

Next we summarize the speed-ups for all model schemes no. 1 to no. 26 (denoted as M_{01} – M_{26}) for $n = 6$. All listed restrictions for outcomes follow from the application of partial associativity.

$$M_{01}–M_{03}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{01}: 1 \in a +_h a, 1 \in b +_h b, 1 \in c +_h c, 1 \in d +_h d;$$

$$M_{02}: 1 \in a +_h a, 1 \in b +_h b, 1 \in c +_h d;$$

$$M_{03}: 1 \in a +_h b, 1 \in c +_h d.$$

$$M_{04}–M_{05}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, c, d\}.$$

$$M_{04}: 1 \in a +_h a, 1 \in b +_h b, c +_h c, 1 \in c +_h d;$$

$$c +_h c \not\subseteq \{0, a\}, \{0, b\};$$

$$M_{05}: 1 \in a +_h b, c +_h c, 1 \in c +_h d;$$

$$c +_h c \not\subseteq \{0, a\}, \{0, b\}.$$

$$M_{06}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c, d\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{06}: 1 \in a +_h a, 1 \in b +_h c, b +_h d, 1 \in d +_h d;$$

$$b +_h d \not\subseteq \{0, a\}, \{0, c, d\}, \{0, b\}, \{0, b, d\}.$$

$$M_{07}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, b, c\}, d +_h 0 \subseteq \{0, b, c, d\}.$$

$$M_{07}: 1 \in a +_h a, b +_h b, b +_h c, 1 \in b +_h d, 1 \in c +_h c;$$

$$b +_h b \not\subseteq \{0, a\};$$

$$b +_h c \not\subseteq \{0, a\}, \{0, b, c\}, \{0, b\}.$$

$$M_{08}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{08}: 1 \in a +_h a, 1 \in b +_h b, b +_h d, c +_h c, 1 \in c +_h d;$$

$$b +_h d \not\subseteq \{0, a\}, \{0, b, d\}, \{0, c, d\}, \{0, b, c\};$$

$$c +_h c \not\subseteq \{0, a\}, \{0, b, d\}, \{0, b, c\}.$$

$$M_{09}–M_{10}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, b, c\}, d +_h 0 \subseteq \{0, a, d\}.$$

$$M_{09}: a +_h b, 1 \in a +_h c, 1 \in b +_h d;$$

$$a +_h b \not\subseteq \{0, b, c\}, \{0, a, d\}, \{0, a\}, \{0, b\}.$$

$$M_{10}: a +_h a, 1 \in a +_h d, b +_h b, 1 \in b +_h c.$$

$$a +_h a \not\subseteq \{0, b, c\}, \{0, b\}, \{0, a\};$$

$$b +_h b \not\subseteq \{0, a, d\}, \{0, b\}, \{0, a\}.$$

$$M_{11}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, b, c\}, d +_h 0 \subseteq \{0, a, b, d\}.$$

$$M_{11}: a +_h b, 1 \in b +_h c, b +_h b, 1 \in b +_h d;$$

$$a +_h b \not\subseteq \{0, b, c\}, \{0, a\}, \{0, b\};$$

$$b +_h b \not\subseteq \{0, a\}, \{0, b\}.$$

$$M_{12}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, b, c\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{12}: a +_h b, 1 \in a +_h c, 1 \in b +_h d, d +_h d;$$

$$a +_h b \not\subseteq \{0, b, c\}, \{0, a, d\}, \{0, a\}, \{0, b, d\};$$

$$d +_h d \not\subseteq \{0, b, c\}, \{0, a\}, \{0, a, d\}.$$

$$M_{13}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, a, b, c\}, d +_h 0 \subseteq \{0, a, b, d\}.$$

$$M_{13}: a +_h a, a +_h b, 1 \in a +_h c, b +_h b, 1 \in b +_h d;$$

$$a +_h a \not\subseteq \{0, a\}, \{0, b\};$$

$$a +_h b \not\subseteq \{0, a\}, \{0, b\};$$

$$b +_h b \not\subseteq \{0, a\}, \{0, b\}.$$

$$M_{14}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, a, b, c\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{14}: a +_h a, a +_h b, 1 \in a +_h c, 1 \in b +_h d, d +_h d;$$

$$a +_h a \not\subseteq \{0, b, d\}, \{0, a\};$$

$$a +_h b \not\subseteq \{0, b, d\}, \{0, a\}, \{0, a, d\};$$

$$d +_h d \not\subseteq \{0, a, b, c\}, \{0, a, d\}, \{0, a\}.$$

$$M_{15}–M_{16}: a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b, c, d\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$$

$$M_{15}: 1 \in a +_h b, a +_h c, a +_h d, 1 \in c +_h c, 1 \in d +_h d;$$

$$\begin{aligned} a +_h c &\not\subseteq \{0, b, c, d\}, \{0, a\}, \{0, a, d\}, \{0, a, c\}; \\ a +_h d &\not\subseteq \{0, b, c, d\}, \{0, a\}, \{0, a, d\}, \{0, a, c\}; \\ d +_h d &\not\subseteq \{0, a, b, c\}, \{0, a, d\}, \{0, a\}. \end{aligned}$$

M_{16} : $1 \in a +_h b, a +_h c, a +_h d, 1 \in c +_h d.$

$$\begin{aligned} a +_h c &\not\subseteq \{0, b, c, d\}, \{0, a\}, \{0, a, d\}, \{0, a, c\}; \\ a +_h d &\not\subseteq \{0, b, c, d\}, \{0, a\}, \{0, a, d\}, \{0, a, c\}. \end{aligned}$$

M_{17} : $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b, c, d\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, c, d\}.$

M_{17} : $1 \in a +_h b, a +_h c, a +_h d, c +_h c, 1 \in c +_h d.$

$$\begin{aligned} a +_h c &\not\subseteq \{0, b, c, d\}, \{0, a\}; \\ a +_h d &\not\subseteq \{0, b, c, d\}, \{0, b\}, \{0, a, c\}, \{0, a, c, d\}; \\ c +_h c &\not\subseteq \{0, a\}. \end{aligned}$$

$M_{18}-M_{19}$: $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$

M_{18} : $1 \in a +_h a, a +_h d, 1 \in b +_h b, b +_h d, c +_h c, 1 \in c +_h d.$

$$\begin{aligned} a +_h d &\not\subseteq \{0, a, d\}, \{0, b, d\}, \{0, c, d\}, \{0, a, b, c\}; \\ b +_h d &\not\subseteq \{0, a, d\}, \{0, b, d\}, \{0, c, d\}, \{0, a, b, c\}; \\ c +_h c &\not\subseteq \{0, a, d\}, \{0, b, d\}, \{0, a, b, c\}. \end{aligned}$$

M_{19} : $1 \in a +_h b, a +_h d, b +_h d, c +_h c, 1 \in c +_h d.$

$$\begin{aligned} a +_h d &\not\subseteq \{0, b, d\}, \{0, a, d\}, \{0, c, d\}, \{0, a, b, c\}; \\ b +_h d &\not\subseteq \{0, b, d\}, \{0, a, d\}, \{0, c, d\}, \{0, a, b, c\}; \\ c +_h c &\not\subseteq \{0, b, d\}, \{0, a, d\}, \{0, a, b, c\}. \end{aligned}$$

$M_{22}-M_{23}$: $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$

M_{22} : $1 \in a +_h a, a +_h d, 1 \in b +_h b, b +_h d, b +_h c.$

$$\begin{aligned} a +_h d &\not\subseteq \{0, a, d\}, \{0, b, c\}, \{0, b, d\}, \{0, a, c\}; \\ b +_h c &\not\subseteq \{0, a, d\}, \{0, b, d\}, \{0, c, d\}, \{0, a, b, c\}. \end{aligned}$$

M_{23} : $a +_h a, 1 \in a +_h c, b +_h b, 1 \in b +_h d, c +_h d.$

$$\begin{aligned} a +_h a &\not\subseteq \{0, b, d\}, \{0, a, d\}, \{0, b, c\}; \\ b +_h b &\not\subseteq \{0, a, c\}, \{0, a, d\}, \{0, b, c\}; \\ c +_h d &\not\subseteq \{0, a, c\}, \{0, b, d\}, \{0, a, d\}, \{0, b, c\}. \end{aligned}$$

M_{24} : $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b, c\}, c +_h 0 \subseteq \{0, c\}, d +_h 0 \subseteq \{0, d\}.$

M_{24} : $1 \in a +_h a, a +_h d, 1 \in b +_h b, b +_h c, c +_h c, 1 \in c +_h d;$

$$\begin{aligned} a +_h d &\not\subseteq \{0, a, d\}, \{0, b, c, d\}, \{0, a, c\}; \\ b +_h c &\not\subseteq \{0, a, d\}, \{0, b, c\}, \{0, a, c\}; \\ c +_h c &\not\subseteq \{0, a, d\}, \{0, a, c\}. \end{aligned}$$

M_{25} : $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, a, b\}, c +_h 0 \subseteq \{0, a, b, c\}, d +_h 0 \subseteq \{0, a, b, c, d\}.$

M_{25} : $a +_h a, a +_h b, a +_h c, 1 \in a +_h d, b +_h b, 1 \in b +_h c.$

$$\begin{aligned} a +_h a &\not\subseteq \{0, a\}; \\ a +_h b &\not\subseteq \{0, a\}, \{0, a, b\}; \\ a +_h c &\not\subseteq \{0, a\}, \{0, a, b\}, \{0, a, b, c\}; \\ b +_h b &\not\subseteq \{0, a\}, \{0, a, b\}. \end{aligned}$$

M_{26} : $a +_h 0 \subseteq \{0, a\}, b +_h 0 \subseteq \{0, b\}, c +_h 0 \subseteq \{0, b, c\}, d +_h 0 \subseteq \{0, d\}.$

M_{26} : $a +_h b, a +_h c, 1 \in a +_h d, b +_h b, 1 \in b +_h c, d +_h d.$

$$\begin{aligned} a +_h b &\not\subseteq \{0, b, c, d\}, \{0, a, b\}, \{0, a, d\}; \\ a +_h c &\not\subseteq \{0, b, c, d\}, \{0, a, b, c\}, \{0, a, b\}, \{0, a, d\}; \\ b +_h b &\not\subseteq \{0, a, b\}, \{0, a, d\}; \\ d +_h d &\not\subseteq \{0, b, c, d\}, \{0, a, b, c\}, \{0, a, b\}. \end{aligned}$$