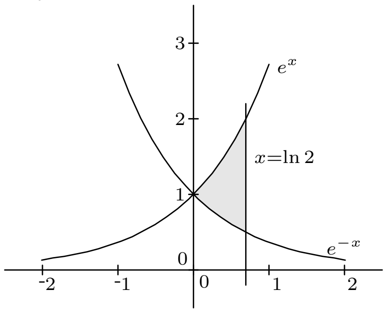
Na interval $\langle -2, 5\rangle$ dominuje $-x^2 + 5x + 14$, preto pre obsah plochy ohraničenej krivkami platí:

$\int_{-2}^{5}(-x^2 + 5x + 14 - (x^2 - x - 6))\,\mathrm{d}x = \int_{-2}^{5}(-2x^2 + 6x + 20)\,\mathrm{d}x = \left[-\frac{2}{3}x^3 + 3x^2 + 20x\right]_{-2}^{5} =$

$-\frac{2}{3}(125 + 8) + 3(25 - 4) + 20(5 - (-2)) = -\frac{266}{3} + 203 = \frac{343}{3} = 114\frac{1}{3}$

$$\boxed{114\tfrac{1}{3}}$$

$y = e^x, y = e^{-x}, x = \ln 2$



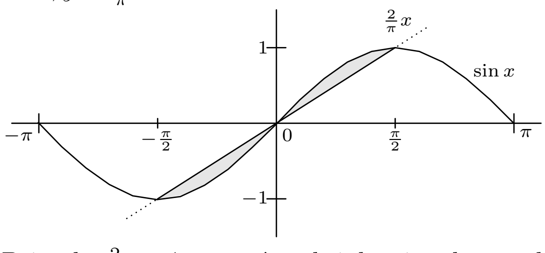
Krivky e^x a e^{-x} sa pretínajú v jedinom bode $x = 0$, čo je dolná hranica.

Horná je daná priamkou $x = \ln 2$. e^x dominuje na intervale $\langle 0, \ln 2\rangle$. Plocha sa rovná:

$\int_0^{\ln 2}(e^x - e^{-x})\,\mathrm{d}x = [e^x + e^{-x}]_0^{\ln 2} = e^{\ln 2} + e^{-\ln 2} - e^0 - e^{-0} = 2 + \frac{1}{2} - 2 = \frac{1}{2}$

$$\boxed{\tfrac{1}{2}}$$

$y = \sin x, y = \frac{2}{\pi}x$



Priamka $\frac{2}{\pi}x$ môže pretínať krivku $\sin x$ len vtedy, keď jej hodnoty sú v intervale $\langle -1, 1\rangle$, t.j. $x \in \langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle$.

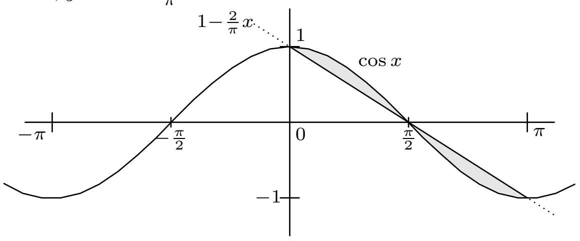
Body prieseku sú $\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\}$. Oblasť je symetrická podľa počiatku súradnicového systému, preto sa stačí obmedziť len na interval $\langle 0, \frac{\pi}{2}\rangle$ a výsledný objem bude dvojnásobkom. Na intervale $\langle 0, \frac{\pi}{2}\rangle$ dominuje $\sin x$.

$V = 2 \cdot \pi \int_0^{\frac{\pi}{2}}(\sin^2 x - \frac{4}{\pi^2}x^2)\,\mathrm{d}x = 2\pi \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\mathrm{d}x - \frac{8}{\pi}\int_0^{\frac{\pi}{2}} x^2\mathrm{d}x = \pi \int_0^{\frac{\pi}{2}}(1 - \cos 2x)\,\mathrm{d}x - \frac{8}{\pi}\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} =$

$= \pi \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} - \frac{8}{\pi}\frac{\pi^3}{8\cdot 3} = \pi^2(\frac{1}{2} - \frac{1}{3}) = \frac{\pi^2}{6}$

$\boxed{\frac{\pi^2}{6}}$

$y = \cos x, y = 1 - \frac{2}{\pi}x$



Priamka $1-\frac{2}{\pi}x$ môže pretínať krivku $\cos x$ len vtedy, keď jej hodnoty sú v intervale $\langle -1, 1\rangle$, t.j. $x \in \langle 0, \pi\rangle$.

Body prieseku sú $\{0, \frac{\pi}{2}, \pi\}$. Oblasť je symetrická podľa bodu $[\frac{\pi}{2}, 0]$, preto sa stačí obmedziť len na interval $\langle 0, \frac{\pi}{2}\rangle$ a výsledný objem bude dvojnásobkom. Na intervale $\langle 0, \frac{\pi}{2}\rangle$ dominuje $\cos x$.

$V = 2\cdot\pi\int_0^{\frac{\pi}{2}}(\cos^2 x - (1 - \frac{2}{\pi}x)^2)\mathrm{d}x = 2\pi\int_0^{\frac{\pi}{2}} \frac{\cos 2x + 1}{2}\,\mathrm{d}x - 2\pi\int_0^{\frac{\pi}{2}}(1 - \frac{4}{\pi}x + \frac{4}{\pi^2}x^2)\,\mathrm{d}x = \pi\left[\frac{\sin 2x}{2} + x\right]_0^{\frac{\pi}{2}} - 2\pi\left[x - \frac{2}{\pi}x^2 + \frac{4}{3\pi^2}x^3\right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{2} - (\pi^2 - \pi^2 + \frac{\pi^2}{3}) = \pi^2(\frac{1}{2} - \frac{1}{3}) = \frac{\pi^2}{6}$

Pozn: Na výpočet $\int_0^{\frac{\pi}{2}}(1-\frac{2}{\pi}x)^2\mathrm{d}x$ možno použit aj substitučnú metódu. Dôjde k podstatnému zjednodušeniu výpočtov.

$\boxed{\frac{\pi^2}{6}}$