1. $\int \frac{\mathrm{d}x}{(1+x^2)\operatorname{arctg} x} = \left| \begin{array}{l} t = \operatorname{arctg} x \\ \mathrm{d}t = \frac{\mathrm{d}x}{1+x^2} \end{array} \right| = \int \frac{\mathrm{d}t}{t} = \ln|t| + C = \ln|\operatorname{arctg} x| + C$

$$\boxed{\ln|\operatorname{arctg} x| + C}$$

2. $\int \frac{3x^3-x^2-7x+6}{(x^2-2x+2)x}\mathrm{d}x = \int \frac{3\cdot x(x^2-2x+2)+5x^2-13x+6}{x(x^2-2x+2)}\mathrm{d}x = \int (3 + \frac{5x^2-13x+6}{x(x^2-2x+2)})\,\mathrm{d}x$

$\frac{5x^2-13x+6}{x(x^2-2x+2)} = \frac{A}{x} + \frac{Bx+C}{x^2-2x+2} = \frac{A(x^2-2x+2)+(Bx+C)x}{x(x^2-2x+2)} = \frac{(A+B)x^2+(-2A+C)x+2A}{x(x^2-2x+2)}$, t.j. $\begin{array}{c} A+B=5,\ 2A=6 \\ -2A+C=-13 \end{array}$

$$A=3, B=2, C=7$$

$I = \int(3+\frac{3}{x}+\frac{2x+7}{x^2-2x+2})\,\mathrm{d}x = 3x+3\ln|x|+\int\frac{2x-2+9}{x^2-2x+2}\,\mathrm{d}x = 3x+3\ln|x|+\ln(x^2-2x+2)+\int\frac{9}{x^2-2x+2}\,\mathrm{d}x =$

$= \ldots + 9\int\frac{\mathrm{d}x}{(x-1)^2+1} = \ldots + 9\operatorname{arctg}(x-1)+C = 3x+3\ln|x|+\ln(x^2-2x+2)+9\operatorname{arctg}(x-1)+C$

$$\boxed{3x+3\ln|x|+\ln(x^2-2x+2)+9\operatorname{arctg}(x-1)+C}$$

3. $\int \frac{\ln x}{x^{\frac{7}{8}}}\,\mathrm{d}x = \int \ln x \cdot x^{-\frac{7}{8}}\,\mathrm{d}x = \left| \begin{array}{ll} \ln x & x^{-\frac{7}{8}} \\ \frac{1}{x} & \frac{x^{\frac{1}{8}}}{\frac{1}{8}} = 8x^{\frac{1}{8}} \end{array} \right| = 8x^{\frac{1}{8}}\ln x - 8\int x^{-\frac{7}{8}}\,\mathrm{d}x = 8x^{\frac{1}{8}}\ln x - 8\cdot 8x^{\frac{1}{8}} + C =$

$= 8x^{\frac{1}{8}}(\ln x - 8) + C$

$$\boxed{8x^{\frac{1}{8}}(\ln x - 8) + C}$$

4. $V_{o_x} : y = \frac{6x}{\pi}, y = \sin 3x$

Funkcia sin nadobúda hodnoty z intervalu $\langle -1, 1\rangle$, preto ako hraničné body prichádzajú do úvahy len tie, pre ktoré platí, že $|\frac{6x}{\pi}| \le 1$, t.j. $x \in \langle -\frac{\pi}{6}, \frac{\pi}{6}\rangle$. Naozaj jediné priesečníky sú $-\frac{\pi}{6}, 0, \frac{\pi}{6}$. Oblasť je symetrická podľa počiatku súradnicového systému, preto stačí vypočítať objem na intervale $\langle 0, \frac{\pi}{6}\rangle$, čím dostaneme polovicu hľadaného objemu. Na tomto intervale funkcia $\sin 3x$ dominuje $y = \frac{6x}{\pi}$, preto platí:

$V_{o_x} = 2\cdot\pi\int_0^{\frac{\pi}{6}}(\sin^2 3x - (\frac{6x}{\pi})^2)\,\mathrm{d}x = 2\pi\int_0^{\frac{\pi}{6}}(\frac{1-\cos 6x}{2} - \frac{36}{\pi^2}x^2)\,\mathrm{d}x = 2\pi\left[\frac{1}{2}x - \frac{\sin 6x}{12} - \frac{36}{\pi^2}\cdot\frac{x^3}{3}\right]_0^{\frac{\pi}{6}} =$

$= \frac{\pi^2}{6} - \frac{\pi^2}{9} = \frac{3\pi^2}{54} = \frac{\pi^2}{18}$

$$\boxed{\frac{\pi^2}{18}}$$

5. $P : y = 2x^3, y^2 = 4x$

Priesečníky kriviek: $y^2 = 4x = 4x^6$, preto $x(1-x^5) = 0$, t.j. 0, 1. Na intervale $\langle 0, 1\rangle$ dominuje $y^2 = 4x$, t.j. $y = 2\sqrt{x}$.

$P = \int_0^1 (2\sqrt{x} - 2x^3)\,\mathrm{d}x = \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^4\right]_0^1 = \frac{4}{3} - \frac{1}{2} = \frac{8-3}{6} = \frac{5}{6}$

$$\boxed{\frac{5}{6}}$$