

Náznaky riešení:

Pozn: 1. Pri hľadaní partikulárneho riešenia v úlohách stačí nájsť jedno z nich. Teda pri určovaní neurčitého integrálu sa nemusí uviesť obligátne $+C$.

Pozn: 2. Odstraňovanie absolútnych hodnôt popísané v riešeniach sa prevádza obmedzením sa na vhodné intervaly. V niektorých náznakoch riešenia sa rovno uvádzajú tvary bez absolútnych hodnôt, t.j. predpokladá sa, že tie boli už odstránené.

1. $10^x - 10^{-y}y' = 0$

$$y = \log_{10} \frac{1}{C-10^x}$$

2. $1 - 2x - y^2y' = 0$

$$y = \sqrt[3]{3x - 3x^2 + C}$$

3. $\frac{1}{x^2} + \frac{y'}{1+y^2} = 0$

$$y = \operatorname{tg}(C + \frac{1}{x})$$

4. $\frac{x}{\sqrt{1-x^2}} + \frac{yy'}{\sqrt{1-y^2}} = 0, y(0) = \frac{\sqrt{3}}{2}$

$$\sqrt{1-y^2} + \sqrt{1-x^2} = \frac{3}{2}, \text{ resp. } y = \sqrt{x^2 + 3\sqrt{1-x^2} - \frac{9}{4}}$$

5. $1 + y^2 + xyy' = 0$

$$y = \pm \sqrt{\frac{K-x^2}{x^2}}$$

6. $-1 + e^{-y}(1 + y') = 0$

$$y = \ln \frac{1}{1+Ke^x}$$

7. $e^{x+y} - y' = 0$

$$y = \ln \frac{1}{C-e^x}$$

8. $2y - x^3y' = 0$

$$y = Ke^{-\frac{1}{x^2}}$$

9. $(y-1)(y-2) - y' = 0$

$$y = \frac{Ke^x - 2}{Ke^x - 1}$$

10. $(1 + e^x)yy' = e^x$

$$y = \pm \sqrt{C + 2\ln(1 + e^x)}$$

11. $y'x^3 + xy = 0$

$$y = Ke^{\frac{1}{x}}, K \in \mathbb{R}$$

12. $\frac{x}{y+1} - \frac{yy'}{1+x} = 0, y(0) = 1$

$$3y^2 + 2y^3 = 3x^2 + 2x^3 + 5$$

13. $\frac{dy}{dx} = \operatorname{tg} y \cot g x, y(\frac{\pi}{2}) = \frac{\pi}{6}$

$$y = \arcsin(\frac{1}{2} \sin x)$$

14. $e^{x-y} - y' = 0, y(0) = 1$

$$y = \ln(e^x + e - 1)$$

15. $y' - y \operatorname{tg} x = 0$

16. $y' - y(x \sin x - \cos x)$

$$y = Ke^{-x \cos x}, K \in \mathbb{R}$$

17. $y' + \frac{1}{x^2}y = 0$

$$y = Ke^{\frac{1}{x}}, K \in \mathbb{R}$$

18. $y' + 3y = x$

$$y = \frac{1}{9}(3x - 1) + Ke^{-3x}, K \in \mathbb{R}$$

19. $x^2y' + xy = -1$

$$y = -\frac{\ln x}{x} + \frac{K}{x}, K \in \mathbb{R}$$

20. $xy' + y = x^3$

$$y = \frac{1}{4}x^3 + \frac{K}{x}, K \in \mathbb{R}$$

21. $y' + \frac{1}{x+1}y = \sin x$

$$y = -\cos x + \frac{\sin x}{x+1} + \frac{K}{x+1}, K \in \mathbb{R}$$

22. $(1 + x^2)y' + y = \operatorname{arctg} x$

$$y = \operatorname{arctg} x - 1 + Ke^{-\operatorname{arctg} x}$$

23. $y' \cos x + 2y \sin x = 2 \sin x$

$$y = 1 + K \cos^2 x$$

24. $x \ln(x)y' - 2y = \ln x$

$$y = C \ln^2 x - \ln x, C \in R, x \in R^+ \setminus \{0\}$$

25. $y' - xy = xe^{x^2}$

$$y = e^{x^2} + Ke^{\frac{x^2}{2}}, K \in R$$

26. $y' - \frac{xy}{1-x^2} = \arcsin x + x$

$$y = \frac{K}{\sqrt{1-x^2}} + \frac{1}{4} \frac{\arcsin^2 x}{\sqrt{1-x^2}} + \frac{1}{2} x \arcsin x + \frac{1}{4} \sqrt{1-x^2} - \frac{1}{8} \frac{1}{\sqrt{1-x^2}} - \frac{1}{3} (1-x^2), K \in R$$

27. $xy' - y = x^2 \cos x$

$$y = x \sin x + Kx, K \in R$$

28. $y' + 2xy = xe^{-x^2}$

$$y = \frac{1}{2} x^2 e^{-x^2} + Ke^{-x^2}, K \in R$$

29. $y' + x^2 y = x^2, y(2) = 1$

$$y = 1$$

30. $y' + y = \cos x, y(0) = 1$

$$y = \frac{1}{2} (e^{-x} + \cos x + \sin x)$$

31. $y' + \frac{n}{x} y = \frac{a}{x^n}, n = 2, 3, \dots, a > 0, y(1) = 0$

$$y = \frac{a}{x^n} (1-x)$$

32. $y' + y \cotg x = \sin x, y(\frac{\pi}{2}) = 1$

$$y = \frac{1}{2} \cos x - \frac{1}{2} \frac{x}{\sin x} + \frac{3}{4} \frac{\pi}{\sin x}$$

33. $y' \sqrt{1-x^2} + y = \arcsin x, y(0) = 0$

34. $y' - \frac{y}{x \ln x} = x \ln x, y(e) = \frac{e^2}{2}$

$$y = \frac{1}{2} x^2 \ln x$$

35. $y' \sin x - y \cos x = 1, y(\frac{\pi}{2}) = 0$

$$y = -\cos x$$