Model schemes

Note: ① means the candidate set must contain the element 1, \bigcirc means any non-empty set not containing the element 1. The field containing the symbol – means that result of the $+_h$ operation is not defined. Red elements are superfluous due to commutativity, i.e. it suffices to generate just black results. Since $0 +_h x$ is defined for all elements and $x \in 0 +_h x$, we denote any set containing x by x.

For each n we assume that elements of the hyper effect algebra are denoted as $\{0, a, b, \ldots, 1\}$.

n = 1

$+_h$	0
0	{0}

n = 2

$+_h$	0	1
0	{0}	1
1	1	-

n = 3

$+_h$	0	a	1
0	{0}	a	1
а	(a)	1	-
1	1	-	-

n = 4

Relation no. 1:

	0	a	b	1
0	1	1	1	1
а		1		1
b			1	1
1				1

Model scheme no. 1, a' = a, b' = b:

$+_h$	0	a	b	1
0	{0}	a		
a	(a)	1	-	-
b		-	1	-
1	1	-	-	-

Model scheme no. 2, a' = b, b' = a:

$+_h$	0	a	b	1
0	{0}	(a)		1
а	(a)	-	1	-
b	b	1	-	-
1	1	-	-	-

Relation no. 2:

	0	a	b	1
0	1	1	1	1
а		1	1	1
b			1	1
1				1

Model scheme no. 3, a' = b, b' = a:

$+_h$	0	a	b	1
0	{0}	(a)	(b)	1
a	(a)	0	1	-
b		1	-	-
1	1	-	-	-

n = 5

Relation no. 1:

\leq	0	a	b	с	1
0	1	1	1	1	1
a		1			1
b			1		1
С				1	1
1					1

Model scheme no. 1, a' = a, b' = b, c' = c:

$+_h$	0	a	b	с	1
0	{0}	(a)		\bigcirc	1
a	(a)	1	-	-	-
b	${}$	-	1	-	-
С	\bigcirc	-	-	1	-
1	1	-	-	-	-

Model scheme no. 2, a' = a, b' = c, c' = b:

$+_h$ 0	0	a	b	С	1
0	{0}	(a)		\bigcirc	1
а	(a)	1	-	-	-
b	b	-	-	1	-
С	\bigcirc	-		-	-
1	1	-	-	-	-

Relation no. 2:

\leq	0	a	b	с	1
0	1	1	1	1	1
a		1			1
b			1	1	1
с				1	1
1					1

Model scheme no. 3, a' = a, b' = c, c' = b:

$+_h$	0	a	b	с	1
0	{0}	(a)		\bigcirc	1
a	(a)	1	-	-	-
b		-	0	1	-
С	\bigcirc	-	1	-	-
1	1	-	-	-	-

Relation no. 3:

\leq	0	а	b	с	1
0	1	1	1	1	1
а		1		1	1
b			1		1
С			1	1	1
1					1

Model scheme no. 4, a' = b, b' = a, c' = c:

$+_h$	0	a	b	С	1
0	{0}	(a)		\bigcirc	1
a	(a)	-	1	\bigcirc	-
b	b	1	-	-	-
С	\bigcirc	\bigcirc	-	1	-
1	1	-	-	-	-

Relation no. 4:

\leq	0	a	b	с	1
0	1	1	1	1	1
a		1	1	1	1
b			1	1	1
С				1	1
1					1

Model scheme no. 5, a' = c, b' = b, c' = a:

$+_h$	0	а	b	с	1
0	{0}	(a)		\bigcirc	1
a	(a)	\bigcirc	0	1	-
b	b	\bigcirc	1	-	-
С	\bigcirc	1	-	-	-
1	1	-	-	-	-

Relation no. 5:

\leq	0	a	b	с	1
0	1	1	1	1	1
а		1	1		1
b			1	1	1
с		1		1	1
1					1

Model scheme no. 6, a' = a, b' = c, c' = b:

$+_h$	0	a	b	с	1
0	{0}	(a)		\bigcirc	1
a	(a)	1	-	0	-
b	b	-	0	1	-
С	\bigcirc	0	1	-	-
1	1	-	-	-	-

n = 6

Relation no. 1:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1				1
b			1			1
С				1		1
d					1	1
1						1

Model scheme no. 1, a' = a, b' = b, c' = c, d' = d:

$+_h$	0	a	b	С	d	1
0	{0}	(a)		\bigcirc	\mathcal{Q}	1
a b	(a)	1				-
b	b		1			-
С	\bigcirc			1		-
d	d				1	-
1	1	-	-	-	-	-

Model scheme no. 2, $a^\prime=a,\,b^\prime=b,\,c^\prime=d,\,d^\prime=c:$

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	d	1
a	a	1				-
b	b		1			-
С	\bigcirc				1	-
d	d			1		-
1	1	-	-	-	-	-

Model scheme no. 3, a' = b, b' = a, c' = d, d' = c:

	0	0	b	с	d	1
$+_h$	-	a				T
0	{0}	(a)		\bigcirc	(d)	1
a	(a)		1			-
b		1				-
с	\bigcirc				1	-
d	d			1		-
1	1	-	-	-	-	-

Relation no. 2:

\leq	0	а	b	С	d	1
0	1	1	1	1	1	1
а		1				1
b			1			1
С				1	1	1
d					1	1
1						1

Model scheme no. 4, a' = a, b' = b, c' = d, d' = c:

$+_h$	0	a	b	С	d	1
0	{0}	(a)		\bigcirc	d	1
a	(a)	1				-
b			1			-
С	\bigcirc			0	1	-
d	d			1		-
1	1	-	-	-	-	-

Model scheme no. 5, a' = b, b' = a, c' = d, d' = c:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	(a)		1			-
b	b					-
С	\bigcirc			0	1	-
d	(d)			1		-
1	1	-	-	-	-	-

Relation no. 3:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1				1
b			1		1	1
С				1		1
d				1	1	1
1						1

Model scheme no. 6, $a^\prime=a,\,b^\prime=c,\,c^\prime=b,\,d^\prime=d$:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	d	1
a	(a)	1				-
b	b			1	0	-
С	\bigcirc		1			-
d	d		0		1	-
1	1	-	-	-	-	-

Relation no. 4:

\leq	0	а	b	с	d	1
0	1	1	1	1	1	1
а		1				1
b			1	1	1	1
с				1	1	1
d					1	1
1						1

Model scheme no. 7, a' = a, b' = d, c' = c, d' = b:

$+_h$	0	a	b	С	d	1
0	{0}	(a)		\bigcirc	\mathcal{Q}	1
a	a	1				-
b	b		0	0	1	-
С	\bigcirc		0	1		-
d	d		1			-
1	1	-	-	-	-	-

Relation no. 5:

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1
a 1	
	T
b 1 1	1
c 1 1	1
d 1 1	1
1	1

Model scheme no. 8, $a^\prime=a,\,b^\prime=b,\,c^\prime=d,\,d^\prime=c:$

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	(a)	1				-
b			1		0	-
С	\bigcirc			0	1	-
d	d		\bigcirc	1		-
1	1	-	-	-	-	-

Relation no. 6:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1			1	1
b			1	1		1
С				1		1
d					1	1
1						1

Model scheme no. 9, a' = c, b' = d, c' = a, d' = b:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	d	
a	(a)		0	1		-
b		0			1	-
С	\bigcirc	1				-
d	d		1			-
1	1	-	-	-	-	-

Model scheme no. 10, a' = d, b' = c, c' = b, d' = a:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	d	
a	(a)	0			1	-
b	b		0	1		-
С	\bigcirc		1			-
d	d					-
1	1	-	-	-	-	-

Relation no. 7:

\leq	0	а	b	с	d	1
0	1	1	1	1	1	1
а		1			1	1
b			1	1	1	1
с				1		1
d					1	1
1						1

Model scheme no. 11, a' = c, b' = d, c' = a, d' = b:

$+_h$	0	a	b	С	d	1
0	{0}	(a)		\bigcirc	d	1
a	a		0	1		-
b		0	0		1	-
С	\bigcirc	1				-
d	d		1			-
1	1	-	-	-	-	-

Relation no. 8:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1			1	1
b			1	1		1
С				1		1
d			1		1	1
1						1

Model scheme no. 12, a' = c, b' = d, c' = a, d' = b:

$+_h$	0	а	b	с	d	1
0	{0}	(a)		\bigcirc	(d)	1
a	a		0	1		-
b	b	0			1	-
С	\bigcirc	1				-
d	d		1		\bigcirc	-
1	1	-	-	-	-	-

Relation no. 9:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1	1	1
b			1	1	1	1
с				1		1
d					1	1
1						1

Model scheme no. 13, a' = c, b' = d, c' = a, d' = b:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	d	1
a	(a)	0	0	1		-
b		0	0		1	-
С	\bigcirc	1				-
d	(d)		1			-
1	1	-	-	-	-	-

Relation no. 10:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1	1	1
b			1	1		1
с				1		1
d			1		1	1
1						1

Model scheme no. 14, $a^\prime=c,\,b^\prime=d,\,c^\prime=a,\,d^\prime=b$:

$+_h$	0	а	b	с	d	1
0	{0}	(a)		\bigcirc	\mathcal{Q}	1
a	(a)	0	0	1		-
b	b	0			1	-
С	\bigcirc	1				-
d	d		1		0	-
1	1	-	-	-	-	-

Relation no. 11:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1	1	1
b			1			1
с			1	1		1
d			1		1	1
1						1

Model scheme no. 15, a' = b, b' = a, c' = d, d' = c:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	d	1
a	(a)		1	0	\bigcirc	-
b		1				-
С	\bigcirc	\bigcirc		1		-
d	d	0			1	-
1	1	-	-	-	-	-

Model scheme no. 16, a' = b, b' = a, c' = d, d' = c:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	d	1
a	(a)		1	0	0	-
b	b					-
С	\bigcirc	0			1	-
d	d	0		1		-
1	1	-	-	-	-	-

Relation no. 12:

\leq	0	а	b	с	d	1
0	1	1	1	1	1	1
а		1		1	1	1
b			1			1
С			1	1	1	1
d			1		1	1
1						1

Model scheme no. 17, $a^\prime=b,\,b^\prime=a,\,c^\prime=d,\,d^\prime=c:$

$+_h$	0	a	b	с	d	1
0	{0}	(a)		\bigcirc	(d)	1
a	(a)		1	0	0	-
b		1				-
с	\bigcirc	0		0	1	-
d	d	0		1		-
1	1	-	-	-	-	-

Relation no. 13:

≤ 0	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1		1
b			1	1		1
с				1	1	1
\mathbf{d}		1	1		1	1
1						1

Model scheme no. 18, a' = a, b' = b, c' = d, d' = c:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	(a)	1			\bigcirc	-
b			1		\bigcirc	-
С	\bigcirc			\bigcirc	1	-
d	d	0	0	1		-
1	1	-	-	-	-	-

Model scheme no. 19, a' = b, b' = a, c' = d, d' = c:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	d	1
a	(a)		1		0	-
b	b				\bigcirc	-
С	\bigcirc			0	1	-
d	(d)	0	0			-
1	1	-	-	-	-	-

Relation no. 14:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1		1
b			1	1		1
С				1		1
d		1	1	1	1	1
1						1

Model scheme no. 20, $a^\prime=a,\,b^\prime=b,\,c^\prime=d,\,d^\prime=c:$

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	(d)	1
a	(a)	1			0	-
b			1		0	-
с	\bigcirc				1	-
d	d	0	0		0	-
1	1	-	-	-	-	-

Model scheme no. 21, a' = b, b' = a, c' = d, d' = c:

$+_h$	0	а	b	С	d	1
0	{0}	(a)		\bigcirc	d	1
a	(a)		1		\bigcirc	-
b		1			\bigcirc	-
С	\bigcirc				1	-
d	d	0	0	1	0	-
1	1	-	-	-	-	-

Relation no. 15:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
a		1		1		1
b			1		1	1
с			1	1		1
d		1			1	1
1						1

Model scheme no. 22, a' = a, b' = b, c' = d, d' = c:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	(a)	1			0	-
b			1	0		-
С	\bigcirc		0		1	-
d	d	0		1		-
1	1	-	-	-	-	-

Model scheme no. 23, a' = c, b' = d, c' = a, d' = b:

$+_h$	0	a	b	С	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	a	0		1		-
b	b		0		1	-
С	\bigcirc	1			0	-
d	d		1	0		-
1	1	-	-	-	-	-

Relation no. 16:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1		1		1
b			1		1	1
С			1	1	1	1
d		1			1	1
1						1

Model scheme no. 24, a' = a, b' = b, c' = d, d' = c:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	d	1
a	a	1			\bigcirc	-
b	b		1	0		-
с	\bigcirc		0	0	1	-
d	d	0		1		-
1	1	-	-	-	-	-

Relation no. 17:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1	1	1	1	1
b			1	1	1	1
с				1	1	1
d					1	1
1						1

Model scheme no. 25, a' = d, b' = c, c' = b, d' = a:

$+_h$	0	a	b	с	d	1
0	{0}	a		\bigcirc	\mathcal{Q}	1
a	(a)	0	\bigcirc	0	1	-
b	b	0	0	1		-
С	\bigcirc	0	1			-
d	d					-
1	1	-	-	-	-	-

Relation no. 18:

\leq	0	a	b	с	d	1
0	1	1	1	1	1	1
а		1	1	1		1
b			1	1	1	1
с				1	1	1
d		1			1	1
1						1

Model scheme no. 26, a' = d, b' = c, c' = b, d' = a:

$ +_h $ 0	0	a	b	с	d	1
0	{0}	a		\bigcirc	(d)	1
a	(a)		0	0	1	-
b		0	0	1		-
С	\bigcirc	0	1			-
d	(d)				0	-
1	1	-	-	-	-	-