1. $\int \frac{\mathrm{d}x}{x\sqrt{x-4}} = \begin{vmatrix} t=\sqrt{x-4} & x=t^2+4 \\ \mathrm{d}t=\frac{\mathrm{d}x}{\sqrt{x-4}} & \end{vmatrix} = \int \frac{\mathrm{d}t}{t^2+4} = \frac{1}{4}\int \frac{\mathrm{d}t}{1+(\frac{t}{2})^2} = \begin{vmatrix} u=\frac{t}{2} \\ \mathrm{d}u=\frac{\mathrm{d}t}{2} \end{vmatrix} = \frac{1}{2}\int \frac{\mathrm{d}u}{1+u^2} = \frac{1}{2}\operatorname{arctg} u + C =$

$= \frac{1}{2}\operatorname{arctg}\frac{t}{2} + C = \frac{1}{2}\operatorname{arctg}\frac{\sqrt{x-4}}{2} + C$

$$\boxed{\frac{1}{2}\operatorname{arctg}\frac{\sqrt{x-4}}{2} + C}$$

2. $\int \frac{\mathrm{d}x}{(x-1)x^2}$

$\frac{1}{(x-1)x^2} = \frac{A}{x-1} + \frac{B}{x} + \frac{C}{x^2} = \frac{Ax^2+Bx(x-1)+C(x-1)}{(x-1)x^2} = \frac{(A+B)x^2+(-B+C)x-C}{x^2(x-1)}$, t.j. $C=-1, B=-1, A=1$

$I = \int(\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2})\,\mathrm{d}x = \ln|x-1| - \ln|x| + \frac{1}{x} + C = \ln|\frac{x-1}{x}| + \frac{1}{x} + C$

$$\boxed{\ln|\frac{x-1}{x}| + \frac{1}{x} + C}$$

3. $\int x^2 \operatorname{arctg} x\,\mathrm{d}x = \begin{vmatrix} \operatorname{arctg} x & x^2 \\ \frac{1}{1+x^2} & \frac{x^3}{3} \end{vmatrix} = \frac{1}{3}x^3 \operatorname{arctg} x - \frac{1}{3}\int \frac{x^3}{1+x^2}\,\mathrm{d}x = \ldots - \frac{1}{3}\int \frac{x(x^2+1)-x}{1+x^2}\,\mathrm{d}x =$

$= \ldots - \frac{1}{3}\int x\,\mathrm{d}x + \frac{1}{3}\int \frac{x\,\mathrm{d}x}{1+x^2} = \ldots - \frac{1}{6}x^2 + \frac{1}{6}\int \frac{2x}{1+x^2}\,\mathrm{d}x = \ldots + \frac{1}{6}\ln(1+x^2) + C =$

$= \frac{1}{3}x^3 \operatorname{arctg} x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C$

$$\boxed{\frac{1}{3}x^3 \operatorname{arctg} x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C}$$

4. $P: y=2x^2, y=x^2, y=1$

Priesečník priamky $y=1$ a paraboly $y=2x^2$, je bod $\frac{\sqrt{2}}{2}$, podobne s parabolou $y=x^2$ je to $x=1$.

Paraboly sa pretínajú v bode $x=0$. T.j. máme 3 hraničné body, pričom na $\langle 0, \frac{\sqrt{2}}{2}\rangle$ dominuje $y=2x^2$ nad $y=x^2$ a na $\langle \frac{\sqrt{2}}{2}, 1\rangle$ dominuje $y=1$ nad $y=x^2$. Pre obsah plochy ohraničenej krivkami teda platí:

$P = \int_0^{\frac{\sqrt{2}}{2}} (2x^2 - x^2)\,\mathrm{d}x + \int_{\frac{\sqrt{2}}{2}}^1 (1-x^2)\,\mathrm{d}x = \Big[\frac{x^3}{3}\Big]_0^{\frac{\sqrt{2}}{2}} + \Big[x - \frac{x^3}{3}\Big]_{\frac{\sqrt{2}}{2}}^1 = \frac{\sqrt{2}}{12} + (1-\frac{1}{3}) - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{12}) =$

$= \frac{2-\sqrt{2}}{3}$

$$\boxed{\frac{2-\sqrt{2}}{3}}$$

5. $V_{o_x}: x=0, x=\frac{1}{2}, y=3\cos(\pi x)$

Hranice sú 0 a $\frac{1}{2}$. Objem sa rovná:

$V_{o_x} = \pi\int_0^{\frac{1}{2}} 9\cos^2(\pi x)\,\mathrm{d}x = 9\pi\int_0^{\frac{1}{2}} \frac{1+\cos(2\pi x)}{2}\,\mathrm{d}x = 9\pi\left[\frac{1}{2}x + \frac{\sin(2\pi x)}{2\cdot 2\pi}\right]_0^{\frac{1}{2}} = \frac{9}{4}\pi$

$$\boxed{\frac{9}{4}\pi}$$