

1.  $\lim_{(x,y) \rightarrow (0,0)} (1+xy)^{|x|+|y|}$

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2.  $f(x, y) = \frac{x+y}{x^2+y}, a = \infty, b = \infty$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y = 1, \lim_y \lim_x = 0$

3.  $f(x, y) = \frac{y^x}{1+y^{2x}}, a = 0, b = \infty$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y = 0, \lim_y \lim_x = \frac{1}{2}$

4.  $f(x, y) = \frac{\sin(\pi x)}{2x+y}, a = \infty, b = \infty$

$\lim_{xy} = 0, \lim_x \lim_y = 0, \lim_y \lim_x = 0$

5.  $f(x, y) = \frac{x+y}{x-y}, a = 0, b = 0$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y = 1, \lim_y \lim_x = -1$

6.  $f(x, y) = \frac{x^2+y^2}{1+(x-y)^4}, a = \infty, b = \infty$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y = 0, \lim_y \lim_x = 0$

7.  $f(x, y) = (x+y) \operatorname{tg} \frac{1}{x} \operatorname{tg} \frac{1}{y}, a = 0, b = 0$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y \text{ neexistuje, } \lim_y \lim_x \text{ neexistuje}$

8.  $f(x, y) = \begin{cases} 4-x-y & x \neq 2, y \neq 1 \\ 3 & x = 2, y = 1 \end{cases}, a = 2, b = 1$

$\lim_{xy} = 1, \lim_x \lim_y = 1, \lim_y \lim_x = 1$

9.  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}, a = 0, b = 0$

$\lim_{xy} \text{ neexistuje, } \lim_x \lim_y = 0, \lim_y \lim_x = 0$

Určte body nespojitosti:

10.  $f(x, y) = \sin \frac{1}{x-y}$

$D(f) = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$ , spojitá na  $D(f)$

11.  $f(x, y) = \frac{x^2+3y^2+5}{y^2-2x}$

$D(f) = \{(x, y) \in \mathbb{R}^2 : y^2 \neq 2x\}$ , spojitá na  $D(f)$

12.  $f(x, y) = \begin{cases} \cos \frac{1}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

$D(f) = \mathbb{R}^2$ , spojitá na  $\mathbb{R}^2 - \{(0, 0)\}$

13.  $z = \frac{x+y}{x-y}$

$D(f) = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$ , spojitá na  $D(f)$

14.  $z = \sin \frac{1}{|x|-|y|}$

$D(f) = \{(x, y) : |x| \neq |y|\}$ , spojitá na  $D(f)$

15.  $z = \ln |1-x^2-y^2|$

$D(f) = \{(x, y) \in \mathbb{R}^2 : x^2+y^2 \neq 1\}$ , spojitá na  $D(f)$

16.  $f(x, y, z) = \frac{3z}{x-2y+3z}$

$D(f) = \{(x, y, z) \in \mathbb{R}^3 : x-2y+3z \neq 0\}$ , spojitá na  $D(f)$

17.  $f(x, y) = x^2 - 2xy - 3y^2, A = (-1, 1)$

$Df(\mathbf{x}, A) = -4x - 4y$

18.  $f(x, y) = e^{xy}, A = (0, 0)$

$df(\mathbf{x}, A) = 0$

19.  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}, A = (0, 0)$

$df(\mathbf{x}, A) = 0$

20.  $f(x, y, z) = x^5 y^3 z^3$

$df(\mathbf{x}, \cdot) = 5x^4 y^3 z^3 dx + 3x^5 y^2 z^3 dy + 3x^5 y^3 z^2 dz$

21.  $f(x, y) = \ln \sqrt{x^2+y^2}$

$df(\mathbf{x}, \cdot) = \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy$

22.  $f(x, y) = e^{\alpha x} \cos\left(\frac{\beta y}{x}\right)$

$df(\mathbf{x}, \cdot) = e^{\alpha x} \left( \alpha \cos\left(\frac{\beta y}{x}\right) + \beta \frac{y}{x^2} \sin\left(\frac{\beta y}{x}\right) \right) dx - e^{\alpha x} \cdot \frac{\beta}{x} \cdot \sin\left(\frac{\beta y}{x}\right) dy$

23.  $f(x, y, z) = \operatorname{arctg}\left(\frac{yz}{x^2}\right)$

$df(\mathbf{x}, \cdot) = -2 \frac{x}{yz} dx + \frac{y^2}{y^2 z} dy + \frac{x^2}{yz^2} dz$

24.  $\sqrt{3,03^2 + 9,01^2}$

$\frac{30,06}{\sqrt{10}} \doteq 9,5058066 - 9,5058403$

25.  $1,05^{2,01}$

$1,1 - 1,103038$

26.  $\sin 151^\circ \cotg 41^\circ$

$\frac{1}{2} + \frac{8-\sqrt{3}}{360}\pi \doteq 0,554698 - 0,5577097$

27.  $\ln(\sqrt{0,96} + \sqrt[3]{1,02} + 2)$

$2 \ln 2 + \frac{1}{600} - 0,005 \doteq 1,38296 - 1,38279$

28.  $f(x, y) = e^{xy}, A = (0, 0)$

$d^2 f(\mathbf{x}, A) = 2xy$

29.  $f(x, y, z) = xy + yz + xz, A = (1, 1, 0)$

$d^2 f(\mathbf{x}, A) = 2(xy + xz + yz - x - y - 2z + 1)$

30.  $f(x, y, z) = \frac{z}{x^2+y^2}, A = (1, 1 - 2)$

$d^2 f(\mathbf{x}, A) = -x^2 - y^2 + 4xy - xz - yz - 4x - 4y + 2z + 6$

31.  $f(x, y) = x^2 y^2, A = (1, 1)$

$d^2 f(\mathbf{x}, A) = 2x^2 + 8xy + 2y^2 - 12x - 12y + 12$

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32.  $f(x, y, z) = xyz, A = (7, 11, -10)$

$d^3 f(\mathbf{x}, A) = 6(x - 7)(y - 11)(z + 10)$

33.  $f(x, y) = \cos(x^2 + y^2), n = 5$

$T(f, 5, (0, 0)) = 1 - \frac{1}{2}(x^2 + y^2)^2$

34.  $f(x, y) = e^x \sin y, n = 3$

$T(f, 3, (0, 0)) = y + xy + \frac{1}{2}x^2 y - \frac{1}{3!}y^3$

35.  $f(x, y) = x^3 + y^3 - 2xy, n = 3$

$T(f, 3, (0, 0)) = x^3 + y^3 - 2xy$

36.  $f(x, y) = x^y, A = (1, 1), n = 2$

$T(f, 2, A) = 1 + (x - 1)^2 + (x - 1)(y - 1)$

37.  $f(x, y) = \sin x \cos y, A = (\frac{\pi}{4}, \frac{\pi}{4}), n = 3$

$T(f, 3, A) = \frac{1}{2} + \frac{1}{2}(x - \frac{\pi}{4}) - \frac{1}{2}(y - \frac{\pi}{4}) - \frac{1}{4}(x - \frac{\pi}{4})^2 - \frac{1}{2}(x - \frac{\pi}{4})(y - \frac{\pi}{4}) - \frac{1}{4}(y - \frac{\pi}{4})^2 - \frac{1}{12}(x - \frac{\pi}{4})^3 + \frac{1}{4}(x - \frac{\pi}{4})^2(y - \frac{\pi}{4}) - \frac{1}{4}(x - \frac{\pi}{4})(y - \frac{\pi}{4})^2 + \frac{1}{12}(y - \frac{\pi}{4})^3$

38.  $f(x, y) = 1 + 6x - y^2 - xy - x^2$

$(4, -2)_{max}$

39.  $f(z, t) = 5 + 6z - 4z^2 - 3t^2$

$(\frac{4}{3}, 0)_{max}$

40.  $f(x, y) = x^3 + y^3 - 18xy + 215$

$(0, 0)_{sedlo}, (6, 6)_{min}$

41.  $f(x, y) = \sqrt{(a-x)(a-y)(x+y-a)}$

$(\frac{2}{3}a, \frac{2}{3}a)_{max}$

42.  $f(x, y, z) = \frac{x}{y+z} + \frac{y}{x+z} + \frac{z}{x+y}$

$(t, t, t)_{min}, t \neq 0$

43.  $z = xy - x + y - 1, \text{ ak } x + y = 1$

$(-\frac{1}{2}, \frac{3}{2})_{max}$

44.  $z = x + y, \text{ ak } \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}, a > 0$

$(-\sqrt{2}a, -\sqrt{2}a)_{max}, (\sqrt{2}a, \sqrt{2}a)_{min}$

45.  $z = x^2 + y^2, \text{ ak } \frac{x}{p} + \frac{y}{q} = 1$

$(\frac{pq^2}{p^2+q^2}, \frac{p^2q}{p^2+q^2})_{min}$

46.  $u = \cos x \cos y \cos z, \text{ ak } x + y + z = -\pi$

Maximá:  $(-\frac{2\pi}{3} + k\pi, -\frac{2\pi}{3} + l\pi, -(k+l - \frac{1}{3})\pi), (\frac{2\pi}{3} + k\pi, \frac{2\pi}{3} + l\pi, -(k+l + \frac{7}{3})\pi),$   
 $(\frac{2\pi}{3} + 2k\pi, -\frac{\pi}{3} + 2l\pi, -(2k+2l + \frac{4}{3})\pi), (-\frac{2\pi}{3} + 2k\pi, \frac{\pi}{3} + 2l\pi, -(2k+2l + \frac{2}{3})\pi),$   
 $(-\frac{\pi}{3} + 2l\pi, \frac{2\pi}{3} + 2k\pi, -(2k+2l + \frac{2}{3})\pi), (\frac{\pi}{3} + 2l\pi, -\frac{2\pi}{3} + 2k\pi, -(2k+2l + \frac{4}{3})\pi), k, l \in Z$   
 Minimá:  $(k\pi, l\pi, -(1+k+l)\pi), k, l \in Z$

47.  $u = xyz$ , ak  $x^2 + y^2 + z^2 = 3$  Maximá:  $(1, 1, 1)$ ,  $(1, -1, -1)$ ,  $(-1, 1, -1)$ ,  $(-1, -1, 1)$ , hodnota 1  
Minimá:  $(-1, -1, -1)$ ,  $(-1, 1, 1)$ ,  $(1, -1, 1)$ ,  $(1, 1, -1)$ , hodnota  $-1$
48.  $u = x^2 + y^2 + z^2$ , ak  $x + y - 3z + 7 = 0$ ,  $x - y + z - 3 = 0$   $(0, -1, 2)_{min}$
49.  $f(x, y) = x^2 - 2y^2 + 4xy - 6x - 1$  na oblasti  $x \geq 0$ ,  $y \geq 0$ ,  $y \leq -x + 3$   $(0, 3)_{min}, f = -19$ ,  $(0, 0)_{max}, f = -1$
50.  $f(x, y) = xy^2(4 - x - y)$  na oblasti ohraničenej krivkami  $x = 0$ ,  $y = 0$ ,  $x + y = 6$   $(1, 2)_{max}, f(1, 2) = 4$ ;  $(2, 4)_{max}, f(2, 4) = -64$
51.  $f(x, y) = x^3 + y^3 - 3xy$  na obdĺžniku s vrcholmi  $A = (0, -1)$ ,  $B = (2, -1)$ ,  $C = (2, 2)$ ,  $D = (0, 2)$  Minimum:  $(0, -1)$ ,  $(1, 1)$ , hodnota  $-1$ ; Maximum:  $(2, -1)$ , hodnota  $13$
52.  $f(x, y) = \cos x \cos y \cos(x + y)$  na štvorci  $A = (0, 0)$ ,  $B = (\pi, 0)$ ,  $C = (\pi, \pi)$ ,  $D = (0, \pi)$  Minimum:  $(\frac{\pi}{3}, \frac{\pi}{3})$ , hodnota  $-\frac{1}{8}$ , Maximum:  $(0, 0)$ ,  $(\pi, 0)$ ,  $(\pi, \pi)$ ,  $(0, \pi)$ , hodnota  $1$
53.  $f(x, y) = x^2 + y^2$ , na kruhu  $x^2 + y^2 \leq 4$   $\{(x, y) : x^2 + y^2 = 4\}_{max}$ ,  $(0, 0)_{min}$
54.  $f(x, y, z) = x + y + z$ , na oblasti  $1 \geq x \geq y^2 + z^2$  Minimum:  $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$ ,  $f = -\frac{1}{2}$ , Maximum:  $(1, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ ,  $f = 1 + \sqrt{2}$