1. $\int \frac{x^3+x-1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{x(x^2+1)-1}{x(x^2+1)}\,\mathrm{d}x = \int (1 - \frac{1}{x(x^2+1)})\,\mathrm{d}x.$

$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)} = \frac{(A+B)x^2+Cx+A}{x(x^2+1)}$, t.j. $A=1, B=-1, C=0$

Potom sa hľadaný integrál rovná:

$I = x - \int (\frac{1}{x} - \frac{x}{x^2+1})\mathrm{d}x = x - \ln|x| + \frac{1}{2}\int \frac{2x}{x^2+1}\mathrm{d}x = x - \ln|x| + \frac{1}{2}\ln(x^2+1) + C.$

$$\boxed{x - \ln|x| + \frac{1}{2}\ln(x^2+1) + C}$$

2. $\int \frac{\sqrt{x}+\sqrt[4]{x}+\sqrt[3]{x}}{2(x+\sqrt[6]{x^7})}\,\mathrm{d}x$

Najmenší spoločný násobok odmocnín sa rovná : $\mathrm{nsn}(2,4,3,1,6) = 12$, preto sa zvolí substitúcia $t = \sqrt[12]{x}$, resp. $x = t^{12}$.

$I = \left| \begin{array}{c} x = t^{12} \\ \mathrm{d}x = 12t^{11}\,\mathrm{d}t \end{array} \right| = \int \frac{t^6+t^3+t^4}{2(t^{12}+t^{14})} \cdot 12t^{11}\mathrm{d}t = 6\int \frac{t^6+t^4+t^3}{t(1+t^2)}\mathrm{d}t = 6\int \frac{t^3\cdot t(t^2+1)+t(t^2+1)-t}{t(t^2+1)}\mathrm{d}t =$

$= 6\int (t^3 + 1 - \frac{1}{t^2+1})\,\mathrm{d}t = \frac{3}{2}t^4 + 6t - 6\,\mathrm{arctg}\,t + C = \frac{3}{2}\sqrt[3]{x} + 6\sqrt[12]{x} - 6\,\mathrm{arctg}\sqrt[12]{x} + C$

$$\boxed{\frac{3}{2}\sqrt[3]{x} + 6\sqrt[12]{x} - 6\,\mathrm{arctg}\sqrt[12]{x} + C}$$

3. $\int \mathrm{arctg}(\sqrt{x})\mathrm{d}x = \left| \begin{array}{cc} \mathrm{arctg}(\sqrt{x}) & 1 \\ \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}} & x \end{array} \right| = x\,\mathrm{arctg}(\sqrt{x}) - \int \frac{x}{2(1+x)\sqrt{x}}\,\mathrm{d}x = x\,\mathrm{arctg}(\sqrt{x}) - \int \frac{\sqrt{x}}{2(1+x)}\mathrm{d}x =$

$= \left| \begin{array}{c} t = \sqrt{x} \\ \mathrm{d}t = \frac{\mathrm{d}x}{2\sqrt{x}} \end{array} \right| = \ldots + \int \frac{\sqrt{x}}{2(1+x)} \cdot 2\sqrt{x}\,\mathrm{d}t = \ldots + \int \frac{x}{1+x}\,\mathrm{d}t = \ldots + \int \frac{t^2}{1+t^2}\mathrm{d}t = \ldots + \int \frac{1+t^2-1}{1+t^2}\mathrm{d}t =$

$= \int (1 - \frac{1}{1+t^2})\mathrm{d}t = t - \mathrm{arctg}\,t + C = \sqrt{x} - \mathrm{arctg}(\sqrt{x}) + C$

$$\boxed{\sqrt{x} - \mathrm{arctg}(\sqrt{x}) + C}$$

4. $S_{o_x} : y^2 = 4ax, 0 \le x \le 3a$

$S = 2\pi \int |f(x)|\sqrt{1+(f'(x))^2}\,\mathrm{d}x$

Hranice sú od 0 po $3a$. Funkcia $f(x) = \sqrt{4ax} = 2\sqrt{ax}$, $f'(x) = 2\sqrt{a}\cdot\frac{1}{2\sqrt{x}} = \frac{\sqrt{a}}{\sqrt{x}}$.

$S = 2\pi\int_0^{3a} 2\sqrt{ax}\sqrt{1+(\frac{\sqrt{a}}{\sqrt{x}})^2}\,\mathrm{d}x = 4\pi\int_0^{3a}\sqrt{ax}\sqrt{\frac{a+x}{x}}\,\mathrm{d}x = 4\pi\sqrt{a}\int_0^{3a}\sqrt{a+x} = 4\pi\sqrt{a}\cdot\left[\frac{2}{3}(a+x)^{\frac{3}{2}}\right]_0^{3a} =$

$= \frac{8}{3}\pi\sqrt{a}[(4a)^{\frac{3}{2}} - a^{\frac{3}{2}}] = \frac{8}{3}\pi\sqrt{a}(2^3a\sqrt{a} - a\sqrt{a}) = \frac{56}{3}\pi a^2$

$$\boxed{\frac{56}{3}\pi a^2}$$

5. $P : xy = 4, x + y = 5$

Tieto dve krivky sa pretínajú : $y = 5 - x, x(5-x) = 4, -x^2 + 5x - 4 = 0 = -(x-1)(x-4)$, preto sú hranice rovné 0 a 4. Na tomto intervale $\langle 0, 4\rangle$ dominuje krivka $x + y = 5$, t.j. $P = \int_1^4 ((5-x) - \frac{4}{x})\,\mathrm{d}x = \left[5x - \frac{x^2}{2} - 4\ln|x| + C\right]_1^4 = 20 - 8 - 4\ln 4 - 5 + \frac{1}{2} = \frac{15}{2} + 8\ln 2$

$$\boxed{\frac{15}{2} + 8\ln 2}$$